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Hydraulic Conductivity Estimation: Comparison of Empirical Formulas Based on New Laboratory Experiments

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Abstract: Hydraulic conductivity (*K*) is one of the most important characteristics of soils in terms of groundwater movement and the formation of aquifers. Generally, it indicates the ease of infiltration and penetration of water in the soil. It depends on various factors, including fluid viscosity, pore size, grain size, porosity ratio, mineral grain roughness, and soil saturation level. Each of the empirical formulas used to calculate *K* includes one or more of the influencing parameters. In this study, pumping tests from an aquifer were performed by using a hydrology apparatus. Laboratory experiments were conducted on six types of soil with different grain sizes, ranging from fine sand to coarse sand, to obtain *K*. The experimental-based *K* values were compared with that of empirical formulas. The results demonstrate that Breyer and Hazen (modified) formulas adequately fit the laboratory values. The novelty of the present study is the comparison of the experimental formulas in completely similar conditions of the same sample, such as porosity, viscosity, and grain size, using the pumping test in a laboratory method, and the results show that the Hazen and the Breyer formulas provide the best results. The findings of this work will help in better development of groundwater resources and aquifer studies.

Keywords: hydraulics; hydraulic conductivity; empirical formulas; laboratory experiments

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1. Introduction

Hydraulic conductivity is one of the most important parameters of an aquifer and is used, along with storage capacity, in groundwater resource management plans [1]. It indicates the ability of soil and sediments to transmit water through the existing pores in the soil. In permeable soils, the hydraulic conductivity is high. To be more specific, the less permeable a soil is, the smaller its hydraulic conductivity is [2]. By definition, the hydraulic conductivity, denoted by *K*, is the parameter at which the soil can pass a volume unit of water when the hydraulic gradient is equal to one. Therefore, it has the dimension of length divided by time, the same as the dimension of flow velocity [3].

Groundwater flow occurs obeying Darcy's Law, which is currently the most widely used theory of seepage when considering laminar flow [4]. It can properly approximate the empirical relationship between flow velocity and hydraulic gradient [2]. Generally, *K* relies on the three factors of groundwater properties, groundwater relative quantity, and changes in pore structure [3]. Water occupies the voids in the soil or any other porous medium. However, soil porosity, which indicates the volume of empty spaces, does not represent groundwater flow. Therefore, the effective porosity of the soil, which is equal to the ratio of the volume of voids to the total volume of soil, indicates water movement in the soil. It is important in environmental investigations, water sanitation processes by soil, and groundwater pollution caused by sewer pipe defects [5]. The knowledge of soil porosity is essential for a wide range of applications; for example, water resources management, irrigation systems, hydrogeology, and hydrology, as well as environmental protection [6]. Since the shape and size of empty spaces are not easily calculated and, on the other hand, the size of soil particles is easily measurable, the particle size distribution can be used as an approximation for calculating the hydraulic conductivity. In addition to particle size, the viscosity of water is also a parameter that affects hydraulic conductivity.

The pumping test is a common method used to estimate the hydraulic conductivity of aquifers [7]. Consequently, many researchers have tried to correctly calculate K from pumping test experiments [8]. On the other hand, many empirical formulas have been proposed to approximate hydraulic conductivity, such as the Kozeny–Carman (K-C), Breyer, Terzaghi, Slichter, USBR, Alyamani and Sen (A&S), and Hazen formulas. Other methods used to calculate hydraulic conductivity are the flow tracing methods, slug test methods, and pumping test methods. The results obtained from these models have lower errors with increasing scale, and as such the value of hydraulic conductivity will be closer to reality [9].

Since much groundwater research requires the measurement of soil permeability, some researchers are interested in measuring hydraulic conductivity using various methods. In some of these methods, hydraulic conductivity is theoretically measured based on Darcy's formula. In this method, the data obtained from the experiment are placed in a relationship derived from Darcy's formula and, consequently, the hydraulic conductivity is obtained. Many empirical formulas have also been presented based on particle size and soil type [10], as hydraulic conductivity and soil parameters play an important role in the amount of penetration, permeability, and infiltration of soils, influencing environmental factors [11] and the amount of runoff [12].

