



Two pseudo-Goldstone bosons in the Gildener–Weinberg model

Parsa Ghorbani^a

Physics Department, Faculty of Science, Ferdowsi University of Mashhad, Mashhad, Iran

Received: 14 March 2024 / Accepted: 12 September 2024
© The Author(s) 2024

Abstract In a dimensionless multi-scalar extension of the Standard Model, Gildener and Weinberg assumed that along a flat direction, there is only one classically massless scalar, known as the *scalon*, which acquires mass through radiative corrections à la Coleman–Weinberg, while all other scalars remain heavy. In this paper, by introducing a toy model with four scalar degrees of freedom we demonstrate the existence of *two* scalons along a specific flat direction that we construct. We present the effective potential for the model and provide the masses of the heavy scalars and two radiatively light pseudo-Goldstone bosons.

1 Introduction

Gildener and Weinberg (GW) [7] came up with a generic dimensionless multi-scalar model as an extension to the Standard Model Higgs potential for which the theory undergoes a dynamical electroweak symmetry breaking through the radiative corrections à la Coleman–Weinberg [2].

In the approach of GW, a quartic multi-scalar potential $V = \lambda_{ijkl}\phi_i\phi_j\phi_k\phi_l$ can be represented in *radial form* as $V = \phi^4\lambda_{ijkl}N_iN_jN_kN_l$, if we replace the scalar fields ϕ_i by $\phi_i = \phi N_i$ with $\phi \in (0, \infty)$ and N_i being respectively the radial field, and a unit vector as a ray in the field space from origin to a point on the surface of the unit sphere. Then at a given mass scale Λ , a flat direction $\mathbf{N} = \mathbf{n}$ for which $\mathbf{n} \cdot \mathbf{n} = 1$, is found such that the potential and its minimum vanish; i.e. $\lambda_{ijkl}n_in_jn_kn_l = 0$ and $\lambda_{ijkl}n_jn_kn_l = 0$. According to Coleman–Weinberg mechanism, loop quantum corrections to the effective potential lead to the generation of non-zero vacuum expectation value (VEV) for the radial field ϕ giving VEV to all scalars through $\langle\phi_i\rangle = \langle\phi\rangle n_i$. Whether or not a scalar field in the model takes a VEV, depends on the choice

of the flat direction. This non-zero $\langle\phi\rangle$ breaks symmetries of the underlying theory.

In their seminal paper [7], Gildener and Weinberg assume, without providing any proof, that at tree level there is only one massless scalar (regardless of the total number of scalars) due to the scale symmetry breaking. They name this scalar the “scalon” which gains a slight mass due to radiative corrections. All other scalars remain massive. Note that any other spontaneously broken symmetry which is space-time independent, would result in additional massless Goldstone bosons. Subsequently, the mass matrix $P_{ij} = 1/2\lambda_{ijkl}n_kn_l$ (see Eq. (3.8) in [7]), will have more than one zero eigenvalues.¹ Here, we are not considering any global symmetry of this type.

In this paper we question the uniqueness of the scalon in scale invariant multi-scalar potentials. Note that applying the Goldstone theorem for spontaneously broken space-time dependent symmetries including the scale invariance, is not straightforward [3]. The counting rule, $n_{GB} = \dim G - \dim H$ with n_{GB} , $\dim G$ and $\dim H$ being respectively, the number of Goldstone bosons, the number of generators before symmetry breaking, and the number of generators of unbroken symmetry, does not always hold [3, 11, 12]. As discussed in [12], the number of Goldstone bosons in a spontaneous symmetry breaking is equal to or greater than the number of generators in broken symmetry. In this work, we are not going to provide a formal proof of the number of Goldstone bosons for spontaneous scale symmetry breaking. Instead, we aim to show that in the absence of any space-time independent global symmetry in the model we will provide in four-dimensional space-time, there exist two Goldstone bosons. We emphasize that these massless modes are not related to any space-time independent global symmetry such as global $U(1)$ symmetry.

