



Letter

More on β -symmetry

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ABSTRACT

Recent work has proposed a method for imposing T-duality on the metric, B -field, and dilaton of the classical effective action of string theory without using Kaluza-Klein reduction. Specifically, the D -dimensional effective action should be invariant under global $O(D, D)$ transformations, provided that the partial derivatives along the β -parameters of the non-geometrical elements of the $O(D, D)$ group become zero. In this paper, we speculate that the global β -symmetry can be utilized to identify both bulk and boundary couplings. We demonstrate that the Gibbons-Hawking term at the leading order of α' is reproduced by this symmetry. Additionally, for closed spacetime manifolds at order α' , we show that the 20 independent geometrical couplings at this order are fixed by this symmetry up to field redefinitions. Specifically, we show that the invariance of the most general covariant and gauge invariant bulk action at order α' under the most general covariant deformed β -transformation at order α' fixes the action up to 13 parameters. These parameters reflect the field redefinitions freedom for the closed spacetime manifolds. For particular values of these parameters, we recover the effective action in the Metsaev-Tseytlin and in the Meissner schemes.

1. Introduction

The classical effective action of string theory and its non-perturbative objects can be determined by imposing gauge symmetries and T-duality. This duality was first observed in the spectrum of string theory when compactified on a circle [1,2]. It has been shown in [3,4] that the Kaluza-Klein reduction of the classical spacetime effective action of string theory on a torus $T^{(d)}$ is invariant under rigid $O(d, d)$ -transformations at all orders of α' . However, the non-perturbative objects D_p -brane/ O_p -plane are not invariant under the rigid $O(d, d)$ -transformations, but they are transformed covariantly under a \mathbb{Z}_2 -subgroup of the $O(d, d)$ -group [5]. By assuming that the effective actions at the critical dimension are background independent, one can consider a particular background that includes one circle. Then, one can use the non-geometrical \mathbb{Z}_2 -subgroup of the rigid $O(1, 1)$ -group to construct the higher-derivative corrections to the bulk and boundary spacetime effective actions of string theory [6–15]. Furthermore, it has been shown in [16–21] that the \mathbb{Z}_2 -constraint can be used to construct the higher-derivative corrections to the bulk and boundary D_p -brane/ O_p -plane actions. The \mathbb{Z}_2 -transformations in the base space are the Buscher rules [22,23], plus their α' -corrections [24] that depend on the scheme of the effective action [6]. The constant parameter of the $O(1, 1)$ -group corresponds to the trivial geometrical re-scaling transformation that receives no α' -corrections.

The rigid $O(d, d)$ -group contains geometrical subgroups consisting of rigid diffeomorphisms with d^2 parameters and shifts on the B -field with $d(d-1)/2$ parameters, as well as the following non-geometrical β -transformations:

$$\begin{aligned}\delta E_{\mu\nu} &= -E_{\mu\rho}\beta^{\rho\sigma}E_{\sigma\nu}, \\ \delta\Phi &= \frac{1}{2}E_{\mu\nu}\beta^{\mu\nu}.\end{aligned}\quad (1)$$

Here, Φ is the dilaton, $E_{\mu\nu} = G_{\mu\nu} + B_{\mu\nu}$, and $\beta^{\mu\nu}$ is a constant antisymmetric bi-vector with $d(d-1)/2$ parameters [25]. A diffeomorphism invariant and B -field gauge invariant effective action at the critical dimension D is naturally invariant under the geometrical subgroups of the $O(D, D)$ -group with non-constant parameters. The invariance of the D -dimensional covariant action under the β -transformations then demands that the effective action is invariant under the rigid $O(D, D)$ -transformations. Recently, in [26], it has been proposed that the universal sector of the effective actions of string theories at any order of α' should be invariant under the β -transformations with the following constraint on the partial derivatives:

$$\beta^{\mu\nu}\partial_\nu(\dots) = 0. \quad (2)$$

The β -transformations (1) should be deformed at higher orders of α' , as in the Buscher rules.

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Using the frame formalism, in which the frames e^a_μ are defined as $e^a_\mu e^b_\nu \eta_{ab} = G_{\mu\nu}$ and are the independent fields, it has been shown in [26] that the following effective action at the leading order of α' is invariant under the β -transformations:

$$\mathbf{S}^0 = -\frac{2}{\kappa^2} \int d^D x \sqrt{-G} e^{-2\Phi} \left(R - 4\nabla_\mu \Phi \nabla^\mu \Phi + 4\nabla_\mu \nabla^\mu \Phi - \frac{1}{12} H^2 \right). \quad (3)$$

Here, H is the field strength of the B -field. The existence of this symmetry at the supergravity level was already noted in [27]. It has been observed in [26] that the β -transformations (1) in combination with the local Lorentz transformations satisfy a closed algebra. It has also been shown that the effective action at order α' in a particular scheme, namely the two-parameter generalized Bergshoeff-de Roo scheme, in which the local Lorentz transformations and the B -field gauge transformations receive a particular α' -correction [28], is invariant under a deformed β -transformations. The corresponding deformations at order α' for the β -transformations (1) have been found. It has also been shown that the deformed local transformations and the deformed β -transformations satisfy a closed algebra [26].

Assuming that the effective actions of string theory at the critical dimension are background-independent [14], we can expect that the global symmetry observed in the effective action of closed spacetime manifolds also applies to open spacetime manifolds. In the latter case, we have both bulk and boundary actions, denoted as S and ∂S respectively. In certain schemes, the sum of these two actions may exhibit invariance under global transformations, represented as:

$$\delta(S + \partial S) = 0. \quad (4)$$

In other schemes, each action may possess separate invariance:

$$\delta S = 0 \quad ; \quad \delta(\partial S) = 0. \quad (5)$$

In the former cases, the bulk action remains invariant under the global symmetry, albeit with residual total derivative terms. Similarly, the boundary action is also invariant, but may contain anomalous terms. However, these anomalous terms can be canceled out by the residual total derivative terms in the bulk through the utilization of Stokes' theorem. In the latter cases, both the bulk and boundary actions lack residual total derivative terms and anomalous terms respectively. In this paper, we are going to study bulk and boundary covariant couplings by imposing invariance under the β -transformations.