Schulze-Makuch et al. [13] evaluated hydraulic conductivity using the results of pumping tests by the Theis and Cooper–Jacob methods. They assessed the changes in K with an increase in the range of the aquifer affected by pumping. They stated that the changes are independent of the measuring method of the hydraulic conductivity [13]. Li et al. [14] developed a dimensionless analytical solution for variable rate pumping experiments that includes piecewise constant approximations for variable pumping rates. The analysis of time–drawdown curves was consistent with the Theis curve. Vukovic and Soro [15] summarized several empirical methods from former studies and presented a general formula. Botros et al. [16] compared various methods for measuring soil hydraulic conductivity. The methods included empirical formulas, infiltration tests, and pumping from wells. They reported that infiltration tests are the most robust method for measuring K. They also suggested that empirical formulas can be used if infiltration testing is not possible or there are limitations to conducting such tests [16]. El-Daly et al. [17] conducted research on the empirical formulas of Hazen, Shefferd, and A&S and suggested that among these formulas, Hazen's formula gives more accurate approximations. Odong [18] used the Hazen, K-C, Breyer, Slichter, Terzaghi, USBR, and A&S formulas on four types of soils with different grain sizes, from medium sand to gravel, and calculated their hydraulic conductivity. He also obtained the hydraulic conductivity from the pumping test of a well to compare the results obtained from empirical formulas with actual field values. He recommended the K-C formula and the Breyer formula for gravel [18]. In 2008, Cheong et al. [19] evaluated various methods of using empirical formulas and extracting from aquifers to calculate the hydraulic conductivity. They stated that the calculated hydraulic conductivity of the aquifer obtained by the empirical formulas was 3.3 times higher than the pumping test. They also measured the hydraulic conductivity from a borehole test which was 10 to 100 times less than the results obtained by the empirical formulas and pumping tests [19]. Geostatistical methods are widely used to determine lost or ungauged parameters and can also be used for aquifer investigations in areas with extensive wells to estimate groundwater parameters [20]. Sun et al. [21] used steady-state and transient groundwater flow models to evaluate the hydraulic conductivity estimated by simulating pumping experiments that were not used for parameter estimation. They showed that the inverse modeling of geostatistical pumping tests and highly parameterized hydraulic tomography models could provide robust estimates of K and specific storage. Such estimations are useful for simulating steady-state and transient groundwater flow. They discussed that the inverse modeling approaches provide the best reduction prediction in steady and transient conditions. Based on their study, it appears that inverse modeling and data assimilation are essential steps in predicting accurate groundwater flow behaviors [21]. For unsteady seepage flow, Richards' equation, as a nonlinear partial differential equation, can be used [22]. Koka et al. [6] investigated the values of soil hydraulic conductivity using different steady and unsteady flow methods using cumulative infiltration data of three soils of sandy loam, loam, and clay.

Previous studies have compared laboratory-derived hydraulic conductivity with the permeability of soil by using a permeameter. It is a plastic cylinder in which the sand sample is subjected to upward vertical seepage flow, while the lower part of the permeameter is connected to a water supply and the upper part to an outlet pipe [23]. The purpose of this study is to evaluate the hydraulic conductivity of aquifers using grain size methods. The findings of this study will help in the better development of water resources and their subsequent management.

This study reveals an approximation of an aquifer, measuring K as it is calculated by using Darcy's law and Theim's equation and pumping from a well, adopting a laboratory model named a "hydrology apparatus". The application of grain size formulas depends on the type of soil for which the hydraulic conductivity is estimated. Furthermore, few formulas provide a reliable estimate of the results due to the difficulty of including all possible variables of the porous medium. Therefore, another goal of this paper is to evaluate the applicability and reliability of some common empirical formulas for determining the hydraulic conductivity of soil materials. All the previous laboratory studies have been conducted using Darcy's test with cylindrical soil samples. The novelty of the present study is that it compares the experimental formulas in completely similar conditions of the same sample such as porosity, viscosity, and grain size using the pumping test in a laboratory-based method.

2. Materials and Methods

2.1. Model Structure

The research method involved conducting experiments on a hydrology apparatus with various capabilities for modeling rainfall, generating runoff, examining hydrographs and sediment transport, and extracting water from wells and groundwater flows.

The hydrology apparatus has a length of 2.4 m, 1 m in width, and 1.8 m in height (total height of the device). The soil reservoir in the device has a height of 25 cm and an approximate volume of 0.5 m^3. To avoid width limitations, the width of the apparatus was considered to be more than ten times the well diameter, as demonstrated in related experiments [24].

Considering the shape of the hydraulic model shown in Figure 1, the water reservoir is located at the bottom of the device and can hold 200 L of water. The water inside the reservoir is transferred to the intermediate reservoir, where the soil is placed by a pump. The soil reservoir is limited by two overflow outlets on both sides, which are 20 cm away from the edges of the device, serving as the entry and exit points for water.

Figure 1. Hydrology apparatus.

Figure 2 presents a schematic view of the distances between wells and piezometers, which is used in the Theim equation [25]. The distances between piezometers, which are represented as r in Theim's equation, are also presented in Figure 3. Each number shows the distance between the related piezometers, numbered from 1 to 20, which is needed in the Theim equation. For example, the distance between piezometers 1 and 20 in the physical model is 200 cm. The distances between the piezometer and the wells, for two wells, are denoted as r1 and r2, which will appear later in Equation (2).

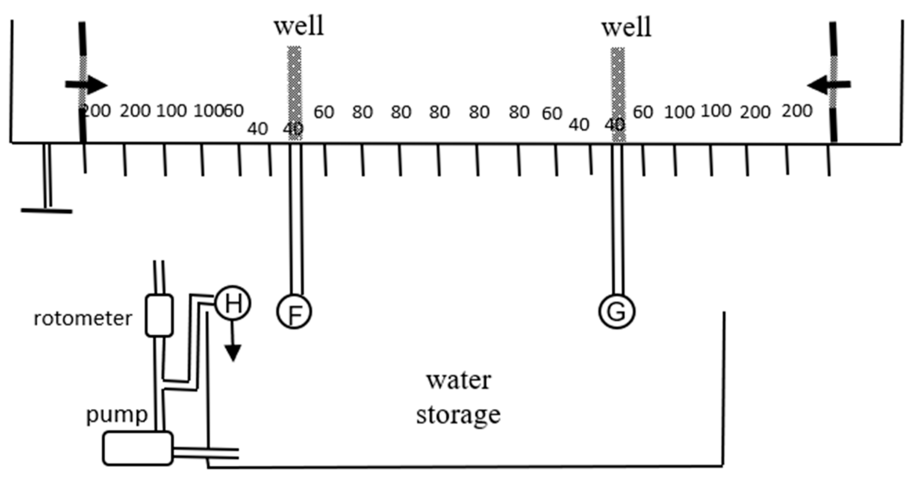


Figure 2. A schematic view of the distances between wells and the piezometer.