¹ An example is the global $U(1)$ symmetry in complex singlet scalar extension of the SM, see e.g. [1].

^ae-mail: parsaghorbani@gmail.com (corresponding author)

We will argue that despite the GW approach, it is not necessary to be restricted to only one radial field or flat direction. In fact, instead of having a single flat direction n_i with $n_i n_i = 1$, we will identify two flat directions, n_j and l_k , such that $n_j n_j + l_k l_k = 1$. Essentially, this implies that instead of replacing ϕ_i by ϕN_i for all scalar fields in the potential, we choose two radial fields, say ρ and η . For a subset of scalars, we have $\phi_j = \rho N_j$, and for the remaining ones, $\phi_k = \eta L_k$, where N_j and L_k represent two arbitrary directions in the field space. Having defined two flat direction in terms of the radial fields ρ and η , the potential takes the general form as follows

$$V = \rho^4 \lambda_{ijkl} N_i N_j N_k N_l + \eta^4 \lambda'_{ijkl} L_i L_j L_k L_l + \rho^2 \eta^2 \lambda''_{ijkl} N_i N_j L_k L_l. \tag{1}$$

Note that the original GW conditions must still be satisfied; that is along both flat directions one must have

$$V_{\text{tr}}(\mathbf{N}, \mathbf{L}) \Big|_{\substack{\mathbf{N}=\mathbf{n} \\ \mathbf{L}=\mathbf{l}}} = 0$$

$$\frac{\partial V_{\text{tr}}(\mathbf{N}, \mathbf{L})}{\partial N_i} \Big|_{\substack{\mathbf{N}=\mathbf{n} \\ \mathbf{L}=\mathbf{l}}} = \frac{\partial V_{\text{tr}}(\mathbf{N}, \mathbf{L})}{\partial L_i} \Big|_{\substack{\mathbf{N}=\mathbf{n} \\ \mathbf{L}=\mathbf{l}}} = 0. \tag{2}$$

where $\mathbf{N} = \mathbf{n}$ and $\mathbf{L} = \mathbf{l}$ indicate two flat directions.

2 Gildener–Weinberg approach

To employ the GW approach with one scalon, let us examine a straightforward \mathbb{Z}_2 symmetric potential featuring two real scalars. The tree-level potential is given by

$$V(h_1, h_2) = \frac{1}{4} \lambda_1 h_1^4 + \frac{1}{2} \lambda_{12} h_1^2 h_2^2 + \frac{1}{4} \lambda_2 h_2^4. \tag{3}$$

This structure could represent a scale-invariant real singlet scalar extension to the Higgs potential

$$V(H_1, h_2) = \lambda_1 \left(H_1^\dagger H_1 \right)^2 + \lambda_{12} \left(H_1^\dagger H_1 \right) h_2^2 + \frac{1}{4} \lambda_2 h_2^4, \tag{4}$$

where $H_1^\dagger = (0, h_1)/\sqrt{2}$ denotes the Higgs doublet in the unitary gauge. However, our intention is not to complicate the model with internal symmetries that are broken after the fields acquire VEV. Instead, we focus solely on the potential presented in Eq. (3) involving two singlet scalars. The flat direction (n_1, n_2) is determined by the conditions

$$\lambda_{12}^2 = \lambda_1 \lambda_2, \quad \frac{\langle h_1 \rangle^2}{\langle h_2 \rangle^2} \equiv \frac{n_1^2}{n_2^2} = -\frac{\lambda_{12}}{\lambda_1}, \quad \lambda_{12} < 0. \tag{5}$$

with $n_1^2 + n_2^2 = 1$. This is equivalent to having

$$\lambda_1 = -\lambda_{12} \frac{\langle h_2 \rangle^2}{\langle h_1 \rangle^2}, \tag{6a}$$

$$\lambda_2 = -\lambda_{12} \frac{\langle h_1 \rangle^2}{\langle h_2 \rangle^2}. \tag{6b}$$

