A new optimization problem in FSO communication system

Mohammad Ali Amirabadi, and Vahid Tabataba Vakili

***Abstract*—According to the physical phenomena of atmospheric channels and wave propagation, performance of wireless communication system can be optimized by simply adjusting its natural parameters. This way is economically more favorable than consuming more power or using additional processing techniques. In this paper for the first time an optimization problem is developed on the performance of a free-space optical multi-input multi-output (FSO-MIMO) communication system. Also it is the first time that optimization of FSO system is developed under saturated atmospheric turbulence. In order to get closer to the actual results, the effect of pointing error is taken into considerations. Assuming MPSK, DPSK modulation schemes, new closed-form expressions are derived for Bit Error Rate (BER) and outage probability (P_{out}) of the proposed structure. Furthermore, an optimization problem is developed taking into account the equivalent beam waist as variable parameter, and BER as objective function, there is no constraint in this system. Obtained results can be a useful outcome for FSO-MIMO system designers in order to mitigate effects of pointing error as well as atmospheric turbulence and thus achieve optimum performance.**

***Index Terms*—Free Space Optical Communication, Multi-input Multi-output, Saturate Atmospheric Turbulence, Pointing Error.**

I. Introduction

DUE to considerable demand for capacity and data rate in the next generation communication systems, communicating over the optical domain, the so called FSO system, with unlimited, unlicensed spectrum, has been proposed as an alternative for conventional wireless systems [1]. FSO system has large bandwidth. In addition, because of a very narrow equivalent beam waist, FSO is highly secure and contains no interference. Besides these advantages, constraints such as high sensitivity to atmospheric turbulence and trans-receiver misalignment severely limits FSO practical applications, therefore, FSO is not reliable [2].

A solution for this problem is implementing FSO-MIMO structure [3]–[5]. In MIMO, using spatial diversity, different copies of original signal can be obtained at the receiver. These copies are encountered with different fading, and combining them causes better recovery of the original data. There are various methods for combination, in which Maximum Ratio Combiner (MRC), Equal Gain Combiner (EGC) and Selection Combiner (SC) are some ways of better extracting original data [6].

Even at clear weather, FSO system is uncounted with atmospheric turbulence. This effect is like fading in RF system and causes random fluctuations in signal intensity [7]. Following statistical distributions have been developed to investigate this effect: Exponential-Weibull [8], Generalized Malaga [9], Lognormal [10], Gamma-Gamma [11], and Negative Exponential [12]. Among them Negative Exponential has high accompany with experimental results for saturated atmospheric turbulence.

trans-receiver misalignment can be caused by winds, thermal expansions, and earthquakes. Under the influence this effect high-rise buildings sway in three directions of along wind, across wind, and torsional. trans-receiver misalignment is a random process, and affects system performance by means of the pointing error [13].

In FSO communication systems, often Intensity Modulation / Direct Detection (IM / DD) based on on-off keying (OOK). OOK is simple, and its detection threshold is adopted based on atmospheric turbulence intensity, this makes it suitable for areas with varying turbulence intensity. Pulse Position Modulation (PPM) is another modulation used in FSO system, which does not need adaptive detection threshold. Subcarrier Intensity Modulation (SIM) does not require adaptive detection threshold and compared with PPM has higher spectral efficiency [14].

Several investigations have been developed on optimization of FSO system. A minimization model for transmitter power and optimization model for divergence angle in a given BER are developed in [13]. However, it has not provided closed-form expressions. Two optimization models for FSO systems are presented in [15], and wavelength is taken as varying parameter. A FSO system in atmospheric turbulence and pointing error is considered in [16], beam width, pointing error variance, and detector size are taken into account; lognormal and gamma-gamma atmospheric turbulences are considered. The BER expression for an intensity-modulation/direct detection (IM/DD) FSO system in strong atmospheric turbulence and pointing error is derived in [17]. [18] assumed IM/DD in the general model of misalignment given in [16]. It did not consider any atmospheric turbulence effects.