	1	2	3	4	5	6	W1	7	8	9	10	11	12	13	14	W2	15	16	17	18	19	20
1	0	20	40	50	60	66	70	74	80	88	96	104	112	120	126	130	134	140	150	160	180	200
2	20	0	20	30	40	46	50	54	60	68	76	84	92	100	106	110	114	120	130	140	160	180
3	40	20	0	10	20	26	30	34	40	48	56	64	72	80	86	90	94	100	110	120	140	160
4	50	30	10	0	10	16	20	24	30	38	46	54	62	70	76	80	84	90	100	110	130	150
5	60	40	20	10	0	6	10	14	20	28	36	44	52	60	66	70	74	80	90	100	120	140
6	66	46	26	16	6	0	4	8	14	22	30	38	46	54	60	64	68	74	84	94	114	134
W1	70	50	30	20	10	4	0	4	10	18	26	34	42	50	56	60	64	70	80	90	110	130
7	74	54	34	24	14	8	4	0	6	14	22	30	38	46	52	56	60	66	76	86	106	126
8	80	60	40	30	20	14	10	6	0	8	16	24	32	40	46	50	54	60	70	80	100	120
9	88	68	48	38	28	22	18	14	8	0	8	16	24	32	38	42	46	52	62	72	92	112
10	96	76	56	46	36	30	26	22	16	8	0	8	16	24	30	34	38	44	54	64	84	104
11	104	84	64	54	44	38	34	30	24	16	8	0	8	16	22	26	30	36	46	56	76	96
12	112	92	72	62	52	46	42	38	32	24	16	8	0	8	14	18	22	28	38	48	68	88
13	120	100	80	70	60	54	50	46	40	32	24	16	8	0	6	10	14	20	30	40	60	80
14	126	106	86	76	66	60	56	52	46	38	30	22	14	6	0	4	8	14	24	34	54	74
W2	130	110	90	80	70	64	60	56	50	42	34	26	18	10	4	0	4	10	20	30	50	70
15	134	114	94	84	74	68	64	60	54	46	38	30	22	14	8	4	0	6	16	26	46	66
16	140	120	100	90	80	74	70	66	60	52	44	36	28	20	14	10	6	0	10	20	40	60
17	150	130	110	100	90	84	80	76	70	62	54	46	38	30	24	20	16	10	0	10	30	50
18	160	140	120	110	100	94	90	86	80	72	64	56	48	40	34	30	26	20	10	0	20	40
19	180	160	140	130	120	114	110	106	100	92	84	76	68	60	54	50	46	40	30	20	0	20
20	200	180	160	150	140	134	130	126	120	112	104	96	88	80	74	70	66	60	50	40	20	0

Figure 3. Heat map of locations of wells and piezometers in the hydrology apparatus.

2.2. Materials

Soil has different types with different characteristics. There are various methods for soil classification. Most simply, soils can be classified based on their solid particles. According to the unified soil classification system (USCS), coarse-grained soils are known as gravel, whereas fine-grained soils are known as sand [26]. According to standards, the types of soils are

(i). Gravel (particles with dimensions of 2 to 63 mm),
(ii). Sand (particles with dimensions of 0.063 to 2 mm),
(iii). Silt (particles with dimensions of 0.002 to 0.063 mm), and
(iv). Clay (particles smaller than 0.002 mm).

Particle size is usually evaluated for the initial diagnosis of soil properties, such as permeability, compressibility, shear strength, etc. Nevertheless, particle size is not an accurate measure to determine soil properties.

The gradation curve is a graphical tool to show the size distribution of particles in soils. The dimensions of a soil's constituent particles are usually drawn in its granulation curve. This graph displays the percentage by weight of particles smaller than a certain size. The slope of the curve represents the uniformity of the soil.

Various quantities are determined using the grading curve to determine the particle size distribution. There are points used to display the qualitative characteristics of grading in the grading curve, such as the following:

(i). d_{10} (the diameter of which the dimensions of 10% of the sieved particles are smaller. As such, 90% of particles are larger than this size. d_{10} is known as "Effective size"),
(ii). d_{20} (the diameter of which 20% of the sieved particles are smaller), and
(iii). d_{50} (the diameter of which 50% of the sieved particles are smaller).

The sharp slope of the granulation curve indicates the presence of uniform particles, and its gentle slope reveals the presence of particles with very different dimensions.

The experiment was applied to a 25 cm-thick aquifer of homogeneous soil with almost uniform particle size distribution in the soil tank. The amount of the d_n parameter can be obtained from the soil gradation curve. The index n is up to 100, and d_n is the sieve size with n percent of the particles passing or being smaller than that size. The K values for soils A, B, C, D, E, and F are 4.02, 5.93, 6.95, 7.73, 8.19, and 8.78 mm/s, respectively. The experiments were done on soils in the range of fine sand to gravel, which resulted in relatively high transmissivity.

