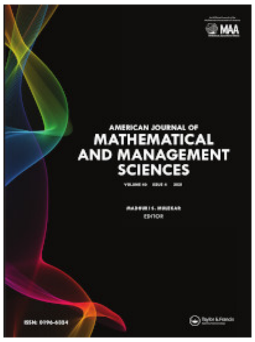

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Erratum to: Reliability of a Coherent System in a Multicomponent Stress-Strength Model

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ABSTRACT

This article considers the stress-strength modeling for calculating of the stress-strength reliability of a coherent system as a function of the stress-strength reliabilities of its components. The system components may experience the same or different stress levels. We have found some mistakes in examples given by Bhattacharya and Roychowdhury (2013) due to misapplication of the system reliability when the system components were subjected to a common stress level. We show that the stress-strength reliability of a system with different stress levels does not include the case when the system components are subjected to a common stress level as a special case and give a correct argument. Their result for the case of different stress levels is correct.

KEYWORDS AND PHRASES

Coherent system; minimal cut sets; minimal path sets; stress-strength reliability

1. Introduction

Stress-strength models are important in reliability literature and engineering applications. A system or unit may be subjected to randomly occurring environmental stress such as pressure, temperature, and humidity and survival of the system depends on its resistance. In the simplest setup of stress-strength models, a unit functions if its strength is greater than the stress imposed on it. The reliability of the unit is then defined as $R = P(X > Y)$, where X and Y represent the random values of strength of the unit and stress placed on the unit, respectively. The estimation of R has been widely studied under various distributional assumptions on X and Y (see e.g., Kotz et al. (2003)). These models have also been studied for systems that consist of more than one component. Most literature on this subject are concerned with case of the system components are subjected to a common stress level. Eryilmaz (2008) considered a multivariate stress-strength model for a coherent system. He assumed that the components are subjected to a common random stress. Eryilmaz (2010) expressed the system stress-strength reliability in terms of those of series systems and presented some approximations for system reliability. He assumed that the stress level imposed on the components is common. Eryilmaz (2013) studied the stress-strength reliability of a system with a time-dependent (dynamic) strength and a static and common random value of the stress.

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Bhattacharya and Roychowdhury (2013) studied the stress-strength reliability of a system with different stress levels and claimed their results include the case when the system components are subjected to a common stress level as a special case. In the following section, we point out that their claim is not correct and give a correct argument.

2. Main Result

Let ϕ be the structure function of a coherent system with n components whose random strengths are $X_1, ..., X_n$ and suppose the components are subjected to the stress levels $Y_1, ..., Y_n$, respectively. The ith component fails if the imposed stress exceeds its strength at any time, i.e., if $Y_i \geq X_i$. Thus $p_i = P(X_i > Y_i)$ gives the stress-strength reliability of the ith component. We define the status of components as follows.

$$Z_i = \begin{cases} 1 & \text{if} \quad X_i > Y_i \\ 0 & \text{if} \quad X_i \leq Y_i \quad i = 1, 2, ..., n \end{cases} \tag{1}$$

where we assume that $Y_1, ..., Y_n$ are independent and Y_i has a continuous distribution function G_i. Also, assume that $X_1, ..., X_n$ are independent random variables and X_i has a continuous distribution function F_i. We also assume that F_i and G_i are independent distributions. Then, the reliability of the coherent system ϕ under the above mentioned stress-strength setup is given by,

$$R_\phi = Pr\{\phi(Z_1, ..., Z_n) = 1\}$$

where $\phi(\mathbf{z})$ indicates the state of the system. Note, that the binary random variables defined by (1) are independent.

In the following, using minimal path(cut) sets of the system, we obtain a general expression for R_ϕ (for details on the coherent structures, minimal path(cut) sets, etc., see e.g., Barlow and Proschan (1975)). Suppose now that the coherent system has p minimal path sets given by $P_1, ..., P_p$ and c minimal cut sets $C_1, ..., C_c$. It is known that,

$$\begin{aligned} \phi(\mathbf{z}) &= \max_{1 \leq i \leq p} \min_{j \in P_i} z_j = \min_{1 \leq i \leq c} \max_{j \in C_i} z_j \\ &= 1 - \prod_{i=1}^{p} \left(1 - \prod_{j \in P_i} z_j\right) = \prod_{i=1}^{c} \left[1 - \prod_{j \in C_i} (1 - z_j).\right] \end{aligned}$$

Lemma 1. *We have*

$$R_\phi = Pr\{\cap_{i=1}^{c} [\cup_{j \in C_i} (X_j > Y_j)]\} = Pr\left\{\cup_{i=1}^{p} [\cap_{j \in P_i} (X_j > Y_j)]\right\} \tag{2}$$

Proof. The first equality was proven by Bhattacharya and Roychowdhury (2013). The second equality can be similarly proven. In the following Remarks, we consider some special cases for Equation (2) and also point out the examples of mistakes given by Bhattacharya and Roychowdhury (2013).