To determine the bulk and boundary actions with β -symmetry at any order of α' in an arbitrary covariant scheme, one must consider the most general bulk and boundary covariant and gauge invariant couplings, excluding total derivative terms, with arbitrary coefficients. The next step involves fixing the parameters by ensuring the actions remain invariant under the deformed β -transformations and verifying that the combined deformed β -transformations and standard local transformations satisfy a closed algebra. However, in this paper, our focus is solely on imposing the invariance of effective actions and determining the extent to which this constraint can fix the parameters within the bulk effective action. Such a calculation in bosonic string theory at order α' has been performed in [6] by imposing the invariance of the action under the deformed Buscher rules.

The outline of the paper is as follows: In section 2, we demonstrate that in the presence of a boundary, the combination of the leading-order bulk effective action and the Gibbons-Hawking boundary term satisfies the β -symmetry. In section 3, we investigate the bulk effective action at order α' . We assume no deformation for the local transformations and use the most general covariant deformations at order α' for the β -transformations given in Eq. (1). We show that the invariance of the most general bulk couplings under the β -transformations fixes the couplings up to 13 parameters. The couplings are exactly the same as those found by the deformed Buscher rules. In section 4, we provide a

brief discussion of our results. We have used the “xAct” package [32] for performing our calculations in this paper.

2. The β -symmetry at leading order

In [26], it was shown that the leading-order bulk action given in Eq. (3) is completely invariant under the β -transformation, meaning that there are no residual total derivative terms. However, this action is not the standard spacetime effective action of string theory. For closed spacetime manifolds, where there is no boundary, one can use an integration by parts to convert the effective action to the standard form, which has no Laplacian of the dilaton, as

$$\mathbf{S}^0 = -\frac{2}{\kappa^2} \int d^D x \sqrt{-G} e^{-2\Phi} \left(R + 4\nabla_\mu \Phi \nabla^\mu \Phi - \frac{1}{12} H^2 \right). \quad (6)$$

However, for spacetime manifolds with boundaries, one expects both boundary and bulk couplings. At the leading order of α' , there is the Gibbons-Hawking boundary term for the spacetime action [29]. However, as we will see shortly, this term is not invariant under the β -transformations. Therefore, the β -symmetry does not allow us to add the Gibbons-Hawking term to the bulk action given in Eq. (3). In [8], it was shown that if one adds the Gibbons-Hawking term to the standard form of the bulk action given in Eq. (6), then the resulting action is invariant under the Buscher rules. Therefore, we expect this combination to be invariant under the β -transformations as well.

We consider a spacetime M^D with a boundary ∂M^D . Using the transformations given in Eq. (1) and the constraint given in Eq. (2), one can derive the following transformations for the different terms in Eq. (6):

$$\begin{aligned} \delta(\sqrt{-G} e^{-2\Phi}) &= \sqrt{-G} e^{-2\Phi} \left[-2\delta\Phi + \frac{1}{2} G^{\mu\nu} \delta G_{\nu\mu} \right] = 0, \\ \delta(R) &= -2G^{\gamma\delta} \nabla_\gamma \nabla_\delta (B_{\mu\nu} \beta^{\mu\nu}) \\ &\quad + G^{\alpha\gamma} G^{\beta\delta} \nabla_{[\gamma} (B_{\delta]\mu} \beta_1^\mu) \nabla_{[\alpha} (G_{\beta]\nu} \beta_2^\nu) - (\beta_1 \leftrightarrow \beta_2), \\ \delta(\nabla_\mu \Phi \nabla^\mu \Phi) &= G^{\alpha\beta} \nabla_\alpha (B_{\mu\nu} \beta^{\mu\nu}) \nabla_\beta \Phi, \\ \delta(H^2) &= 12G^{\alpha\gamma} G^{\beta\delta} \nabla_{[\gamma} (B_{\delta]\mu} \beta_1^\mu) \nabla_{[\alpha} (G_{\beta]\nu} \beta_2^\nu) - (\beta_1 \leftrightarrow \beta_2), \end{aligned} \quad (7)$$

where we have also defined the vectors β_1^μ, β_2^μ as $\beta^{\mu\nu} = \frac{1}{2}(\beta_1^\mu \beta_2^\nu - \beta_1^\nu \beta_2^\mu)$. It is worth noting that the field $(B_{\mu\nu} \beta^{\mu\nu})$ is a scalar in spacetime. Similarly, the fields $G_{\mu\alpha} \beta_1^\alpha, G_{\mu\alpha} \beta_2^\alpha, B_{\mu\alpha} \beta_1^\alpha,$ and $B_{\mu\alpha} \beta_2^\alpha$ are vectors in spacetime. Therefore, the right-hand side terms in Eq. (7) are covariant and invariant under the gauge transformations corresponding to these vectors.

Upon substituting the transformations given in Eq. (7) into the action given in Eq. (6), one can observe that the transformation of H^2 is canceled by the last two terms in the transformation of R . Similarly, the transformation of $(\nabla\Phi)^2$ combines with the first term in the transformation of R to produce the following covariant derivative:

$$\delta(\mathbf{S}^0) = \frac{4}{\kappa^2} \int d^D x \sqrt{-G} \nabla^\gamma \left(e^{-2\Phi} H_{\gamma\mu\nu} \beta^{\mu\nu} \right). \quad (8)$$

The presence of the total derivative term given in Eq. (8) indicates that for spacetime manifolds with a boundary, there must be some covariant couplings on the boundary that are not invariant under the β -transformations either. Using Stokes' theorem, one can write the anomalous term given in Eq. (8) as:

$$\delta(\mathbf{S}^0) = \frac{4}{\kappa^2} \int d^{D-1} \sigma \sqrt{|g|} n^\gamma \left[e^{-2\Phi} H_{\gamma\mu\nu} \beta^{\mu\nu} \right]. \quad (9)$$

¹ It is worth noting that the terms on the right-hand side of the second and fourth lines in equation (7) can be expressed in terms of $\beta^{\mu\nu}$. As a result, the combination $(\beta_1^\mu \beta_2^\nu + \beta_1^\nu \beta_2^\mu)$, which encompasses $d(d+1)/2$ parameters, does not appear in the transformations stated in equation (7). Therefore, only $d(d-1)/2$ parameters of β_1^μ and β_2^μ appear in the transformations, precisely the same parameters that $\beta^{\mu\nu}$ possesses.