2.3. Experimental Processes

In Figure 4, two pipes (P1 and P3) enter the upper water reservoir on the left side of the device, while two pipes are placed in the lower water reservoir on the right side of the device (P2 and P4). The upper pipes (P1 and P2) transfer the water from the water storage to the aquifer, while the lower pipes (P3 and P4) are used as overflow outlets and return the water to the water storage. Therefore, by adjusting the height of all pipes, the water level can be regulated. A well is embedded in the soil reservoir, which allows water to be extracted from the aquifer.

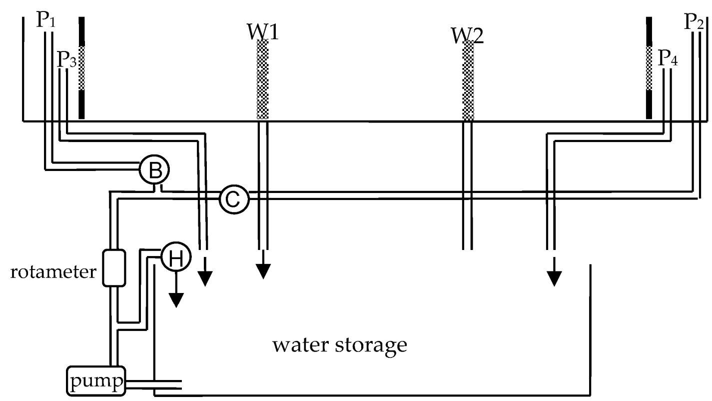


Figure 4. Groundwater flow in the hydrology apparatus.

To perform each experiment, at first, water was directed towards the upper reservoir with the pipe P1 by opening valve B, which created an underground flow moving towards the lower part of the device. By opening the well, the underground water was extracted from the well, causing a decrease in the static water level and the formation of a cone-shaped water level drop. Initially, it was necessary to wait for 40 min to ensure the establishment of a steady flow. This experiment used one well (W_1), while the other well (W_2) was closed during the experiment. Water was extracted by opening a tap under W_1.

The soil was required to be saturated at first through the upper and lower drainage channels. Once the water level in the soil reservoir reached a constant value, the water level in the upper and lower drainage channels was adjusted in a way that the hydraulic gradient in the aquifer was zero, and there was no water flow in the soil. In this case, we can use the Theim equations to examine the effects of the wells. Two wells were embedded in the experimental model which directed the water towards the overflow and then to the water storage reservoir. By measuring the water level in the rectangular overflow, we can calculate the discharge from the well. In this experiment, the discharge from the well was directly measured to achieve more accurate estimations.

2.4. Experimental Formulas

To calculate the hydraulic conductivity with Theim's equation, it was necessary to extract water from the well. In this case, a cone of depression was formed and its dimensions were expanded until the water table was stable and there was no change in water level in piezometers. Then, it could be assumed that it had reached a steady-state condition. The time to reach a steady flow in the desired experiment took approximately 40 min, which was considered in each trial.

Darcy's law describes the flow of a fluid through a porous media. It states that the discharge, Q, is proportional to the gradient in the hydraulic head, the cross-sectional area (A), and K.

$$Q = KA\ (dh/dl), \tag{1}$$

where K is the hydraulic conductivity of the porous media (m/s) and h is water height (L).

By manipulating Theim's equation, the slope of the line in Figure 5 will provide K.

$$Q\ ln(r_2/r_1) = K\pi\left({h_1}^2 - {h_2}^2\right) \tag{2}$$

where h_1 and h_2 are water heights in piezometric wells with the distances of r_1 and r_2 from the pumping well (m).

Theim's equation is a generalization of Darcy's law and is written as follows for the cone of depression under a steady state condition when the water is extracted from a well:

$$Q = K\pi \frac{{h_2}^2 - {h_1}^2}{ln\frac{r_2}{r_1}}, \tag{3}$$

Twenty piezometers were installed in the lower part of the device to observe the water level. Therefore, with the height of water in each piezometer and the distance from the center of the extraction well to the piezometer, the hydraulic conductivity can be calculated from Theim's equation.

The USBR equation shown in Equation (4) provides the hydraulic conductivity in terms of d_{20} [27]. It does not consider soil porosity. According to the studies of Cheng and Chen [28], the USBR formula is suitable for soils with medium grading and a coefficient of uniformity less than 5.

$$K = \frac{g}{v} \times 4.8 \times 10^{-4} {d_{20}}^{0.3} \times {d_{20}}^2 \tag{4}$$

where g is the acceleration of gravity (m/s^2) and ν is the water kinematic viscosity (m^2/s).

Hazen's equation [29] is presented in Equation (5):

$$K = C{d_{10}}^2 \tag{5}$$

where C is Hazen's coefficient.

The modified Hazen's formula [30] is presented in Equation (6):

$$K = \frac{g}{v} \times 6 \times 10^{-4}[1 + 10(n - 0.26)]{d_{10}}^2 \tag{6}$$

where n is the porosity of the soil.

Hazen's equation was introduced for soils with uniform grading, but it also applies to soils with variable grading from fine sand to coarse sand, provided that the coefficient of uniformity of the particles is less than 5 and the effective particle diameter is between 0.1 and 3 mm.

