

# A Comparative Study on Seismic Behavior of Two Adaptive Sliding Isolators

Ali Akbar Shaikhzadeh<sup>1\*</sup>, Abbas Karamoddin<sup>2</sup>

1. PhD Candidate, Ferdowsi University, Mashhad, Islamic Republic of Iran,  
*ali.shaikhzadeh@mail.um.ac.ir*
2. Assistant Professor, Ferdowsi University, Mashhad, Islamic Republic of Iran,  
*akaramodin@yahoo.com*

## Abstract

The seismic response of conventional friction pendulum isolators (FPS) can lead to resonance phenomenon due to their constant isolation frequency. Recent studies have explored the incorporation of passive adaptability to mitigate this issue. Two types of adaptive sliding isolators, Variable Curvature Friction Pendulum System (VCFPS) and Variable Friction Pendulum System (VFPS) from the variable-curvature and variable-friction isolator groups with passive adaptability, respectively, were numerically simulated and compared in this study. The study utilized a velocity-dependent coefficient of friction and modified viscoplasticity model to simulate the non-linear friction force of the isolators in a 2-DOF idealized shear building. The results show that VCFPS effectively controls transmitted acceleration to the superstructure and mitigates resonance in frequency response. On the other hand, VFPS is successful in reducing base displacement but does not perform well in controlling super-structural acceleration and mitigating resonance. Overall, VCFPS demonstrates better performance than VFPS.

**Keywords:** variable curvature; variable frequency; variable friction; friction pendulum system;

## 1- Introduction

The Friction Pendulum System (FPS) isolator has proven effective across various structures and excitation characteristics through extensive analytical and experimental studies (Almazán et al., 1998; Mokha et al., 1991; Tsai, 1997). The sliding surface of a FPS isolator is made spherical, so that the gravitational load of the structure applied on the slider will provide a restoring stiffness that help reduce residual isolator displacement. However, this restoring stiffness, which is proportional to the curvature of the sliding surface, will inevitably introduce a constant isolation frequency to the isolated structure. Due to the existence of this isolation frequency, a resonant problem may occur when FPS is subjected to strong long-period components of an earthquake, such as near-fault ground motions (Lu et al., 2004; Lu et al., 2006). This limitation led researchers to incorporate passive adaptability into FPS. Researchers have recently introduced three types of fully passive-adaptive FPS: (1) sliding

isolators with multiple sliding surfaces (SIMSS), (2) sliding isolators with variable friction (SIVF), and (3) sliding isolators with variable curvature (SIVC). Each type is briefly reviewed, below (Lu et al., 2011).

SIMSS isolators have multiple spherical sliding surfaces arranged in different ways to accommodate larger displacement in a smaller size (Fenz, 2008; Fenz & Constantinou, 2008). SIVF isolators have a constant radius sliding surface with a friction coefficient that varies with displacement (Panchal & Jangid, 2008; Ray et al., 2013). SIVC isolators have a sliding surface with variable curvature, allowing for adaptive isolation stiffness (Lu et al., 2011).

The present study numerically simulates and compares the effectiveness of two types of sliding isolators with passive adaptability: (1) Variable Curvature Friction Pendulum System (VCFPS) and (2) Variable Friction Pendulum System (VFPS), which are candidates of SIVC and SIVF groups, respectively. Two aspects of seismic responses, namely, base displacement and super-structural acceleration are considered for comparison purposes. Since the behavior of sliding isolators is highly nonlinear, researchers have proposed different friction models to simulate this nonlinearity in numerical simulation (Jangid, 2005). We have used the modified viscoplasticity friction model which is a continuous model of the frictional force and is based on the Wen equation (Constantinou et al., 1990).