Here, n_μ is the outward-pointing normal vector to the boundary, which is spacelike (timelike) if the boundary is spacelike (timelike). Inside the square root, g is the determinant of the induced metric on the boundary, which is given by:

$$g_{\bar{\mu}\bar{\nu}} = \frac{\partial x^\mu}{\partial \sigma^{\bar{\mu}}} \frac{\partial x^\nu}{\partial \sigma^{\bar{\nu}}} G_{\mu\nu}. \quad (10)$$

Here, the boundary is specified by the functions $x^\mu = x^\mu(\sigma^{\bar{\mu}})$, where $\sigma^{\bar{\mu}}$ are the coordinates of the boundary ∂M^D .

Let us now consider the following boundary term:

$$\partial \mathbf{S}^0 = -\frac{2a}{\kappa^2} \int d^{D-1} \sigma \sqrt{|g|} e^{-2\Phi} G^{\mu\nu} K_{\mu\nu}. \quad (11)$$

Here, a is a constant parameter, and $K_{\mu\nu}$ is the extrinsic curvature given by $K_{\mu\nu} = \nabla_\mu n_\nu \mp n_\mu n^\rho \nabla_\rho n_\nu$. The minus (plus) sign is used for timelike (spacelike) boundaries. We specify the boundary as $x^\mu(\sigma^{\bar{\mu}}) = (\sigma^{\bar{\mu}}, z_*)$, where z_* is fixed on the boundary. The normal vector becomes along the z -direction, and the induced metric (10) becomes $g_{\bar{\mu}\bar{\nu}} = G_{\bar{\mu}\bar{\nu}}$. The measure in (11) becomes invariant under the β -transformations:

$$\delta(\sqrt{|g|} e^{-2\Phi}) = \sqrt{|g|} e^{-2\Phi} \left[-2\delta\Phi + \frac{1}{2} G^{\bar{\mu}\bar{\nu}} \delta G_{\bar{\mu}\bar{\nu}} \right] = 0. \quad (12)$$

Here, $G^{\bar{\mu}\bar{\nu}}$ is the inverse of the induced metric $G_{\bar{\mu}\bar{\nu}}$, and we have used the β -transformations of metric and dilaton:

$$\delta G_{\bar{\mu}\bar{\nu}} = -G_{\bar{\mu}\bar{\alpha}} \beta^{\bar{\alpha}\bar{\beta}} B_{\bar{\beta}\bar{\nu}} - G_{\bar{\nu}\bar{\alpha}} \beta^{\bar{\alpha}\bar{\beta}} B_{\bar{\beta}\bar{\mu}}, \quad \delta\Phi = \frac{1}{2} B_{\bar{\mu}\bar{\nu}} \beta^{\bar{\mu}\bar{\nu}}. \quad (13)$$

These transformations are the same as the transformation (1), in which the following constraint is also imposed:

$$n_\mu \beta^{\mu\nu} = \beta^{z\nu} = 0. \quad (14)$$

This latter constraint is similar to the \mathbb{Z}_2 -constraint for circular reduction, in which it is assumed that the unit vector n_μ has no component along the killing circle [14]. The transformation of the boundary action (11) under the β -transformations (1) and the constraints (2) and (14) becomes:

$$\delta(\partial \mathbf{S}^0) = -\frac{2a}{\kappa^2} \int d^{D-1} \sigma \sqrt{|g|} e^{-2\Phi} n^\gamma H_{\gamma\mu\nu} \beta^{\mu\nu}. \quad (15)$$

The above transformation cancels the anomalous term in (9) for $a = 2$. Hence, the following bulk and boundary actions:

$$\mathbf{S}^0 + \partial \mathbf{S}^0 = -\frac{2}{\kappa^2} \left[\int d^D x \sqrt{-G} e^{-2\Phi} \left(R + 4\nabla_\mu \Phi \nabla^\mu \Phi - \frac{1}{12} H^2 \right) + 2 \int d^{D-1} \sigma \sqrt{|g|} e^{-2\Phi} K \right], \quad (16)$$

are invariant under the β -transformations, i.e., they satisfy (4). Note that the coefficient of the boundary term is exactly the one that appears in the Gibbons-Hawking boundary term [29].

It is worth noting that we could also include the term $n^\alpha \nabla_\alpha \Phi$ in the boundary coupling (11). Such a term would also be consistent with the β -symmetry; however, it would not be consistent with the least action principle [13] if one considers (6) as the bulk action. If we consider the action (3) as the bulk action, then we can include the term $n^\alpha \nabla_\alpha \Phi$ in the boundary action. In fact, the following combination is invariant under the β -transformation:

$$\partial \mathbf{S}^{(0)} = -\frac{2a_1}{\kappa^2} \int d^{D-1} \sigma e^{-2\Phi} \sqrt{|g|} \left(-\frac{1}{2} K + n^\mu \nabla_\mu \Phi \right). \quad (17)$$

The above boundary couplings are consistent with the least action principle when $a_1 = -4$. The sum of the bulk action (3) and the above boundary action, after applying Stokes' theorem, becomes (16).

In the upcoming section, our attention will be directed towards the bulk couplings at order α' for closed spacetime manifolds. We will proceed under the assumption that these bulk couplings remain invariant under the standard local transformations while permitting covariant

deformations of the β -transformations. By adopting this assumption, we will discover that the invariance of the bulk action under the deformed β -transformations serves to determine the bulk couplings, up to the most general field redefinitions.