Kozeny [31] proposed a formula then modified by Carman in 1937 [32] and 1956 [33]. Kozeny–Carman's equation is shown in Equation (7):

$$K = \frac{g}{v} \times 8.3 \times 10^{-3} \left[\frac{n^3}{(1 - n)^2}\right] {d_{10}}^2 \tag{7}$$

The K-C equation is widely used. It is not suitable for silty soils and soils with an effective diameter greater than 3 mm.

Breyer's equation [34] is typically used for soils with a coefficient of uniformity between 1 and 20 and an effective particle diameter ranging from 0.06 to 0.6 mm. Breyer's equation is given in Equation (8):

$$K = \frac{g}{v} \times 6 \times 10^{-4}\, log \frac{500}{U} {d_{10}}^2 \tag{8}$$

where U (also known as C_U) is the coefficient of uniformity (d_{60}/d_{10}).

Equation (9) presents Slichter's equation [35], which is used for soils with particle diameters ranging from 0.01 to 5 mm:

$$K = \frac{g}{v} \times 1 \times 10^{-2} n^{3.287} {d_{10}}^2 \tag{9}$$

Terzaghi's equation [36] is shown in Equation (10):

$$K = \frac{g}{v} C_t \left(\frac{n - 0.13}{\sqrt[3]{1-n}}\right)^2 {d_{10}}^2 \tag{10}$$

where C_t is the grading coefficient, which needs to satisfy the condition $6.1 \times 10^{-3} < C_t < 10.7 \times 10^{-3}$.

The best application of Terzaghi's equation is predicted for coarse sand [28].

Equation (11) presents the A&S equation:

$$K = 1300[I_o + 0.025(d_{50} - d_{10})]^2 \tag{11}$$

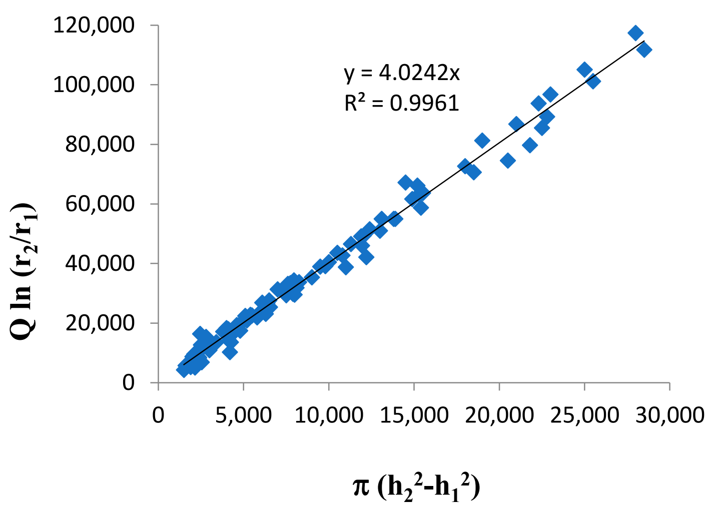
In the A&S equation [37], the hydraulic conductivity is calculated in meters per day. Furthermore, I_o is the Y-intercept of the line passing through the points d_{10}, the effective particle size, and d_{50} is the median particle diameter.

Sauerbrey's formula [38] is presented in Equation (12):

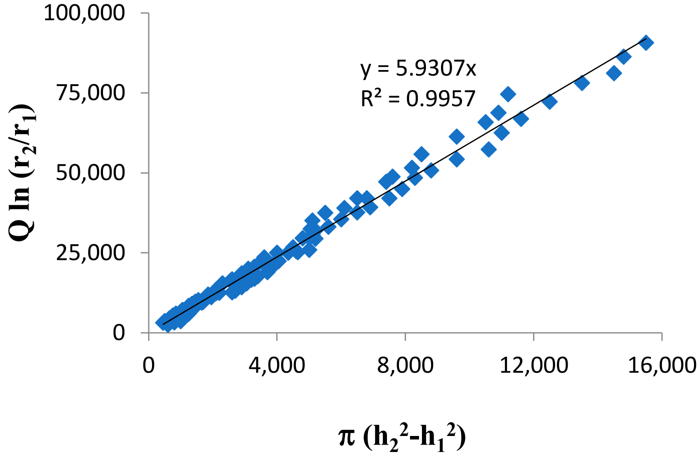
$$K = \frac{g}{v} C_z \frac{n^3}{(1-n)^2} {d_{17}}^2 \tag{12}$$

where C_z is a coefficient equal to 3.75×10^{-3}.

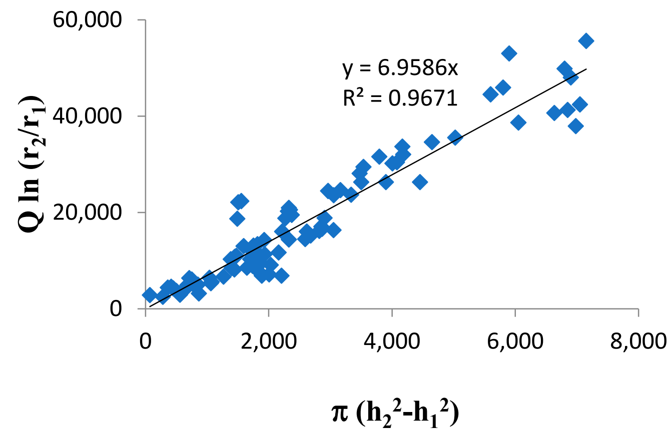
Equation (12) can be used for soils with an effective grain diameter of their porous medium (d_e) up to 5 mm.