with $\lambda_{12} < 0$. Note that the condition $\lambda_{12} < 0$ is needed, along with $\lambda_1 > 0$ and $\lambda_2 > 0$, for the potential to be bounded from below along the flat direction. The potential in terms of a single radial field is

$$V_{\text{tr}}(\mathbf{N}) = \rho^4 \left(\frac{1}{4} \lambda_1 N_1^4 + \frac{1}{4} \lambda_2 N_2^4 + \frac{1}{2} \lambda_{12} N_1^2 N_2^2 \right). \tag{7}$$

where $\mathbf{N} = (N_1, N_2)$ is an arbitrary direction in the field space. Along the flat direction given by Eq. (5) or Eq. (6), the tree-level potential in Eq. (7) satisfies the conditions

$$V_{\text{tr}}(\mathbf{N}) \Big|_{\mathbf{N}=\mathbf{n}} = \frac{\partial V_{\text{tr}}(\mathbf{N})}{\partial N_i} \Big|_{\mathbf{N}=\mathbf{n}} = 0. \tag{8}$$

The mass matrix from the tree level potential is not diagonal; the scalar masses are the eigenvalues of the mass matrix as

$$m_1^2 = 0, \quad m_2^2 = -2\lambda_{12} \left(\langle h_1 \rangle^2 + \langle h_2 \rangle^2 \right). \tag{9}$$

As anticipated by the GW scheme, there exists one massless scalar or scalon at tree level, while the other state is massive. The corresponding one-loop effective potential, using dimensional regularization in the $\overline{\text{MS}}$ scheme, is given by [1, 6],

$$V_{\text{eff}}(\mathbf{n}) \equiv V_{\text{tr}}(\mathbf{n}) + V_{1\text{-loop}}(\mathbf{n}) = B(\mathbf{n}) \rho^4 \left(\log \frac{\rho^2}{\langle \rho \rangle^2} - \frac{1}{2} \right), \tag{10}$$

with

$$B(\mathbf{n}) = \frac{m_2^4}{64\pi^2 \langle \rho \rangle^4}. \tag{11}$$

This leads to radiative correction to the scalon mass as

$$\delta m_1^2 = \frac{m_2^4}{8\pi^2 \langle \rho \rangle^2}, \tag{12}$$

where m_2^2 is known from Eq. (9).

3 Two-scalon approach

Finding the flat direction with one scalon is simple for a potential with two real scalars, but it becomes increasingly

complicated as the number of scalars grows [8]. A relevant question is whether we can discover two flat directions corresponding to two scalons, giving rise to two pseudo-Goldstone bosons due to quantum corrections. To address this question, we examine a toy model featuring four real singlet scalar degrees of freedom,

$$\begin{aligned}
 V_{\text{tr}} = & \lambda_0 h_1 h_2 h_3 h_4 + \frac{1}{4} \lambda_1 h_1^4 + \frac{1}{4} \lambda_2 h_2^4 + \frac{1}{4} \lambda_3 h_3^4 + \frac{1}{4} \lambda_4 h_4^4 \\
 & + \frac{1}{4} \lambda_{12} h_1^2 h_2^2 + \frac{1}{4} \lambda_{13} h_1^2 h_3^2 + \frac{1}{4} \lambda_{14} h_1^2 h_4^2 \\
 & + \frac{1}{4} \lambda_{23} h_2^2 h_3^2 + \frac{1}{4} \lambda_{24} h_2^2 h_4^2 + \frac{1}{4} \lambda_{34} h_3^2 h_4^2,
 \end{aligned}
 \tag{13}$$

with the discrete $\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$ symmetry under which the scalar fields transform as

$$\begin{aligned}
 \mathbb{Z}_2 : & h_1 \rightarrow -h_1, \quad h_2 \rightarrow h_2, \quad h_3 \rightarrow -h_3, \quad h_4 \rightarrow h_4, \\
 \mathbb{Z}_2 : & h_1 \rightarrow -h_1, \quad h_2 \rightarrow -h_2, \quad h_3 \rightarrow h_3, \quad h_4 \rightarrow h_4, \\
 \mathbb{Z}_2 : & h_1 \rightarrow h_1, \quad h_2 \rightarrow h_2, \quad h_3 \rightarrow -h_3, \quad h_4 \rightarrow -h_4.
 \end{aligned}
 \tag{14}$$

Note the presence of an ad-hoc *quad-linear* term in the potential. The linear and cubic terms, as well as terms of the form $h_i h_j^3$ for any $i, j = 1, 2, 3, 4$ are absent because of the $\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$ symmetry.