## 2- Description of Isolators Used in Simulation

### 2-1-Variable Curvature Friction Pendulum System (VCFPS)

The mechanical behavior of VCFPS developed by Tsai et al. (Tsai et al., 2003) is very similar to that of the conventional FPS. The difference between the VCFPS and FPS is that the radius of curvature of VCFPS can be lengthened with an increase of the isolator displacement. The following function is used to describe VCFPS sliding surface:

$$y = R - \sqrt{R^2 - x^2} - f(x) \quad (1)$$

Where  $R$  is the radius of curvature at the center of the sliding surface;  $x$  is the horizontal displacement of the isolator;  $f(x)$  is the function to describe the increase of the radius of curvature with an increase of the horizontal displacement. The function  $f(x)$  can be further expressed as

$$f(x) = E \operatorname{sgn}(x)x^3 \quad (2)$$

In which  $E$  is the parameter that describes the variation of curvature of the concave surface. If the restoring force can bring the slider back to the initial position within the radial sliding displacement  $x_0$ , then the parameter  $E$  can be determined as follows:

$$E = \left( \frac{\frac{Wx_0}{\sqrt{R^2 - x_0^2}} - \frac{\mu W}{\cos\theta_0}}{3W \operatorname{sgn}(x_0)x_0^2} \right) \quad (3)$$

Where  $W$  is the weight of the isolator from the structure;  $\theta_0$  is the corresponding angle of the slider displacement  $x_0$  from where it can bring back slider to initial position; and  $\mu$  is the coefficient of friction of the sliding surface.

Having the geometric equation of sliding surface on hand, one can use the following equations (Lu et al., 2011) to establish the force-displacement (hysteresis) diagram of VCFPS isolator.

$$F(x) = W y'(x) + \mu W Z \left( \frac{1 + y'(x)^2}{1 - \mu Z y'(x)} \right) \quad (4)$$

Where  $\mu$  and  $Z$  are defined by Equations (6) and (5), respectively.

$$\dot{Z} = \frac{1}{u_y} \{A - |Z|^\eta [\gamma \operatorname{sgn}(\dot{x}Z) + \beta]\} \dot{x} \quad (5)$$

$$\mu(\dot{x}) = f_{max} - (f_{max} - f_{min})e^{-a|\dot{x}|} \quad (6)$$

In which  $\dot{x}$  stands for the velocity;  $Z$  is a hysteretic dimensionless quantity; and  $\beta$ ,  $\gamma$ ,  $A$ , and  $\eta$  are dimensionless constants. Furthermore,  $u_y$  represents a displacement quantity. Constantinou and Adnane (1987) have shown that when  $A = 1$  and  $\beta + \gamma = 1$ , the model of Equation (5) collapses to a model of viscoplasticity that was proposed by Ozdemir (1976). In this case,  $u_y$  represents the yield displacement while  $\eta$  controls the mode of transition into the inelastic range. (Constantinou et al., 1990). In addition, in Equation (6)  $f_{max}$  is the coefficient of friction at large velocity of sliding,  $f_{min}$  the coefficient of sliding at very low velocity of sliding, and  $a$  is a constant for given bearing pressure and condition of interface.

The first term in Equation (4), which is independent of the coefficient of friction, represents the restoring force and provides the re-centering ability for the isolator. In addition, this term inevitably introduces and isolation frequency. The second term represents the friction force.

One can call Equation (4) as the exact equation for force-displacement relationship of a SIVC. However, if we furthermore assume that  $y'(x) \ll 1$  and  $\mu \ll 1$ , after neglecting the higher order terms, the approximate equation of force-displacement relationship can be expressed as following

$$F(x) = W y'(x) + \mu W Z \quad (7)$$

### 2-2-Variable Friction Pendulum System (VFPS)

The VFPS is very similar to a conventional FPS in regards of details and operation. The difference

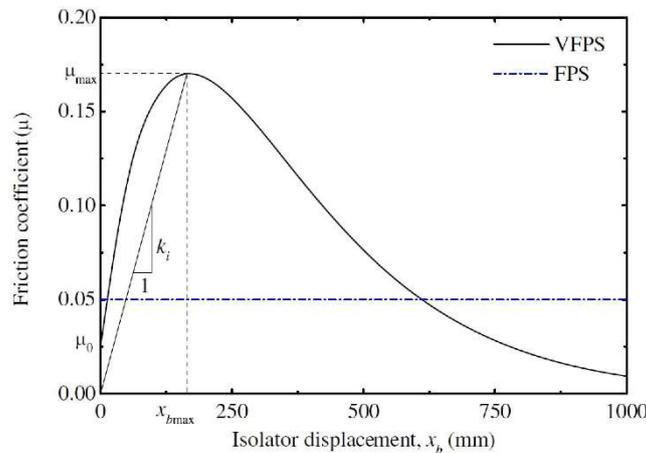


Figure 1: Comparison between friction coefficient of FPS and VFPS

between FPS and VFPS is that the friction coefficient of FPS is considered to be constant whereas the friction coefficient of VFPS is varied in the form of a curve shown in Figure 1 (Panchal & Jangid, 2008).