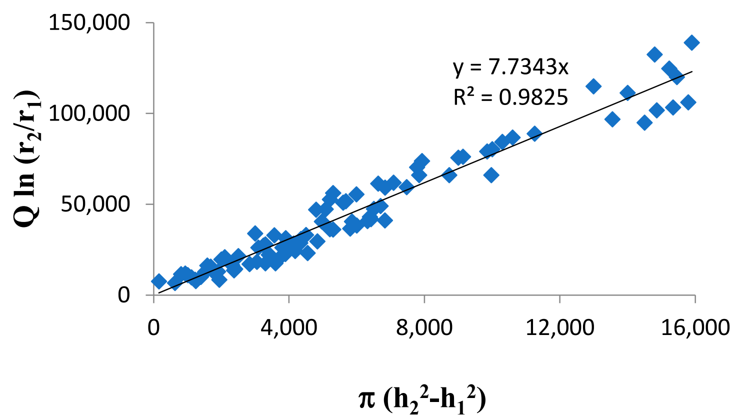
(a)


y = 5.9307x
R² = 0.9957
Q ln (r2/r1)
π (h2²-h1²)
100,000
75,000
50,000
25,000
0
0
4,000
8,000
12,000
16,000

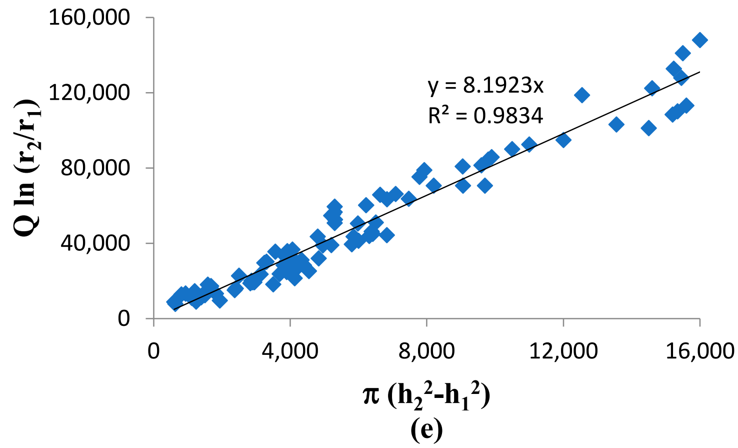
(b)


y = 6.9586x
R² = 0.9671
Q ln (r2/r1)
π (h2²-h1²)
60,000
40,000
20,000
0
0
2,000
4,000
6,000
8,000

(c)


y = 7.7343x
R² = 0.9825
Q ln (r2/r1)
π (h2²-h1²)
150,000
100,000
50,000
0
0
4,000
8,000
12,000
16,000

(d)



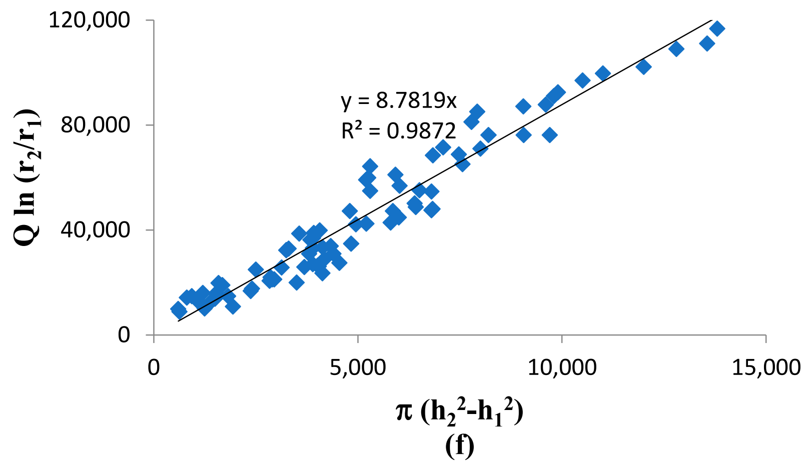


Figure 5. Laboratory-based *K* for different soil types: (**a**) soil A, (**b**) soil B, (**c**) soil C, (**d**) soil D, (**e**) soil E, and (**f**) soil F.

3. Results

The K values were calculated for all six soils. Table 1 shows the hydraulic conductivity of soils A, B, C, D, E, and F. The vertical axis represents $Q.\ln(r_2/r_1)$ of Equation (2) and the horizontal axis represents $\pi\left(h_1{}^2 - h_2{}^2\right)$, so referring to Equation (2), the slope of the line is the hydraulic conductivity. The laboratory calculation of hydraulic conductivity based on Theim's equation is shown in Figure 5. The values of the vertical axis are the amounts of $Q.\ln(r_2/r_1)$ in experimental trials and the values of the horizontal axis show the amount of $\pi\left(h_1{}^2 - h_2{}^2\right)$, where the slope of the trendline gives the hydraulic conductivity values.

Table 1. Laboratory hydraulic conductivity obtained from the slopes of the trendlines in Figure 5.

Soil Type	A	B	C	D	E	F
K (mm/s)	4.02	5.93	6.95	7.73	8.19	8.78

Table 2 shows the hydraulic conductivity obtained from well pumping tests in comparison with those obtained by the empirical formulas for soils A, B, C, D, E, and F. Amounts of the USBR, Hazen, modified Hazen, K-C, Breyer, and Sauerbrey equations for soils A, B, C, D, E, and F were calculated and are compared with experimental values in the table.