Again, it should be noted that our focus here is not on internal symmetries of the underlying theory. However, in order to relate this model to a more phenomenological one beyond the Standard Model framework, consider a potential consisting of two Higgs doublets and two real singlet scalars (For scale-invariant scenarios of 2HDM, refer to [9, 10, 13])

$$\begin{aligned}
 V_{\text{tr}} = & \lambda_0 (H_1^\dagger H_2 + H_2^\dagger H_1) h_3 h_4 \\
 & + \lambda_1 (H_1^\dagger H_1)^2 + \lambda_2 (H_2^\dagger H_2)^2 \\
 & + \lambda'_{12} (H_1^\dagger H_1) (H_2^\dagger H_2) + \lambda''_{12} (H_1^\dagger H_2) (H_2^\dagger H_1) \\
 & + \lambda'''_{12} (H_1^\dagger H_2 + H_2^\dagger H_1)^2 + \frac{1}{2} \lambda_{13} (H_1^\dagger H_1) h_3^2 \\
 & + \frac{1}{2} \lambda_{14} (H_1^\dagger H_1) h_4^2 + \frac{1}{2} \lambda_{23} (H_2^\dagger H_2) h_3^2 \\
 & + \frac{1}{2} \lambda_{24} (H_2^\dagger H_2) h_4^2 + \frac{1}{4} \lambda_{34} h_3^2 h_4^2 + \frac{1}{4} \lambda_3 h_3^4 + \frac{1}{4} \lambda_4 h_4^4,
 \end{aligned}
 \tag{15}$$

where all the couplings are real. In this context, where H_1 and H_2 represent the doublets, and h_3 and h_4 denote the singlets, the discrete symmetry outlined in Eq. (14) manifests as a mechanism to prevent Flavor-Changing Neutral Currents (FCNCs).

If we were to follow the GW prescription in search of a flat direction (n_1, n_2, n_3, n_4) that satisfies the conditions in Eq. (8), the solution would be extremely challenging, if not unattainable. Instead, following the approach outlined above, at a given scale Λ we define two flat directions (n_1, n_2) and (l_1, l_2) that satisfy Eq. (2). The result is the conditions

$$\lambda_{23} = -\lambda_0 \frac{\langle h_1 \rangle \langle h_4 \rangle}{\langle h_2 \rangle \langle h_3 \rangle}
 \tag{16a}$$

$$\lambda_{13} = -\lambda_0 \frac{\langle h_2 \rangle \langle h_4 \rangle}{\langle h_1 \rangle \langle h_3 \rangle}
 \tag{16b}$$

$$\lambda_{14} = -\lambda_0 \frac{\langle h_2 \rangle \langle h_3 \rangle}{\langle h_1 \rangle \langle h_4 \rangle}
 \tag{16c}$$

$$\lambda_{24} = -\lambda_0 \frac{\langle h_1 \rangle \langle h_3 \rangle}{\langle h_2 \rangle \langle h_4 \rangle}
 \tag{16d}$$

$$\lambda_1 = -\frac{\lambda_{12}}{2} \frac{\langle h_2 \rangle^2}{\langle h_1 \rangle^2}
 \tag{16e}$$

$$\lambda_2 = -\frac{\lambda_{12}}{2} \frac{\langle h_1 \rangle^2}{\langle h_2 \rangle^2}
 \tag{16f}$$

$$\lambda_3 = -\frac{\lambda_{34}}{2} \frac{\langle h_4 \rangle^2}{\langle h_3 \rangle^2}
 \tag{16g}$$

$$\lambda_4 = -\frac{\lambda_{34}}{2} \frac{\langle h_3 \rangle^2}{\langle h_4 \rangle^2}
 \tag{16h}$$