Such variation of friction coefficient in VFPS can be achieved by gradually varying the roughness of spherical surface. The curve is selected to exhibit increasing frictional force up to a certain displacement, followed by a decrease. This design provides initial softness for small inputs, stiffness for moderate inputs, and softness again for large inputs. The criterion for curve selection ensures significant reduction in isolator displacement and base shear during near-fault ground motions without significantly altering superstructure acceleration.

The equation adopted to define the curve for friction coefficient,  $\mu$ , of the VFPS is as follows (Panchal & Jangid, 2008):

$$\mu = (\mu_0 + a_1|x|)e^{-a_2|x|} \quad (8)$$

Where  $\mu_0$  is the initial value of friction coefficient;  $a_1$  and  $a_2$  are the parameters that describe the variation of friction coefficient along the sliding surface of VFPS; and  $x$  is the isolator displacement. To find the above parameters, one can approximate the curve by drawing a straight line from the origin up to the peak value of the friction coefficient which is generally kept in the range of 0.15–0.2. The slope of the line gives initial stiffness of the VFPS which controls the initial time period of the VFPS.

The initial stiffness,  $k_i$ , and initial time period,  $T_i$ , of the VFPS are given by

$$k_i = \frac{\mu_{max}W}{x_{max}} \quad (9)$$

$$T_i = 2\pi \sqrt{\frac{M}{k_i}} \quad (10)$$

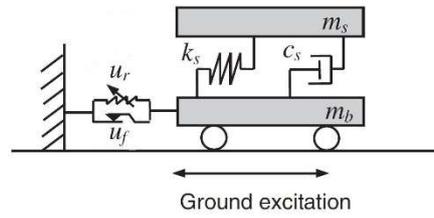
Where  $\mu_{max}$  is the peak friction coefficient of the VFPS;  $x_{max}$  is the isolator displacement corresponding to peak friction coefficient of VFPS;  $W$  is as defined before;  $M$  is the total mass of the base-isolated building. The value of  $x_{max}$  is found out by maximizing the friction coefficient of VFPS and it is expressed by

$$x_{max} = \frac{a_1 - \mu_0 a_2}{a_1 a_2} \quad (11)$$

Knowing the initial value of friction coefficient (i.e.  $\mu_0$  is assumed to be 0.025) and selecting initial time period and peak friction coefficient, the parameters  $a_1$  and  $a_2$  can be evaluated by solving Eqs. (8)-(11).

### 3- Numerical Simulation

To compare the seismic behavior of the two isolators under study, a SDOF structure atop an isolation system is investigated. The SDOF superstructure chosen to be isolated has the same mass, stiffness, and damping properties as the superstructure in the M. Pranesh and R. Sinha research (Pranesh & Sinha, 2000). The mass of the structure and base are taken equal, so that the mass ratio is 0.5.



**Figure 2: The mathematical model for simulating an isolated structure**

Stiffness of the structure is taken such that the time period of fixed-base structure is 0.5 s, while its damping ratio is taken as 2 percent of critical value. The coefficient of friction used for VCFPS and FPS is 0.05 at high speed and half of that at slow speed. Rate parameter is chosen to be  $a = 100 \text{ sec/m}$ . Also, the parameters of the plasticity model assigned are  $u_y = 0.10 \text{ mm}$ ,  $A = 1$ ,  $\eta = 2$ ,  $\beta = 0.1$ , and  $\gamma = 0.9$ .

A set of seven near-fault earthquake excitations, recommended for evaluation of smart base isolated building (Narasimhan et al., 2006) are considered for evaluation purposes.