Table 2. Comparison of empirical formulas based on laboratory-based hydraulic conductivity.

K (mm/s)	Soil A	Soil B	Soil C	Soil D	Soil E	Soil F	RMSE
USBR	1.53	1.69	1.89	2.15	3.93	4.3	4.46
Hazen	3.6	3.88	4.12	4.62	5.29	8.46	2.26
Modified Hazen	4.19	4.54	5.03	5.44	6.12	9.6	1.62
K-C	2.36	2.61	2.98	3.11	3.68	5.24	3.74
Breyer	4.01	4.53	4.8	5.07	6.71	9.53	1.65
Sauerbrey	1.68	1.77	2.19	2.34	2.89	4.01	1.95
Laboratory data (this study)	4.02	5.93	6.95	7.73	8.19	8.78	-

By calculating hydraulic conductivity values in the tests of pumping from a well in an aquifer and comparing them with those obtained by the empirical formulas, the performances of different equations are assessed. The results demonstrate that both the modified Hazen's equation and Breyer's equation have the best approximation of the hydraulic conductivity.

A Taylor diagram is a mathematical diagram that graphically shows which of the approximate representations of a phenomenon is the most realistic one. It is used to quantify the degree of agreement between estimated and observed behaviors (Figure 6). The correlation of the laboratory data was considered to be the benchmark, while the hydraulic conductivity values obtained by empirical formulas were compared to the laboratory-based hydraulic conductivity values. The Taylor diagram shown in Figure 6 indicates that the formula of Sauerbrey has the highest correlation (i.e., 0.83) with the laboratory data, whereas Hazen's equation has the lowest correlation (i.e., 0.74) with the laboratory data.

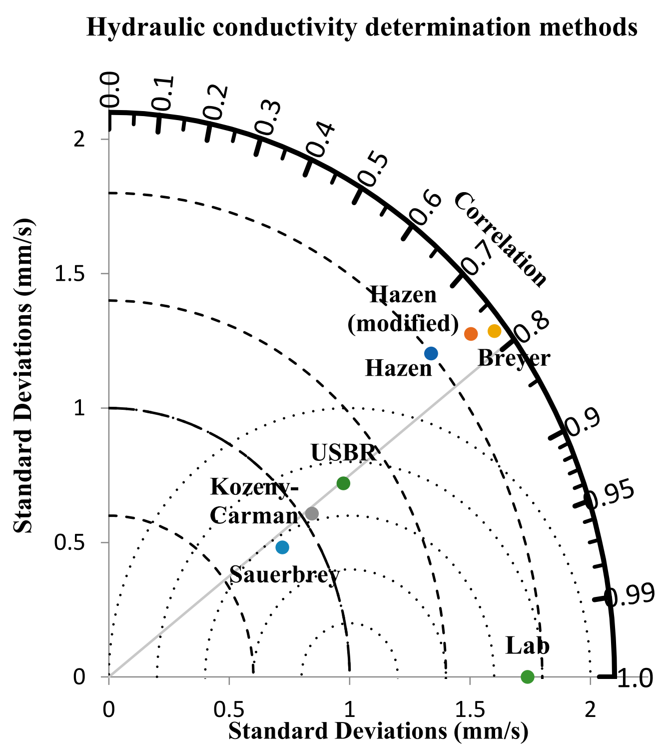


Figure 6. Comparison of different methods for determining hydraulic conductivity based on laboratory data.

The cones of depression of the water table in the experiments for all six types of soils are depicted in Figure 7. The graphs have been extracted by simulating pumping from the well so that the aquifer water storage discharged from the well.

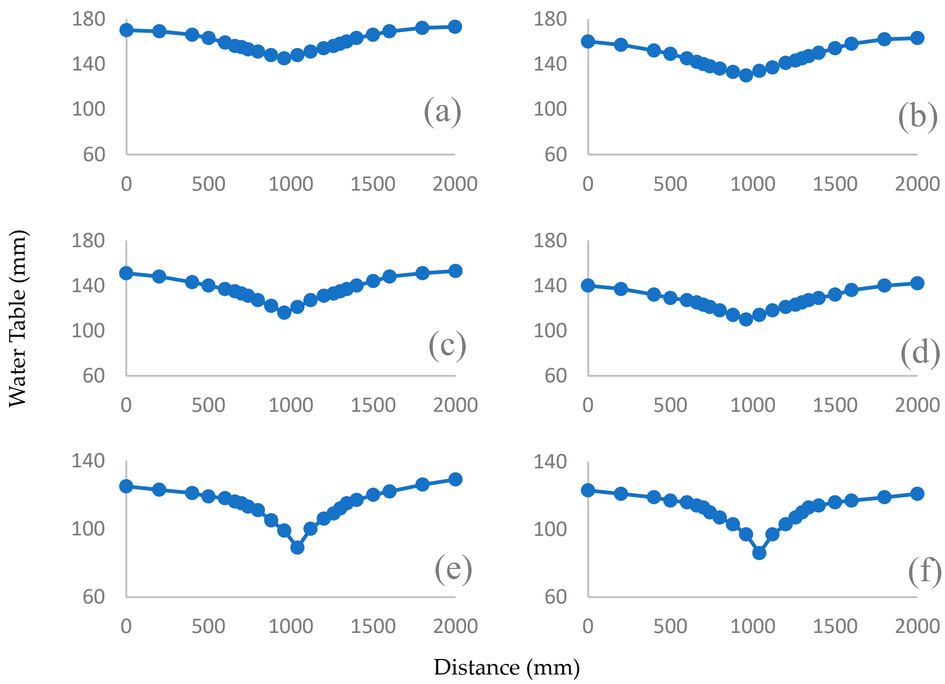


Figure 7. The cones of depression of the water table due to discharge from pumping wells for different soil types: (**a**) soil A, (**b**) soil B, (**c**) soil C, (**d**) soil D, (**e**) soil E, and (**f**) soil F.