In this paper a FSO-MIMO communication system is investigated under the effect of saturated atmospheric turbulence with pointing error. Presented works on FSO optimization, have considered single input single out put structure, to the best of the authors knowledge, it is the first time an optimization model is developed over FSO-MIMO structure. FSO optimization at weak to strong atmospheric turbulences regimes have been investigated in literatures; it is the first time that an FSO optimization problem is developed at saturated regime. Assuming MPSK, DPSK modulation schemes, new closed-form expressions are derived for BER and P_{out} of the proposed structure. Furthermore, an optimization is developed taking into account the equivalent beam waist as the variable

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parameter, and BER as the objective function, there is no constraint in this system.

II. SYSTEM MODEL

Consider a FSO system with N transmit and M receive apertures. Assume x as transmitted signal from all transmit apertures; it is affected by atmospheric turbulence and receiver input noise. Received signal at $i-th, i=1,..,M$ receive aperture, is in the following form:

$$y_i = \eta \sum_{j=1}^{N} I'_{i,j} x + e_i \tag{1}$$

where e_i is additive white Gaussian noise with zero mean and σ^2 variance, $I'_{i,j}$ is atmospheric turbulence intensity between $j-th, j=1,..,N$ transmit and $i-th$ receive aperture. η is the optical to electrical conversion efficiency; it is assumed $\eta=1$, and $E[|x|^2]=E_x$, where $E[\cdot]$ stands for the expectation. The EGC is used to combine received electrical signals as [19]:

$$y = \sum_{i=1}^{M} y_i = \sum_{i=1}^{M}\sum_{j=1}^{N} I'_{i,j} x + \sum_{i=1}^{M} e_i. \tag{2}$$

It is assumed that pointing error and atmospheric turbulence affect the transmitted signal; therefore, received power is multiplication of transmitter power (P_T), transmitter and receiver telescope gains (G_T, G_R), and losses and is given as:

$$P_R = (P_T h \sum_{i=1}^{M}\sum_{j=1}^{N} I'_{i,j}) \eta_T \eta_R (\frac{\lambda}{4\pi d^2})^2 G_T G_R L_A L_T, \tag{3}$$

where h is random variable indicating pointing error, η_T is transmitter optical efficiency and η_R is receiver optical efficiency, λ is the wavelength, d is transmitter to receiver distance, L_A is the atmospheric loss, and L_T is the transmitter pointing loss factor. The term in parentheses is the free-space loss [13]. In this paper gains and optical efficiency are assumed to have unit value, and losses are omitted; therefore only the terms in the first parenthesis remain, i.e.

$$P_R \approx P_T h \sum_{i=1}^{M}\sum_{j=1}^{N} I'_{i,j} = P_T h I'. \tag{4}$$

At the detector, P_R is converted to electric current I_R. The relation between them can be expressed as:

$$I_R = \rho P_R + \rho P_b + I_d + n, \tag{5}$$

where P_b is the received background radiation, I_d is the dark current in photo-diode, n is the receiver noise, and ρ is detector responsibility [15]. The effect due to P_b and I_d can usually be compensated with a proper set-up, thus I_R becomes as follows:

$$I_R \approx \rho P_R + n = \rho P_T h I' + n = \rho P_T I + n. \tag{6}$$

Assuming a Gaussian spatial intensity profile of equivalent beam waist on the receiver plane at distance z from the transmitter and a circular aperture of radius r, the probability density function (pdf) of h is given by:

$$f_h(h) = \frac{\xi^2}{A_0^{\xi^2}} h^{\xi^2-1}; 0 \le \xi \le A_0, \tag{7}$$

where $\xi = w_{z_{eq}}/2\sigma_s$ is the ratio between the equivalent beam waist at the receiver and the pointing error displacement standard deviation at the receiver,$w_{z_{eq}}^2 = w_z^2\sqrt{\pi}erf(\nu)/(2\nu e^{-\nu^2}), \nu = \sqrt{\pi}r/(\sqrt{2}w_z), A_0 = [erf(\nu)]^2$, and $erf(\cdot)$ is the error function, and w_z is beam width [18]. In this paper it is assumed that $\sigma_s = 1/2$, which results in $\xi = w_{z_{eq}}$. Therefore finding optimum ξ is equivalent to find optimum $w_{z_{eq}}$. For simplicity and without loss of generality, Simulation results and analytic expressions are derived in terms of ξ. Considering unit variance Negative Exponential atmospheric turbulence, the pdf of $I_{i,j}$ can be written as follows [20]:

$$f_{I'_{i,j}}(I') = e^{-I'}. \tag{8}$$

Moment Generation Function (MGF) of $I'_{i,j}$, becomes as:

$$M_{I'_{i,j}}(s) = \frac{1}{s+1}. \tag{9}$$

Considering independent identically distributed FSO path, the MGF of $I' = \sum_{i=1}^{M}\sum_{j=1}^{N} I'_{i,j}$ becomes as follows:

$$M_{I'}(s) = (\frac{1}{s+1})^{MN}. \tag{10}$$

Therefore, the pdf of I' becomes as follows:

$$f_{I'}(I') = \frac{I'^{MN-1}}{\Gamma(MN)} e^{-I'}. \tag{11}$$

According that $I = hI'$, the pdf of I becomes equal to:

$$\begin{aligned} f_I(I) &= \int_0^\infty f_{I'}(I') f_h(\frac{I}{I'}) dI' \\ &= \int_0^\infty \frac{\xi^2}{A_0^{\xi^2}\Gamma(MN)} (\frac{I}{I'})^{\xi^2-1} I'^{MN-1} e^{-I'} dI'. \end{aligned} \tag{12}$$

Using [21, Eq.06.05.02.0001.01], the pdf and Cumulative Distribution Function (CDF) of I become equal to:

$$f_I(I) = \frac{\xi^2\Gamma(MN-\xi^2+1)}{A_0^{\xi^2}\Gamma(MN)} I^{\xi^2-1}. \tag{13}$$

$$F_I(I) = \frac{\Gamma(MN-\xi^2+1)}{A_0^{\xi^2}\Gamma(MN)} I^{\xi^2}. \tag{14}$$

III. OUTAGE PROBABILITY

Since OOK modulation is used, x is either 0 or $2P_T$ where P_T is the average transmitted optical power. Received electrical SNR and average electrical SNR, can be defined as [18], [19]:

$$\gamma = \frac{2P_T^2\rho^2 I^2}{\sigma_n^2}, \gamma_{avg} = \frac{2P_T^2\rho^2}{\sigma_n^2}. \tag{15}$$

P_{out} denotes the probability that received electrical SNR falls below a threshold, and can be calculated as follows [18]:

$$Pr(\gamma \leq \gamma_{th}) = F_I(\sqrt{\frac{\gamma_{th}}{\gamma_{avg}}}) = \frac{\Gamma(MN-\xi^2+1)}{A_0^{\xi^2}\Gamma(MN)}(\frac{\gamma_{th}}{\mu})^{\xi^2/2}. \tag{16}$$

According to (16) and values that ξ can take, P_{out} is not so much dependent on number of trans-receiver apertures; in fact the main parameters that could affect and be used for optimization are equivalent beam width and aperture radius that are related to A_0. It has worth to mention that adjusting aperture radius is not as easy as adjusting equivalent beam width.

IV. Bit Error Rate

Assuming $P_b(e|I)$ as the BER conditioned on I, the average BER can be derived from following equation:

$$P_b(e) = \int_0^\infty f_I(I)P_b(e|I)dI, \tag{17}$$

For MPSK, the conditioned BER is as follows [19]:

$$P_b(e|I) = \frac{\zeta_M}{2}\sum_{p=1}^{\tau_M} erfc(\frac{a_p P_T \rho I}{\sigma_n}), \tag{18}$$

where $erfc(\cdot)$ is complimentary error function, $\zeta_M = 2/max(log_2(M),2), a_p = \sqrt{2}sin((2p-1)\pi/M)$, and $\tau_M = max(M/4,1)$ are the MPSK modulation dependent parameters of an constellation containing M-points. Substituting (13) and (18) into (17), and using [21, Eq.06.27.21.0132] the average BER becomes as follows:

$$P_b(e) = \sum_{p=1}^{\tau_M}\frac{\zeta_M}{2}\frac{\xi^2\Gamma(MN-\xi^2+1)}{A_0^{\xi^2}\Gamma(MN)}\int_0^\infty erfc(\frac{a_p P_T\rho I}{\sigma_n}) I^{\xi^2-1}dI = \sum_{p=1}^{\tau_M}\frac{\zeta_M}{2\sqrt{\pi}}\frac{\Gamma(MN-\xi^2+1)\Gamma(0.5(\xi^2+1))}{\Gamma(MN)} (\frac{2}{A_0^2 a_p^2\gamma_{avg}})^{\xi^2/2}. \tag{19}$$

Using [21, Eq.06.05.20.0001.01], BER differentiate will be:

$$\frac{dP_b(e)}{d\xi} = -2\psi(MN-\xi^2+1)+\psi(\frac{\xi^2+1}{2})-ln(\frac{A_0^2 a_p^2\gamma_{avg}}{2}). \tag{20}$$

For DPSK, the instantaneous BER is as follows [14]:

$$P_b(e|I) = \frac{1}{2}e^{-\frac{2P_T^2\rho^2 I^2}{\sigma_n^2}}. \tag{21}$$

Substituting (13) and (21) into (17), and using [21, Eq.06.27.21.0132] the average BER will be:

$$P_b(e) = \frac{\xi^2\Gamma(MN-\xi^2+1)}{A_0^{\xi^2}2\Gamma(MN)}(\int_0^\infty e^{-2(\frac{P_T\rho I}{\sigma_n})^2} I^{\xi^2-1}dI) = \frac{\xi^2\Gamma(MN-\xi^2+1)\Gamma(\xi^2/2)}{2\sqrt{2}\Gamma(MN)}(\frac{1}{A_0^2\gamma_{avg}})^{\xi^2/4}. \tag{22}$$

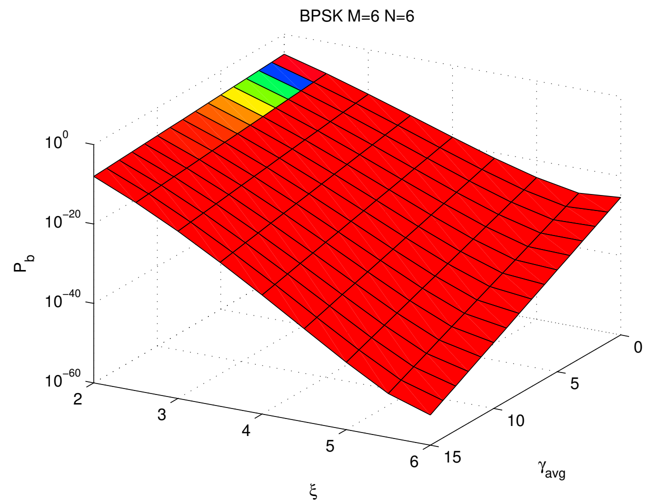

Fig. 1. BER of proposed structure in terms of γ_{avg} and ξ, for BPSK modulation, when number of trans-receiver apertures is $M = N = 6$;

Using [21, Eq.06.05.20.0001.01], differentiate of BER becomes equal to:

$$\frac{dP_b(e)}{d\xi} = \xi^2(\frac{1}{2}\psi(\frac{\xi^2}{2})-\psi(MN-\xi^2+1)) -\frac{1}{2}ln(\frac{\sqrt{2}P_T\rho A_0}{\sigma_n})+1. \tag{23}$$

The same insights of (16) can be confirmed for (19) and (22). These expressions show that average BER is more dependent on changes of ξ as well as A_0 rather than MN, thus it is expected that adjusting equivalent beam width, which is related to both ξ and A_0 affects system performance more than aperture radius.

V. Results and Discussions

In this section results of the optimization problems are discussed. Obtained BER for DPSK and MPSK, considering equivalent beam width as variable parameter can be minimized by finding the root of $dP_b(e)/d\xi = 0$, respectively in (20), and (23). MATLAB $solve(.)$ command can easily solve them.