The idealized structure is simulated using a general mathematical model (Figure 2), depicting a 2-DOF shear building with an isolation system comprising a nonlinear friction element and a variable spring element. The model assumes synchronized seismic motions for all isolators, a consistent representation validated by studies (Lu et al., 2011).

The dynamic equation of motion of the idealized model in Figure 2 can be expressed in state-space form (Lu & Yang, 1997)

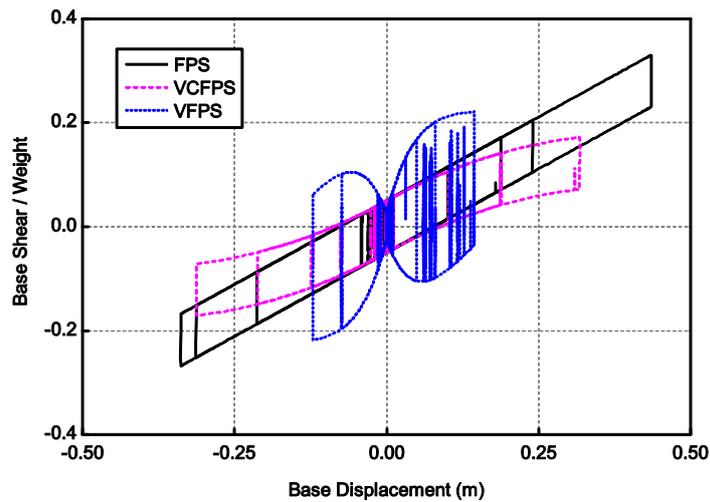
$$\dot{z}(t) = Az(t) + E\ddot{x}_g(t) + BF(t) \tag{12}$$

where  $A$  denotes the system matrix;  $z(t)$  is a vector containing the state variables;  $\ddot{x}_g(t)$  the ground acceleration;  $E$  the excitation distribution matrix;  $B$  the isolator distribution matrix;  $F(t)$  the total isolator shear force.

The ode15s solver in MATLAB efficiently handles stiff systems of first-order ordinary differential equations derived from the state-space formulation of motion equations. It employs a variable order, multi-step algorithm, particularly effective for stiff differential equations. Stiffness arises from the  $Z$  variable, changing slowly during sliding and rapidly during direction reversals or sticking ( $Z$  variable is continuously either +1 or -1). To account for different time steps in the solution algorithm and earthquake acceleration history, linear interpolation calculates acceleration at each solution time step (Fenz, 2008).

**Table 1: Parameter values of isolators used in numerical simulation**

Isolator type		FPS		VCFPS		VFPS	
Parameter values		$R$	1.55 m	$R$	1.55 m	$R$	1.55 m
				$x_0$	0.8 m	$T_i$	1.5 sec
						$\mu_0$	0.025
						$\mu_{max}$	0.15



**Figure 3: Comparison of hysteresis diagrams of FPS, VCFPS, and VFPS under Jiji earthquake**

We selected a 2.5-second isolation period for this study, and Table 1 displays the geometric details of the isolators based on this period. Additionally, we simulated a conventional FPS isolator with the same isolation period for comparing responses between VCFPS, VFPS, and FPS.

#### 4- Results

As the first step in comparing the seismic response of the two isolators under consideration, the hysteresis diagrams of them under the Jiji earthquake have been plotted in Figure 3. As this figure shows, FPS has a constant isolation frequency while VCFPS and VFPS show an adaptive behavior.

In order to study the frequency response of the isolators, the following harmonic ground motion has been considered.

$$u_g(t) = 0.4g \sin wt \quad (13)$$

Figure 4 indicate that VCFPS behaves excellently in mitigating the resonance occurring at the frequency of 0.4 Hz, which is the base isolation frequency. However, VFPS is not successful in this regards and behaves the same as a conventional FPS. Thus, VFPS cannot be considered as an isolator which can mitigate the resonance likely to occur in sliding isolators when subjected to long-period components of near-fault ground motions.

Furthermore, Figure 5 illustrates the maximum seismic response of isolators to seven near-fault earthquakes. Notably, the data for the Jiji earthquake is high for effective presentation, emphasizing other observed earthquake data. VFPS outperforms VCFPS in reducing base displacement, while VCFPS is more effective in controlling transmitted acceleration to the superstructure, except for the intense Jiji earthquake. The choice between VCFPS and VFPS depends on the designer's goal: reducing superstructural acceleration or base displacement, respectively.