Figure 7 demonstrates that for soils with a higher hydraulic conductivity, there are deeper water cones of depression because the soil has considerable transmissivity and there is more flow moving towards the well. On the other hand, the water table in the well drops more and creates a deeper cone of depression. In soils with a lower hydraulic conductivity, there is less flow moving towards the well. Thus, the water table is higher, and a shallow water cone of depression arises.

4. Discussion

This study provides a comprehensive overview of the laboratory calculations and various methods used to calculate an aquifer's hydraulic conductivity. By calculating the hydraulic conductivity in the tests of pumping from a well in the aquifer and comparing it with those obtained by the empirical equations, the performance of each method can be evaluated. Since hydraulic conductivity cannot be easily calculated and the values obtained from various methods are different, the hydraulic conductivity values in the experiments were obtained for six types of soil for comparison purposes (Table 3).

Table 3. Approaches used for comparison from the best to the worst fitting.

	1 (Best)	2	3	4	5	6	7 (Worst)
Cabalar [39]	USBR	Slichter	Chapuis	Terzaghi	A-S	Breyer	Hazen
Ishaku [40]	Terzaghi	K-C	Hazen	Breyer	Slichter	USBR	-
Odong [18]	K-C	Breyer	Slichter	USBR	Terzaghi	A-S	Hazen
Sahu [41]	Hazen	Breyer	Slichter	Terzaghi	K-C	USBR	A-S
Hussain [42]	K-C	Hazen	Breyer	Slichter	Terzaghi	USBR	A-S
Laboratory data (this study)	Hazen (modified)	Breyer	Sauerbrey	Hazen	K-C	USBR	-

As mentioned before, the Hazen formula is suitable for grains smaller than 3 mm, while the USBR formula is adequate for fine and medium sand grains with U < 5. Therefore, for coarse sand where U > 5, neither the USBR and Hazen formulas are applicable. Terzaghi's formula is suitable for coarse-grained sandy soils.

Table (2) demonstrates that the *K* values obtained via USBR and Slichter's equation are lower than those of other methods [43], which is consistent with the results obtained by Vukovic and Soro (1992). These two empirical formulas are usually considered imprecise. On the other hand, Terzaghi's formula yielded lower *K* estimations, which may be due to considering a hypothetical value for C_t ($C_t = 8.4 \times 10^{-3}$).

Breyer's formula is appropriate for homogeneous samples with a uniform granulation. Furthermore, Hazen's formula in some previous studies showed less accuracy than the Kozeny–Carman formula. On the other hand, in the work done by Sahu [39] and this study, Hazen's formula provides more accurate *K* estimations than the Kozeny–Carman formula. Finally, the achieved results in Table 3 can be summarized as follows:

(1) Hazen's formula is suitable for estimating *K* for the soil samples.
(2) Breyer's formula provides acceptable predictions of *K* for soils with $0.06 \leq de \leq 0.6$ mm, and with *CU* ranging from 1 to 20.
(3) Slichter, USBR, and Terzaghi's formulas usually underestimate *K*.
(4) A&S is very sensitive to the shape of the granulation curve and should be used carefully.

This study demonstrates that from the experimental trials, the best overall estimation of hydraulic conductivity was obtained based on Tables 2 and 3. According to Table 2, with the increase of grain size and porosity, the *K* of the borehole soil sample increases. According to Table 3, the hydraulic conductivity calculated by the Hazen (modified) relationship is in closer agreement with the measured values following the Breyer and Sauerbrey relationships. It is noteworthy that Breyer's relation provides a better *K* prediction for sandy loam, whereas other empirical formulas, such as Hazen, Kozeny–Carman, and USBR, clearly underestimate the hydraulic conductivity of soil samples.

5. Conclusions

The present study focuses on the evaluation of seven empirical relationships developed for the estimation of the *K* of borehole soil samples. Estimating hydraulic conductivity using gradation analysis can lead to overestimation or underestimation unless the relevant empirical relationship or the appropriate method is used. Pumping tests from an aquifer were carried out using a hydrological device. Laboratory experiments were conducted on six types of soil with different grain sizes, from fine sand to coarse sand, to obtain *K*. The laboratory-based *K* values were compared with empirical formulas. The results indicate that the formulas of Breyer and Hazen (modified) correspond best to the laboratory values, whereas USBR shows the worst fitting results. In conclusion, the comparative analysis provided in this study provides a new perspective for practical applications of empirical formulas for calculating *K* in various types of sand. These investigations are useful for engineers in the field of groundwater modeling, water resource management, and dewatering to calculate hydraulic conductivity with empirical formulas when digging boreholes is not practically feasible.

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