3. The β -symmetry at order α'

If the local Lorentz and diffeomorphism and B -field gauge transformations are not deformed at order α' , then the most general bulk couplings involve 41 covariant and gauge-invariant terms. However, some of these terms are related by total derivative terms, while others are related by the Bianchi identities. Removing these redundancies, one can identify 20 independent even-parity couplings [30]. That is

$$\mathbf{S}^1 = -\frac{2\alpha'}{\kappa^2} \int d^D x \sqrt{-G} e^{-2\Phi} \left[a_1 H_\alpha^{\delta\epsilon} H^{\alpha\beta\gamma} H_{\beta\delta}{}^\epsilon H_{\gamma\epsilon\epsilon} + a_2 H_{\alpha\beta}{}^\delta H^{\alpha\beta\gamma} H_\gamma{}^{\epsilon\epsilon} H_{\delta\epsilon\epsilon} + a_3 H_{\alpha\beta\gamma} H^{\alpha\beta\gamma} H_{\delta\epsilon\epsilon} H^{\delta\epsilon\epsilon} + a_4 H_\alpha{}^{\gamma\delta} H_{\beta\gamma\delta} R^{\alpha\beta} + a_5 R_{\alpha\beta} R^{\alpha\beta} + a_8 R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta} + a_9 H_\alpha{}^{\delta\epsilon} H^{\alpha\beta\gamma} R_{\beta\gamma\delta\epsilon} + a_{11} R \nabla_\alpha \nabla^\alpha \Phi + a_{10} H_{\beta\gamma\delta} H^{\beta\gamma\delta} \nabla_\alpha \nabla^\alpha \Phi + a_{13} R \nabla_\alpha \Phi \nabla^\alpha \Phi + a_{12} H_{\beta\gamma\delta} H^{\beta\gamma\delta} \nabla_\alpha \Phi \nabla^\alpha \Phi + a_{14} \nabla_\alpha \Phi \nabla^\alpha \Phi \nabla_\beta \nabla^\beta \Phi + a_{15} H_\alpha{}^{\gamma\delta} H_{\beta\gamma\delta} \nabla^\alpha \Phi \nabla^\beta \Phi + a_{16} R_{\alpha\beta} \nabla^\alpha \Phi \nabla^\beta \Phi + a_{17} \nabla_\alpha \Phi \nabla^\alpha \Phi \nabla_\beta \Phi \nabla^\beta \Phi + a_{18} H_\alpha{}^{\gamma\delta} H_{\beta\gamma\delta} \nabla^\beta \nabla^\alpha \Phi + a_{19} \nabla_\beta \nabla_\alpha \Phi \nabla^\beta \nabla^\alpha \Phi + a_{20} \nabla_\alpha H^{\alpha\beta\gamma} \nabla_\delta H_{\beta\gamma}{}^\delta + a_7 R^2 + a_6 R H_{\alpha\beta\gamma} H^{\alpha\beta\gamma} \right], \quad (18)$$

where a_1, \dots, a_{20} are 20 parameters that the gauge symmetries can not fix them.

In this section, we assume the most general covariant deformation for the β -transformations at order α' . By requiring the bulk action to be invariant under the deformed β -transformations, we can then determine the relations between the parameters in (18). Specifically, we consider:

$$\delta\Phi = \delta\Phi^{(0)} + \alpha' \delta\Phi^{(1)} + \dots, \quad \delta G_{\alpha\beta} = \delta G_{\alpha\beta}^{(0)} + \alpha' \delta G_{\alpha\beta}^{(1)} + \dots, \quad \delta B_{\alpha\beta} = \delta B_{\alpha\beta}^{(0)} + \alpha' \delta B_{\alpha\beta}^{(1)} + \dots, \quad (19)$$

where $\delta\Phi^{(0)}$, $\delta G_{\alpha\beta}^{(0)}$, $\delta B_{\alpha\beta}^{(0)}$ are given in (1) and

$$\begin{aligned} \delta\Phi^{(1)} &= f_3 H_{\alpha\beta\gamma} \beta^{\beta\gamma} \nabla^\alpha \Phi + f_1 \beta^{\alpha\beta} \nabla_\gamma H_{\alpha\beta}{}^\gamma + f_2 H_{\alpha\beta\gamma} \nabla^\gamma \beta^{\alpha\beta}, \\ \delta G_{\alpha\beta}^{(1)} &= g_1 (\beta^{\gamma\delta} \nabla_\alpha H_{\beta\gamma\delta} + \beta^{\gamma\delta} \nabla_\beta H_{\alpha\gamma\delta}) + g_4 (H_{\beta\gamma\delta} \nabla_\alpha \beta^{\gamma\delta} + H_{\alpha\gamma\delta} \nabla_\beta \beta^{\gamma\delta}) \\ &\quad + g_7 (H_{\beta\gamma\delta} \beta^{\gamma\delta} \nabla_\alpha \Phi + H_{\alpha\gamma\delta} \beta^{\gamma\delta} \nabla_\beta \Phi) + g_8 H_{\gamma\delta\epsilon} G_{\alpha\beta} \beta^{\delta\epsilon} \nabla^\gamma \Phi \\ &\quad + g_9 (H_{\beta\gamma\delta} \beta_\alpha{}^\delta \nabla^\gamma \Phi + H_{\alpha\gamma\delta} \beta_\beta{}^\delta \nabla^\gamma \Phi) \\ &\quad + g_2 (\beta_\beta{}^\gamma \nabla_\delta H_{\alpha\gamma}{}^\delta + \beta_\alpha{}^\gamma \nabla_\delta H_{\beta\gamma}{}^\delta) \\ &\quad + g_5 (H_{\beta\gamma\delta} \nabla^\delta \beta_\alpha{}^\gamma + H_{\alpha\gamma\delta} \nabla^\delta \beta_\beta{}^\gamma) + g_3 G_{\alpha\beta} \beta^{\gamma\delta} \nabla_\epsilon H_{\gamma\delta}{}^\epsilon \\ &\quad + g_6 H_{\gamma\delta\epsilon} G_{\alpha\beta} \nabla^\epsilon \beta^{\gamma\delta}, \\ \delta B_{\alpha\beta}^{(1)} &= e_5 H_{\gamma\delta\epsilon} H^{\gamma\delta\epsilon} \beta_{\alpha\beta} + e_1 R \beta_{\alpha\beta} \\ &\quad + e_2 (R_\beta{}^\gamma \beta_{\alpha\gamma} - R_\alpha{}^\gamma \beta_{\beta\gamma}) + e_6 (H_\beta{}^{\delta\epsilon} H_{\gamma\delta\epsilon} \beta_\alpha{}^\gamma - H_\alpha{}^{\delta\epsilon} H_{\gamma\delta\epsilon} \beta_\beta{}^\gamma) \\ &\quad + e_7 H_{\alpha\gamma}{}^\epsilon H_{\beta\delta\epsilon} \beta^{\gamma\delta} + e_8 H_{\alpha\beta}{}^\epsilon H_{\gamma\delta\epsilon} \beta^{\gamma\delta} \\ &\quad + e_3 R_{\alpha\beta\gamma\delta} \beta^{\gamma\delta} + e_4 R_{\alpha\gamma\beta\delta} \beta^{\gamma\delta} \\ &\quad + e_9 (\nabla_\alpha \nabla_\gamma \beta_\beta{}^\gamma - \nabla_\beta \nabla_\gamma \beta_\alpha{}^\gamma) \\ &\quad + e_{12} (\nabla_\beta \Phi \nabla_\gamma \beta_\alpha{}^\gamma - \nabla_\alpha \Phi \nabla_\gamma \beta_\beta{}^\gamma) \end{aligned}$$