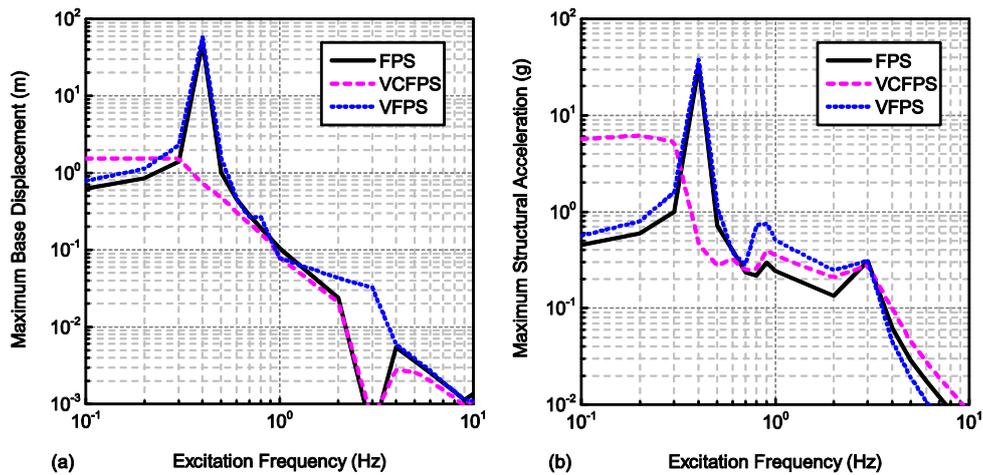


Figure 4: Comparison of frequency response: (a) maximum base displacement; (b) maximum structural acceleration

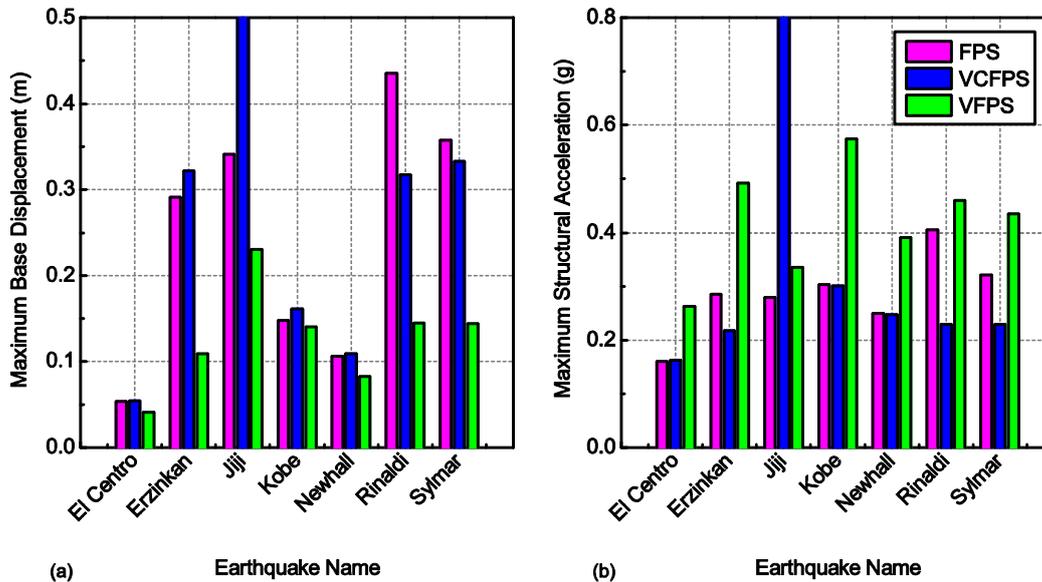


Figure 5: Comparison of maximum responses under the seven earthquakes: (a) base displacement; (b) structural acceleration

## 5- Conclusion

The study investigated the effectiveness of Variable Curvature Friction Pendulum System (VCFPS), Variable Friction Pendulum System (VFPS), and conventional FPS isolators when subjected to near-fault ground motions. The conclusions drawn are as follows:

- 1) VCFPS mitigates resonance phenomenon under harmonic loading, while VFPS behaves similarly to a conventional FPS.
- 2) VCFPS effectively controls transmitted acceleration to the superstructure, but does not perform well in reducing base displacement.
- 3) VFPS excels in reducing base displacement but does not effectively control super-structural acceleration.