In Fig. 1, BER of the proposed FSO-MIMO structure is plotted in terms of average SNR and ξ, for BPSK modulation, when number of transmitter and receiver aperture is $M = N = 6$. As can be seen, BER reduces while increasing equivalent beam width (ξ), this reduction continues till reaching a specific ξ, e.g. at $\gamma_{avg} = 0dB$, this occurs about $\xi = 5.5$. This specific ξ changes at different γ_{avg}. However, it increases while increasing γ_{avg}. Performance of FSO system can be optimized without additional processing, and computation with adjusting system parameters.

In Fig. 2, P_{out} of the proposed FSO-MIMO structure is plotted as a function of normalized SNR and ξ, when number of trans-receiver apertures is $M = N = 6$. It can be seen that reduction in BER is smother than P_{out} while increasing ξ.

In Fig. 3, BER of proposed structure in terms of γ_{avg}, for different number of trans-receiver apertures for DBPSK, and BPSK modulations. As can be seen, performance of BPSK

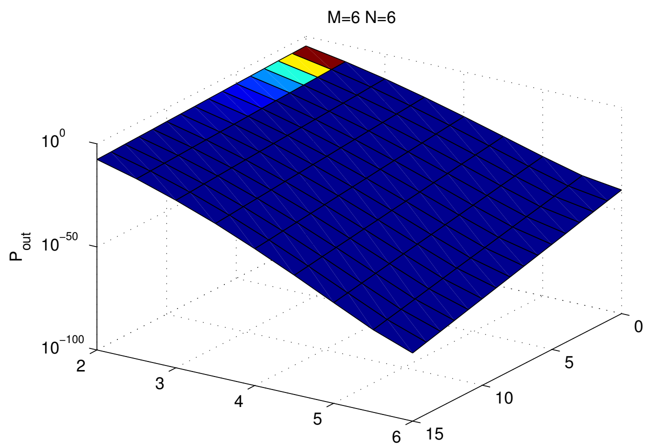

Fig. 2. P_{out} of proposed structure in terms of normalized SNR and ξ, when number of trans-receiver apertures is $M = N = 6$;

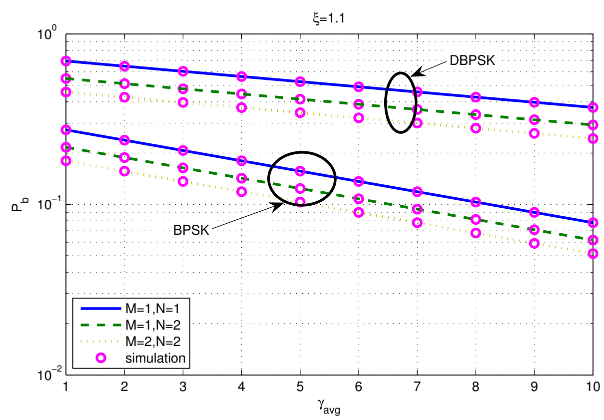

Fig. 3. BER of proposed structure in terms of γ_{avg}, for different number of trans-receiver apertures for DBPSK, and BPSK modulations;

is better than DBPSK, but differential modulations such as DBPSK, are less sensitive to noise and interference and do not require complex processing .

VI. Conclusion

In this paper a FSO-MIMO communication system is considered under the effects of pointing error and saturated atmospheric turbulence. Assuming MPSK, DPSK modulations, new closed-form expressions are derived for BER and outage probability. Furthermore, in order to mitigate effects of pointing error and saturated atmospheric turbulence, an optimization is developed considering BER as objective function and equivalent beam width as variable parameter, there is no constraint assumed.

Results indicate that BER reduces while increasing equivalent beam width, this reduction continues till reaching a specific equivalent beam width, which is different at various average SNRs. Obtained results can be useful outcome for FSO-MIMO system designers in order to achieve the optimum performance by adjusting natural system parameters, without additional processing complexity and latency. This way is more economically favorable than consuming power or using processing techniques.

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