Here, $\langle h_i \rangle$ is defined in terms of the radial field $\rho(\eta)$ and the flat directions $\mathbf{n}(\mathbf{l})$ as follows

$$\begin{aligned}
 \langle h_1 \rangle &= \langle \rho \rangle n_1, \\
 \langle h_2 \rangle &= \langle \rho \rangle n_2, \\
 \langle h_3 \rangle &= \langle \eta \rangle l_1, \\
 \langle h_4 \rangle &= \langle \eta \rangle l_2.
 \end{aligned}
 \tag{17}$$

The tree-level potential, expressed in terms of the radial fields along the arbitrary directions \mathbf{N} and \mathbf{L} as explained above, is given by

$$\begin{aligned}
 V_{\text{tr}} = & \rho^4 \left(\frac{1}{4} \lambda_1 N_1^4 + \frac{1}{4} \lambda_2 N_2^4 + \frac{1}{4} \lambda_{12} N_1^2 N_2^2 \right) \\
 & + \eta^4 \left(\frac{1}{4} \lambda_3 L_1^4 + \frac{1}{4} \lambda_4 L_2^4 + \frac{1}{4} \lambda_{34} L_1^2 L_2^2 \right) \\
 & + \rho^2 \eta^2 \left(\lambda_0 N_1 N_2 L_1 L_2 + \frac{1}{4} \lambda_{13} N_1^2 N_1^2 + \frac{1}{4} \lambda_{14} N_1^2 L_2^2 \right. \\
 & \left. + \frac{1}{4} \lambda_{23} N_2^2 L_1^2 + \frac{1}{4} \lambda_{24} N_2^2 L_2^2 \right).
 \end{aligned}
 \tag{18}$$

The solutions for the flat directions \mathbf{n} and \mathbf{l} ² that satisfy Eq. (16) which itself meets the conditions of Eq. (2) for the tree-level potential in Eq. (18), are

$$\begin{aligned} n_1^2 &= \frac{\lambda_{12}}{\lambda_{12} - 2\lambda_1}, \quad n_2^2 = \frac{2\lambda_1}{2\lambda_1 - \lambda_{12}}, \\ l_1^2 &= \frac{\lambda_{34}}{\lambda_{34} - 2\lambda_3}, \quad l_2^2 = \frac{2\lambda_3}{2\lambda_3 - \lambda_{34}}. \end{aligned} \tag{19}$$

Considering the aforementioned flatness conditions, the requirement for the potential to be bounded from below is simplified to $\lambda_0 < 0$, $\lambda_{12} < 0$ and $\lambda_{34} < 0$. Alternatively, the flatness condition can be expressed as

$$\begin{aligned} \lambda_{12}^2 &= 4\lambda_1\lambda_2, \\ \lambda_{34}^2 &= 4\lambda_3\lambda_4, \\ \lambda_{13}\lambda_{24} &= \lambda_{14}\lambda_{23}. \end{aligned} \tag{20}$$

Now, It can be shown that the eigenvalues of the mass matrix are

$$\begin{aligned} m_1^2 &= 0, \\ m_2^2 &= 0, \\ m_3^2 &= -\left(\langle h_1 \rangle^2 + \langle h_2 \rangle^2\right) \left(\frac{\langle h_3 \rangle \langle h_4 \rangle}{\langle h_1 \rangle \langle h_2 \rangle} \lambda_0 + \lambda_{12}\right), \\ m_4^2 &= -\left(\langle h_3 \rangle^2 + \langle h_4 \rangle^2\right) \left(\frac{\langle h_1 \rangle \langle h_2 \rangle}{\langle h_3 \rangle \langle h_4 \rangle} \lambda_0 + \lambda_{34}\right). \end{aligned} \tag{21}$$

As we asserted earlier, it is clear that there exist two classically massless states along the flat directions, while two other scalars are heavy. Radiative corrections should give small mass corrections to both scalons.