Remark 2. Equation (2) holds true in general even if the independence assumption does not hold. Under independence assumption and according to the form of the minimal cut(path) sets of the system, the first or the second equality in (2) may be easier to use than the other. For example, in a consecutive-k-out-of-n:F system in which the system fails, if at least k out of its n components are consecutively failed, minimal cut sets are $C_i = \{i, i+1, ..., i+k-1\}, i = 1, ..., c = n-k+1$ which are of simple form and easy to use, whereas the minimal path sets of this system do not have such a simple form and determining $p(> c)$ for this system is usually complicated. Hence, the first equality in (2) is easier to use than the second one.

Remark 3. If the minimal cut sets of the system are non-overlapping, the first equality in (2) is then reduced to,

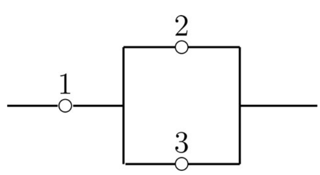
$$R_\phi = \prod_{i=1}^{c} Pr(\cup_{j\in C_i}(X_j > Y_j)).$$

Also, when the minimal path sets are disjointed we then have,

$$R_\phi = 1 - Pr\left\{\cap_{i=1}^{p}\left[\cup_{j\in P_i}(X_j \le Y_j)\right]\right\} = 1 - \prod_{i=1}^{p} Pr(\cup_{j\in P_i}(X_j \le Y_j)).$$

Remark 4. It seems that the situation of common stress level (that is $Y_i = Y$, $i = 1, ..., n$) can be obtained from the case of different stress levels as a particular case, but this is not true in general. Note, that when $Y_i = Y$ the binary random variables, $Z_1, ..., Z_n$ (or equivalently the events $(X_i > Y_i)$) are not independent. For example, in a series system we know that $R_\phi = Pr(minZ_i = 1) = Pr(Z_1 = 1, ..., Z_n = 1) = Pr(X_1 > Y, ..., X_n > Y) = Pr(minX_i > Y) \neq \prod_1^n Pr(X_i > Y)$. Hence, those expressions given by Bhattacharya and Roychowdhury (2013) for the reliability of k-out-of-n systems, a series-parallel system (including Examples 1, 2, and 4), a hi-fi system (Example 5), and a bridge system (Example 6) are not correct. Here, we only give the correct values of those given in Examples 1, 2, and 4.

Example 1. Consider the following series-parallel system.



Its minimal path ses are {1, 2} and {1, 3}. Therefore we have,

$$R = Pr[(X_1 > Y) \cap \{(X_2 > Y) \cup (X_3 > Y)\}]$$

which is equal to

$$Pr(min(X_1, X_2) > Y) + Pr(min(X_1, X_3) > Y) - Pr(min(X_1, X_2, X_3) > Y)$$

Example 2. In Example 1, if X_i is distributed as $\exp(\lambda_i)$, $i = 1, 2, 3$ and Y is distributed as $\exp(\mu)$, and X_i and Y are independent, then,

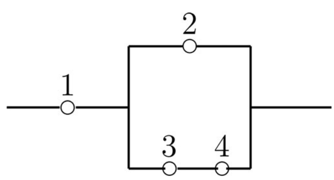
$$P(X_i > Y) = \int_0^{\infty} \left(\int_y^{\infty} \lambda_i e^{-\lambda_i x_i} dx_i \right) \mu e^{-\mu y} dy = \frac{\mu}{\mu + \lambda_i}.$$

Also, when X_i's are independent, it is known that $min\{X_1, ..., X_n\}$ is distributed as $\exp(\sum \lambda_i)$. Hence, the stress-strength reliability of the system is

$$R = \frac{\mu}{\mu + \lambda_1 + \lambda_2} + \frac{\mu}{\mu + \lambda_1 + \lambda_3} - \frac{\mu}{\mu + \lambda_1 + \lambda_2 + \lambda_3}$$

which is quite different with that given in Example 2 of Bhattacharya and Roychowdhury (2013).

Example 3. Now, consider the following series-parallel system in 4 components.



The minimal path sets of this system are {1, 2} and $\{1, 3, 4\}$. Therefore, the stress-strength reliability of the system is,

$$R = P(min\{X_1, X_2\} > Y) + P(min\{X_1, X_3, X_4\} > Y) \\ - P(\min\{X_1, X_2, X_3, X_4\} > Y).$$

Similarly, one can obtain the correct values of the stress-strength system reliability in Examples 5 and 6 of Bhattacharya and Roychowdhury (2013).

References

Barlow, R. E., & Proschan, F. (1975). *Statistical theory of reliability and life testing*. Holt, Rinehart and Winston.

Bhattacharya, D., & Roychowdhury, S. (2013). Reliability of a coherent system in a multicomponent stress-strength model. *American Journal of Mathematical and Management Sciences*, *32*, 40–52.

Eryilmaz, S. (2008). Multivariate stress-strength reliability model and its evaluation for coherent structures. *Journal of Multivariate Analysis*, *99*(9), 1878–1887. https://doi.org/10.1016/j.jmva.2008.01.015

Eryilmaz, S. (2010). On system reliability in stress-strength setup. *Statistics and Probability Letters*, *80*, 834–839.

Eryilmaz, S. (2013). On stress-strength reliability with a time-dependent strength. *Journal of Quality and Reliability Engineering*, *2013*, 1–6. https://doi.org/10.1155/2013/417818

Kotz, S., Lumelskii, Y., & Pensky, M. (2003). *The stress-strength model and its generalizations*. World Scientific.