$$\begin{aligned}
& +e_{10}(\nabla_\gamma \nabla_\alpha \beta_{\beta'}^\gamma - \nabla_\gamma \nabla_\beta \beta_{\alpha'}^\gamma) + e_{17}(\beta_{\beta'}^\gamma \nabla_\gamma \nabla_\alpha \Phi - \beta_{\alpha'}^\gamma \nabla_\gamma \nabla_\beta \Phi) \\
& +e_{11} \nabla_\gamma \nabla^\gamma \beta_{\alpha\beta} \\
& +e_{18} \beta_{\alpha\beta} \nabla_\gamma \nabla^\gamma \Phi + e_{13} \nabla_\gamma \beta_{\alpha\beta} \nabla^\gamma \Phi \\
& +e_{15} \beta_{\alpha\beta} \nabla_\gamma \Phi \nabla^\gamma \Phi + e_{14} (\nabla_\alpha \beta_{\beta\gamma} \nabla^\gamma \Phi - \nabla_\beta \beta_{\alpha\gamma} \nabla^\gamma \Phi) \\
& +e_{16} (\beta_{\beta\gamma} \nabla_\alpha \Phi \nabla^\gamma \Phi - \beta_{\alpha\gamma} \nabla_\beta \Phi \nabla^\gamma \Phi). \tag{20}
\end{aligned}$$

The parameters f_1, f_2, f_3, g_1 through g_9 , and e_1 through e_{18} are all subject to the β -constraint on the effective action (18). The β -transformations $\delta\Phi^{(0)}$, $\delta G_{\alpha\beta}^{(0)}$, and $\delta B_{\alpha\beta}^{(0)}$ should be imposed on the action at order α' to produce $\delta(\mathbf{S}^1)$. Similarly, $\delta\Phi^{(1)}$, $\delta G_{\alpha\beta}^{(1)}$, and $\delta B_{\alpha\beta}^{(1)}$ should be imposed on the leading-order action (6) to produce $\alpha'\Delta(\mathbf{S}^0)$. The latter produces the following terms at order α' :

$$\begin{aligned}
\alpha'\Delta(\mathbf{S}^0) = & -\frac{2\alpha'}{\kappa^2} \int d^D x \sqrt{-G} e^{-2\Phi} \left[-R^{\alpha\beta} \delta G_{\alpha\beta}^{(1)} \right. \\
& -\frac{1}{24} H_{\beta\gamma\delta} H^{\beta\gamma\delta} \delta G^{(1)\alpha}{}_\alpha + \frac{1}{2} R \delta G^{(1)\alpha}{}_\alpha \\
& +\frac{1}{4} H_\alpha{}^{\gamma\delta} H_{\beta\gamma\delta} \delta G^{(1)\alpha\beta} + \frac{1}{6} H_{\alpha\beta\gamma} H^{\alpha\beta\gamma} \delta\Phi^{(1)} \\
& -2R\delta\Phi^{(1)} + 8\nabla_\alpha \Phi \nabla^\alpha \delta\Phi^{(1)} \\
& +2\delta G^{(1)\beta}{}_\beta \nabla_\alpha \Phi \nabla^\alpha \Phi - 8\delta\Phi^{(1)} \nabla_\alpha \Phi \nabla^\alpha \Phi \\
& +\nabla_\beta \nabla_\alpha \delta G^{(1)\alpha\beta} - \nabla_\beta \nabla^\beta \delta G^{(1)\alpha}{}_\alpha \\
& \left. -4\delta G^{(1)}{}_{\alpha\beta} \nabla^\alpha \Phi \nabla^\beta \Phi - \frac{1}{2} H_{\alpha\beta\gamma} \nabla^\gamma \delta B^{(1)\alpha\beta} \right]. \tag{21}
\end{aligned}$$

Upon substituting the deformations (20) into the equation mentioned above, it becomes evident that certain resulting terms are interconnected through total derivatives, while others may be linked by the Bianchi identities. Furthermore, the deformed β -transformation (19), when combined with the local transformations, must satisfy a closed algebra. As a consequence, the parameters in (20) are not all independent. It is possible to determine the relationships between these parameters by imposing the condition of a closed algebra and eliminating the parameters that are related by total derivatives and the Bianchi identities. In such a scenario, the constraint of the effective action's invariance under the resulting deformed β -transformations could fix both the effective action and the deformed β -transformations, up to field redefinitions. However, in this paper, our focus lies in fixing the parameters within the effective action (18). Therefore, we do not impose the aforementioned constraint on the parameters in (20). In general, without imposing the closed algebra constraint, the parameters of the effective action cannot be fixed up to field redefinitions. However, as we will see for the couplings at order α' , they are indeed fixed up to field redefinitions.

By using the transformations (1) and the constraint (2), one can determine the transformation of each coupling in the action (18). The β -constraint requires that the sum of $\delta(\mathbf{S}^1)$ and $\alpha'\Delta(\mathbf{S}^0)$ must be some total derivative terms. Therefore, we include all possible covariant total derivative terms in our calculations. The most general total derivative terms that are invariant under standard diffeomorphisms and B -field gauge transformations involving the spacetime fields and the parameter β are given by:

$$\mathcal{J}^{(1)} = -\frac{2\alpha'}{\kappa^2} \int d^D x \sqrt{-G} \nabla_\alpha \left[e^{-2\Phi} I^{(1)\alpha} \right], \tag{22}$$

where the vector $I^{(1)\alpha}$ is obtained by contracting $R, H, \nabla\Phi, \beta$, and their covariant derivatives at order α' . It contains 48 terms

$$I^{(1)\alpha} = j_1 H^{\gamma\delta\epsilon} R_{\beta\gamma\delta\epsilon} \beta^{\alpha\beta} + \dots + j_{48} H^\alpha{}_{\gamma\delta} \beta^{\beta\gamma} \nabla^\delta \nabla_\beta \Phi, \tag{23}$$

where the parameters j_1, \dots, j_{48} are subject to the β -constraint on the effective action in (18).