4 Effective potential

In general, the radial fields acquire non-zero VEVs, $\langle \rho \rangle$ and $\langle \phi \rangle$, introducing two distinct scales into the theory. The calculation of the effective potential, therefore, differs from the GW approach, which involves only one radial field, and consequently, only one scale. When dealing with the renormalization of a theory with different scales, caution is necessary concerning perturbativity, as these scales may be significantly separated. The intricacies of multi-scale renormalization were elucidated by Einhorn and Jones in [4] (refer also to [5] for multi-scale renormalization involving two fields). To streamline the computations, we assume that the two scales in our model consistently remain within the perturbativity regime. Therefore, we can express the one-loop effective potential in terms of only one scale Λ

² Two flat directions \mathbf{n} and \mathbf{l} may be interpreted as a flat plane.

$$\begin{aligned} V_{\text{eff}}^{1\text{-loop}}(\rho\mathbf{n}, \eta\mathbf{l}) &= A(\mathbf{n})\rho^4 + B(\mathbf{n})\rho^4 \log \frac{\rho^2}{\Lambda^2} \\ &\quad + A'(\mathbf{l})\eta^4 + B'(\mathbf{l})\eta^4 \log \frac{\eta^2}{\Lambda^2}. \end{aligned} \tag{22}$$

where the coefficients are given by

$$\begin{aligned} A(\mathbf{n}) &= \frac{1}{64\pi^2 \langle \rho^4 \rangle} \sum_{i=3,4} m_i^4 \left(-\frac{3}{2} + \log \frac{m_i^2}{\langle \rho \rangle^2}\right) \\ B(\mathbf{n}) &= \frac{1}{64\pi^2 \langle \rho^4 \rangle} \sum_{i=3,4} m_i^4 \\ A'(\mathbf{l}) &= \frac{1}{64\pi^2 \langle \eta^4 \rangle} \sum_{i=3,4} m_i^4 \left(-\frac{3}{2} + \log \frac{m_i^2}{\langle \eta \rangle^2}\right) \\ B'(\mathbf{l}) &= \frac{1}{64\pi^2 \langle \eta^4 \rangle} \sum_{i=3,4} m_i^4 \end{aligned} \tag{23}$$

The one-loop effective potential needs to fulfill the tadpole conditions along two directions,

$$\left. \frac{\partial V_{\text{eff}}^{1\text{-loop}}}{\partial \rho} \right|_{\rho=\langle \rho \rangle} = \left. \frac{\partial V_{\text{eff}}^{1\text{-loop}}}{\partial \eta} \right|_{\eta=\langle \eta \rangle} = 0. \tag{24}$$

These conditions determine the scale Λ as

$$\Lambda = \langle \rho \rangle \exp\left(\frac{A}{2B} + \frac{1}{4}\right) = \langle \eta \rangle \exp\left(\frac{A'}{2B'} + \frac{1}{4}\right), \tag{25}$$

leading to the following effective potential

$$\begin{aligned} V_{\text{eff}}^{1\text{-loop}} &= B(\mathbf{n})\rho^4 \left(\log \frac{\rho^2}{\langle \rho \rangle^2} - \frac{1}{2}\right) \\ &\quad + B'(\mathbf{l})\eta^4 \left(\log \frac{\eta^2}{\langle \eta \rangle^2} - \frac{1}{2}\right). \end{aligned} \tag{26}$$

Following the procedures outlined in [1,6], one can determine the masses of the two scalons resulting from the above effective potential,

$$\delta m_1^2 = \frac{1}{8\pi^2 \langle \rho \rangle^2} \sum_{i=3,4} m_i^4, \quad \delta m_2^2 = \frac{1}{8\pi^2 \langle \eta \rangle^2} \sum_{i=3,4} m_i^4, \tag{27}$$

where m_3^2 and m_4^2 are given by Eq. (21).