- 4) Selection between VCFPS and VFPS depends on whether the main concern is to reduce super-structural acceleration or base displacement.
- 5) Overall, VCFPS outperforms VFPS as it can control super-structural acceleration and mitigate resonance phenomenon likely to occur in sliding isolators during near-fault ground motions.

## 6- References

- Almazán, J. L., De La Llera, J. C., & Inaudi, J. A. (1998). Modelling aspects of structures isolated with the frictional pendulum system. *Earthquake Engineering & Structural Dynamics*, 27(8), 845-867.
- Constantinou, M., Mokha, A., & Reinhorn, A. (1990). Teflon Bearings in Base Isolation II: Modeling. *Journal of Structural Engineering*, 116(2), 455-474.
- Fenz, D. M. (2008). *Development, Implementation and Verification of Dynamic Analysis Models for Multi-spherical Sliding Bearings*. ProQuest.
- Fenz, D. M., & Constantinou, M. C. Spherical Sliding Isolation Bearings with Adaptive Behavior: Experimental Verification. *Earthquake Engineering and Structural Dynamics*, 37(2), 185-205.
- Fenz, D. M., & Constantinou, M. C. (2008). Spherical sliding isolation bearings with adaptive behavior: Theory. *Earthquake Engineering & Structural Dynamics*, 37(2), 163-183.
- Jangid, R. (2005). Computational numerical models for seismic response of structures isolated by sliding systems. *Structural Control and Health Monitoring*, 12(1), 117-137.
- Lu, L.-Y., Shih, M.-H., & Wu, C.-Y. (2004). Near-fault seismic isolation using sliding bearings with variable curvatures. Proceedings of the 13th World Conference on Earthquake Engineering,
- Lu, L.-Y., Wang, J., & Hsu, C.-C. (2006). Sliding isolation using Variable frequency bearings for near-fault ground motions. 4th International Conference on Earthquake Engineering, Taipei, Taiwan,
- Lu, L. Y., Lee, T. Y., & Yeh, S. W. (2011). Theory and experimental study for sliding isolators with variable curvature. *Earthquake Engineering & Structural Dynamics*, 40(14), 1609-1627.
- Lu, L. Y., & Yang, Y. B. (1997). Dynamic response of equipment in structures with sliding support. *Earthquake Engineering & Structural Dynamics*, 26(1), 61-77.
- Mokha, A., Constantinou, M., Reinhorn, A., & Zayas, V. A. (1991). Experimental study of friction-pendulum isolation system. *Journal of Structural Engineering*, 117(4), 1201-1217.
- Narasimhan, S., Nagarajaiah, S., Johnson, E. A., & Gavin, H. P. (2006). Smart base-isolated benchmark building. Part I: problem definition. *Structural Control and Health Monitoring*, 13(2-3), 573-588.
- Panchal, V., & Jangid, R. (2008). Variable friction pendulum system for near-fault ground motions. *Structural Control and Health Monitoring*, 15(4), 568-584.
- Pranesh, M., & Sinha, R. (2000). VFPI: an isolation device for aseismic design. *Earthquake Engineering & Structural Dynamics*, 29(5), 603-627.
- Ray, T., Sarlis, A. A., Reinhorn, A. M., & Constantinou, M. C. (2013). Hysteretic models for sliding bearings with varying frictional force. *Earthquake Engineering & Structural Dynamics*,
- Tsai, C. (1997). Finite element formulations for friction pendulum seismic isolation bearings. *International journal for numerical methods in engineering*, 40(1), 29-49.
- Tsai, C. S., Chiang, T.-C., & Chen, B.-J. (2003). Finite element formulations and theoretical study for variable curvature friction pendulum system. *Engineering Structures*, 25(14), 1719-1730.