The β -constraint on the bulk action dictates that the following relation must be satisfied:

$$\delta(\mathbf{S}^1) + \alpha'\Delta(\mathbf{S}^0) + \mathcal{J}^{(1)} = 0. \tag{24}$$

To solve this equation, we must impose various Bianchi identities as well as the constraint (2). To impose these constraints, we express the curvatures and covariant derivatives in (24) in terms of partial derivatives of the metric and write the field strength H in terms of the B -field. This automatically satisfies all the Bianchi identities. Then, we can write the equation (24) in terms of independent and non-covariant terms. The coefficients of the independent terms must be zero, which produces some algebraic equations involving all parameters that can be easily solved.

Interestingly, we obtain the following seven relations between the parameters of the effective action (18):

$$\begin{aligned}
a_3 = & a_1/3 - a_{10}/8 - (5a_{12})/48 - (5a_{14})/384 - (5a_{15})/144 \\
& - (25a_{17})/2304 + a_{18}/72 + a_{19}/576 \\
& + a_2/9 - (5a_{20})/36, \quad a_4 = -12a_1 - a_{15}/4 \\
& - a_{16}/16 + a_{19}/16 - 4a_2 - a_{20}, \\
a_6 = & a_{10}/2 - a_{11}/8 + a_{12}/4 - (5a_{13})/48 \\
& + a_{14}/12 - a_{16}/48 + (5a_{17})/96, \quad a_8 = 24a_1, \\
a_7 = & a_{11}/2 + a_{13}/4 - a_{14}/8 - a_{17}/16, \quad a_9 = -12a_1, \\
a_5 = & a_{15} + a_{16}/4 - a_{19}/4 + 4a_{20}. \tag{25}
\end{aligned}$$

These relations are identical to the ones obtained by imposing the Buscher rules [6]. Note that a_1 must be non-zero to be consistent with S -matrix elements. This is because upon substituting the above relations into (18), we find that a_1 is the coefficient of the Riemann squared term, which must be non-zero for consistency. The parameters in the deformations (19) and the total derivative terms in (22) are not entirely determined solely by the parameters $a_1, a_2, a_{10}, \dots, a_{20}$. We have obtained the following deformations for the β -transformations:

$$\begin{aligned}
\delta\Phi^{(1)} = & \frac{1}{8} (192a_1 - 192a_2 + 52a_{11} + 52a_{13} + 12a_{14} + 188a_{15} + 13a_{16} \\
& + 24a_{17} - 24a_{18} - a_{19} + 816a_{20}) H_{\alpha\beta\gamma} \beta^{\beta\gamma} \nabla^\alpha \Phi \\
& + \frac{1}{16} (-576a_1 + 64a_2 - 156a_{11} \\
& - 152a_{13} + 64a_{14} - 204a_{15} - 39a_{16} \\
& + 26a_{17} + 8a_{18} + 3a_{19} - 816a_{20}) \beta^{\alpha\beta} \nabla_\gamma H_{\alpha\beta}{}^\gamma \\
& + \frac{1}{32} (1344a_1 + 960a_2 - 624a_{10} \\
& - 260a_{11} - 600a_{12} - 250a_{13} + 50a_{14} \\
& - 460a_{15} - 65a_{16} + 96a_{18} \\
& + 19a_{19} - 1840a_{20}) H_{\alpha\beta\gamma} \nabla^\gamma \beta^{\alpha\beta} + \dots, \\
\delta G_{\alpha\beta}^{(1)} = & (-24a_1 - 8a_2 - \frac{1}{2}a_{15} - \frac{1}{8}a_{16} - a_{18} \\
& + \frac{1}{8}a_{19} + 2a_{20}) (\beta^{\gamma\delta} \nabla_\alpha H_{\beta\gamma\delta} + \beta^{\gamma\delta} \nabla_\beta H_{\alpha\gamma\delta}) \\
& + (-12a_1 - 4a_2 - \frac{3}{4}a_{15} - \frac{3}{16}a_{16} - a_{18} \\
& + \frac{3}{16}a_{19} + a_{20}) (H_{\beta\gamma\delta} \nabla_\alpha \beta^{\gamma\delta} + H_{\alpha\gamma\delta} \nabla_\beta \beta^{\gamma\delta}) \\
& + (-2a_{15} - 8a_{20}) (H_{\beta\gamma\delta} \beta^{\gamma\delta} \nabla_\alpha \Phi \\
& + H_{\alpha\gamma\delta} \beta^{\gamma\delta} \nabla_\beta \Phi) \\
& + (a_{11} + a_{13} + \frac{1}{4}a_{14} + 4a_{15} + \frac{1}{4}a_{16} \\
& + \frac{1}{2}a_{17} + 16a_{20}) H_{\gamma\delta\epsilon} G_{\alpha\beta} \beta^{\delta\epsilon} \nabla^\gamma \Phi
\end{aligned}$$