5 Conclusion

It has been well-accepted that in scale invariant multi-scalar theories, according to Gildener–Weinberg assumption [7], along a flat direction there exist only one classically massless scalar due to scale symmetry breaking dubbed as scalon, while all other scalars are massive. In this paper, we have proposed a toy model with four real singlet scalars that possesses

two classically massless states. This model can be extracted from more phenomenological models such as two Higgs doublet model in addition to two real singlet scalars, but our starting point is a hypothetical potential with no internal or additional global symmetries. Different from the Gildener–Weinberg scheme, we have constructed two flat directions which leads to two classically massless states or two scalons. Note that none of the massless states are due to space-time independent symmetries like global $U(1)$ symmetry. The key point is that one is not obliged to define only one flat direction as in GW approach. We have also calculated the masses of massive states at tree level, and presented the one-loop effective potential and radiative corrections to the masses of two scalons. Incorporating more than two scalons in a renormalizable theory within the framework of the generic GW potential seems not feasible.

Funding There is no Funding information available for this article.

Data Availability Statement This manuscript has no associated data. [Authors' comment: Data sharing not applicable to this article as no datasets were generated or analysed during the current study.]

Code Availability Statement The manuscript has no associated code/software. [Author's comment: Code/Software sharing not applicable to this article as no code/software was generated or analysed during the current study.]

Open Access This article is licensed under a Creative Commons Attribution 4.0 International License, which permits use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons licence, and indicate if changes were made. The images or other third party material in this article are included in the article's Creative Commons licence, unless indicated otherwise in a credit line to the material. If material is not included in the article's Creative Commons licence and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder. To view a copy of this licence, visit <http://creativecommons.org/licenses/by/4.0/>.
Funded by SCOAP³.

References

1. L. Alexander-Nunneley, A. Pilaftsis, The minimal scale invariant extension of the Standard Model. *JHEP* **09**, 021 (2010)
2. S.R. Coleman, E.J. Weinberg, Radiative corrections as the origin of spontaneous symmetry breaking. *Phys. Rev. D* **7**, 1888–1910 (1973)
3. F. Coradeschi, P. Lodone, D. Pappadopulo, R. Rattazzi, L. Vitale, A naturally light dilaton. *JHEP* **11**, 057 (2013)
4. M.B. Einhorn, D.R.T. Jones, A new renormalization group approach to multiscale problems. *Nucl. Phys. B* **230**, 261–272 (1984)
5. C. Ford, C. Wiesendanger, Multiscale renormalization. *Phys. Lett. B* **398**, 342–346 (1997)
6. K. Ghorbani, H. Ghorbani, Scalar dark matter in scale invariant Standard Model. *JHEP* **04**, 024 (2016)
7. E. Gildener, S. Weinberg, Symmetry breaking and scalar bosons. *Phys. Rev. D* **13**, 3333 (1976)
8. K. Kannike, A. Kubarski, L. Marzola, Geometry of flat directions in scale-invariant potentials. *Phys. Rev. D* **99**(11), 115034 (2019)
9. A. Karam, K. Tamvakis, Dark matter and neutrino masses from a scale-invariant multi-Higgs portal. *Phys. Rev. D* **92**(7), 075010 (2015)
10. J.S. Lee, A. Pilaftsis, Radiative corrections to scalar masses and mixing in a scale invariant two Higgs doublet model. *Phys. Rev. D* **86**, 035004 (2012)
11. I. Low, A.V. Manohar, Spontaneously broken space-time symmetries and Goldstone's theorem. *Phys. Rev. Lett.* **88**, 101602 (2002)
12. H.B. Nielsen, S. Chadha, On how to count goldstone bosons. *Nucl. Phys. B* **105**, 445–453 (1976)
13. A.D. Plascencia, Classical scale invariance in the inert doublet model. *JHEP* **09**, 026 (2015)