$$\begin{aligned}
& +(-96a_1 + 32a_2 + 4a_{18} - 8a_{20})(H_{\beta\gamma\delta}\beta_\alpha^{\delta\nabla\gamma}\Phi + H_{\alpha\gamma\delta}\beta_\beta^{\delta\nabla\gamma}\Phi) \\
& +(-72a_1 + 8a_2 - \frac{1}{2}a_{15} - \frac{1}{8}a_{16} \\
& + a_{18} + \frac{1}{8}a_{19} - 2a_{20})(\beta_\beta^{\gamma\nabla\delta}H_{\alpha\gamma}^{\delta} + \beta_\alpha^{\gamma\nabla\delta}H_{\beta\gamma}^{\delta}) \\
& +(-72a_1 + 8a_2 - \frac{1}{2}a_{15} - \frac{1}{8}a_{16} \\
& + \frac{1}{8}a_{19} - 2a_{20})(H_{\beta\gamma\delta}\nabla^\delta\beta_\alpha^\gamma + H_{\alpha\gamma\delta}\nabla^\delta\beta_\beta^\gamma) \\
& +(-\frac{3}{2}a_{11} - \frac{3}{2}a_{13} + \frac{5}{8}a_{14} - 2a_{15} - \frac{3}{8}a_{16} \\
& + \frac{1}{4}a_{17} - 8a_{20})G_{\alpha\beta}\beta^{\gamma\delta}\nabla_\epsilon H_{\gamma\delta}^\epsilon \\
& + (12a_1 + 4a_2 - 3a_{10} - \frac{5}{4}a_{11} - 3a_{12} - \frac{5}{4}a_{13} + \frac{1}{4}a_{14} \\
& - \frac{9}{4}a_{15} - \frac{5}{16}a_{16} + \frac{1}{2}a_{18} + \frac{1}{16}a_{19} - 9a_{20})H_{\gamma\delta\epsilon}G_{\alpha\beta}\nabla^\epsilon\beta^{\gamma\delta} + \dots, \\
\delta B_{\alpha\beta}^{(1)} = & (-4a_1 - \frac{4}{3}a_2 + a_{10} + \frac{1}{4}a_{11} + a_{12} + \frac{1}{4}a_{13} + \frac{1}{24}a_{14} + \frac{3}{4}a_{15} \\
& + \frac{1}{16}a_{16} + \frac{1}{12}a_{17} - \frac{1}{6}a_{18} - \frac{1}{48}a_{19} + 3a_{20})H_{\gamma\delta\epsilon}H^{\gamma\delta\epsilon}\beta_{\alpha\beta} \\
& + (48a_1 + 16a_2 + a_{15} + \frac{1}{4}a_{16} + 2a_{18} \\
& - \frac{1}{4}a_{19} - 4a_{20})(R_{\beta\gamma}^\gamma\beta_{\alpha\gamma} - R_{\alpha\gamma}^\gamma\beta_{\beta\gamma}) \\
& + (24a_1 - 8a_2 - \frac{1}{2}a_{18} + 2a_{20})(H_{\beta}^{\delta\epsilon}H_{\gamma\delta\epsilon}\beta_\alpha^\gamma - H_\alpha^{\delta\epsilon}H_{\gamma\delta\epsilon}\beta_\beta^\gamma) \\
& + 48a_1H_{\alpha\gamma}^\epsilon H_{\beta\delta\epsilon}\beta^{\gamma\delta} \\
& + 192a_1R_{\alpha\gamma\beta\delta}\beta^{\gamma\delta} \\
& + (-2a_{11} - 2a_{13} + \frac{3}{2}a_{14} - \frac{1}{2}a_{16} + a_{17})\beta_{\alpha\beta}\nabla_\gamma\nabla^\gamma\Phi + \dots, \quad (26)
\end{aligned}$$

where dots represent terms in the deformation in which their coefficients are not in terms of the parameters $a_1, a_2, a_{10}, \dots, a_{20}$. The obtained solution also fixes the total derivative terms (23), which we are not concerned with for closed spacetime manifolds.

Upon substituting the relations (25) into (18), the resulting couplings give the effective action of bosonic string theory if we choose $a_1 = 1/96$. There are 12 arbitrary parameters $a_2, a_{10}, a_{11}, \dots, a_{20}$ that reflect the freedom of the action to use arbitrary covariant and gauge invariant field redefinitions in closed spacetime manifolds. For any choice of these parameters, the effective action appears in a specific covariant scheme. For example, if we set $a_2 = -3a_1$ and all other arbitrary parameters to zero, then we obtain the action in the Metsaev-Tseytlin scheme [30], i.e.,

$$\begin{aligned}
\mathbf{S}^1 = & -\frac{\alpha'}{2\kappa^2} \int d^{26}x e^{-2\Phi} \sqrt{-G} \left[R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta} - \frac{1}{2} H_\alpha^{\delta\epsilon} H^{\alpha\beta\gamma} R_{\beta\gamma\delta\epsilon} \right. \\
& \left. + \frac{1}{24} H_{\epsilon\delta\zeta} H^\epsilon{}_\alpha{}^\beta H^\delta{}_\beta{}^\gamma H^\zeta{}_\gamma{}^\alpha - \frac{1}{8} H_{\alpha\beta}{}^\delta H^{\alpha\beta\gamma} H_{\gamma}{}^{\epsilon\zeta} H_{\delta\epsilon\zeta} \right]. \quad (27)
\end{aligned}$$

The corresponding β -transformations are

$$\begin{aligned}
\delta\Phi^{(1)} = & H_{\alpha\beta\gamma}\beta^{\beta\gamma}\nabla^\alpha\Phi - \frac{1}{2}\beta^{\alpha\beta}\nabla_\gamma H_{\alpha\beta}^\gamma - \frac{1}{2}H_{\alpha\beta\gamma}\nabla^\gamma\beta^{\alpha\beta} + \dots, \\
\delta G_{\alpha\beta}^{(1)} = & -2(H_{\beta\gamma\delta}\beta_\alpha^{\delta\nabla\gamma}\Phi + H_{\alpha\gamma\delta}\beta_\beta^{\delta\nabla\gamma}\Phi) \\
& -(\beta_\beta^{\gamma\nabla\delta}H_{\alpha\gamma}^{\delta} + \beta_\alpha^{\gamma\nabla\delta}H_{\beta\gamma}^{\delta}) \\
& - (H_{\beta\gamma\delta}\nabla^\delta\beta_\alpha^\gamma + H_{\alpha\gamma\delta}\nabla^\delta\beta_\beta^\gamma) + \dots, \\
\delta B_{\alpha\beta}^{(1)} = & \frac{1}{2}(H_\beta^{\delta\epsilon}H_{\gamma\delta\epsilon}\beta_\alpha^\gamma - H_\alpha^{\delta\epsilon}H_{\gamma\delta\epsilon}\beta_\beta^\gamma) \\
& + \frac{1}{2}H_{\alpha\gamma}^\epsilon H_{\beta\delta\epsilon}\beta^{\gamma\delta} + 2R_{\alpha\gamma\beta\delta}\beta^{\gamma\delta} + \dots. \quad (28)
\end{aligned}$$

If we choose the arbitrary parameters as

$$\begin{aligned}
a_{10} = & -16a_1, a_{11} = 0, a_{12} = 16a_1, a_{13} = 192a_1, a_{14} = 384a_1, a_{15} = 0, \\
a_{16} = & -384a_1, a_{17} = -384a_1, a_{18} = 48a_1, a_{19} = 0, a_2 = -3a_1, a_{20} = 0, \quad (29)
\end{aligned}$$

then we obtain the action in the Meissner scheme [31], i.e.,

$$\begin{aligned}
\mathbf{S}^1 = & -\frac{\alpha'}{2\kappa^2} \int d^{26}x \sqrt{-G} e^{-2\Phi} \left[R_{GB}^2 + \frac{1}{24} H_\alpha^{\delta\epsilon} H^{\alpha\beta\gamma} H_{\beta\delta}^\epsilon H_{\gamma\epsilon}{}^\delta \right. \\
& - \frac{1}{8} H_{\alpha\beta}{}^\delta H^{\alpha\beta\gamma} H_{\gamma}{}^{\epsilon\epsilon} H_{\delta\epsilon\epsilon} \\
& + \frac{1}{144} H_{\alpha\beta\gamma} H^{\alpha\beta\gamma} H_{\delta\epsilon\epsilon} H^{\delta\epsilon\epsilon} + H_\alpha^{\gamma\delta} H_{\beta\gamma\delta} R^{\alpha\beta} \\
& - \frac{1}{6} H_{\alpha\beta\gamma} H^{\alpha\beta\gamma} R - \frac{1}{2} H_\alpha^{\delta\epsilon} H^{\alpha\beta\gamma} R_{\beta\gamma\delta\epsilon} \\
& - \frac{2}{3} H_{\beta\gamma\delta} H^{\beta\gamma\delta} \nabla_\alpha \nabla^\alpha \Phi + \frac{2}{3} H_{\beta\gamma\delta} H^{\beta\gamma\delta} \nabla_\alpha \Phi \nabla^\alpha \Phi \\
& + 8R \nabla_\alpha \Phi \nabla^\alpha \Phi + 16 \nabla_\alpha \Phi \nabla^\alpha \Phi \nabla_\beta \nabla^\beta \Phi \\
& - 16 R_{\alpha\beta} \nabla^\alpha \Phi \nabla^\beta \Phi - 16 \nabla_\alpha \Phi \nabla^\alpha \Phi \nabla_\beta \nabla^\beta \Phi \\
& \left. + 2H_\alpha^{\gamma\delta} H_{\beta\gamma\delta} \nabla^\beta \nabla^\alpha \Phi \right], \quad (30)
\end{aligned}$$

where R_{GB}^2 is the Gauss-Bonnet couplings. The corresponding β -transformations are

$$\begin{aligned}
\delta\Phi^{(1)} = & -\frac{1}{8} H_{\alpha\beta\gamma} \nabla^\gamma \beta^{\alpha\beta} + \dots, \\
\delta G_{\alpha\beta}^{(1)} = & \frac{1}{4} (H_{\beta\gamma\delta} \nabla_\alpha \beta^{\gamma\delta} + H_{\alpha\gamma\delta} \nabla_\beta \beta^{\gamma\delta}) \\
& - \frac{1}{2} (H_{\beta\gamma\delta} \nabla^\delta \beta_\alpha^\gamma + H_{\alpha\gamma\delta} \nabla^\delta \beta_\beta^\gamma) + \dots, \\
\delta B_{\alpha\beta}^{(1)} = & \frac{1}{4} (H_\beta^{\delta\epsilon} H_{\gamma\delta\epsilon} \beta_\alpha^\gamma - H_\alpha^{\delta\epsilon} H_{\gamma\delta\epsilon} \beta_\beta^\gamma) \\
& + \frac{1}{2} H_{\alpha\gamma}^\epsilon H_{\beta\delta\epsilon} \beta^{\gamma\delta} + 2R_{\alpha\gamma\beta\delta} \beta^{\gamma\delta} + \dots, \quad (31)
\end{aligned}$$

where dots represent terms in the deformation that have unfixed parameters. These parameters may be fixed by requiring the deformed β -transformations, in combination with the local transformations, to close an algebra. Since all the parameters in the effective actions are already completely fixed, we are not interested in this algebra.

4. Discussion

In this paper, we have demonstrated that the standard covariant bulk and boundary NS-NS couplings at the leading order of α' are invariant under the global β -transformations (1). We have then investigated this symmetry at order α' for covariant couplings. Through explicit calculations at this order, we have shown that the effective actions of bosonic string theory in the Metsaev-Tseytlin and Meissner schemes can be obtained by imposing this symmetry. Furthermore, we have discovered the corresponding deformations of the β -transformations up to some unfixed parameters, which may be determined by requiring that the combinations of the deformed β -transformation and local transformations satisfy a closed algebra [26]. In the covariant schemes used in this paper, the action is invariant under β -transformations, but the Lagrangian is not invariant. Hence, to demonstrate that the transformations satisfy a closed algebra, one would need to use the equations of motion in order to observe the closure of the algebra [34].

We have observed that by imposing the constraint of invariance of the effective action under the deformed β -transformations, all 20 independent couplings at order α' are completely fixed, up to field redefinitions. This is achieved without requiring the deformed β -transformations and local transformations to satisfy a closed algebra. If one removes total derivative terms and uses the most general higher-derivative field redefinitions, there would be only 8 couplings at order α' [30]. Starting with these couplings, the β -symmetry can fix all couplings up to one overall factor. However, this is not the case for higher

orders of α' . For example, at order α'^2 , there are 60 independent couplings up to field redefinitions [33]. We have conducted calculations at order α'^2 and found that the β -constraint on the effective action results in only 28 relations among the 60 independent parameters at this order. The remaining parameters need to be determined by imposing the condition that the deformed β -transformations, in combination with the local transformations, satisfy a closed algebra. These 28 relations are consistent with the 59 relations fixed by the Buscher rules. It would be intriguing to investigate whether imposing the closure algebra and the invariance of the effective action under the β -transformations can completely determine all 60 parameters of the effective action, up to one overall factor.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

Declaration of generative AI and AI-assisted technologies in the writing process

During the preparation of this work, the author utilized Poe to enhance the writing. After utilizing this tool/service, the author reviewed and edited the content as necessary, taking full responsibility for the publication's content.

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