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OPTIMIZATION

A new model for facility bus terminal location problem based on modified Kerre's inequality

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Abstract

Recently, Babaie-Kafaki et al. (AsiaPacific J Operat Res 29:1–25, 2012, Appl Soft Comput 46:220–229, 2016) have suggested a model for the fuzzy bus terminal location problem. Here, we propose a new optimization model by improving Babaie-Kafaki et al.'s model. In our model, we define new structures of neighborhoods. Also, we assume that the number of passengers corresponds to the fuzzy nodes. Using modified Kerre's inequality, we propose a new variable neighborhood search algorithm for solving a fuzzy bus terminal location problem. In our algorithm, we consider new types of neighborhoods to have a more realistic fuzzy model. The algorithm is tested on a variety of random generated large-scale fuzzy bus terminal location problems with fuzzy coefficients. In contrast of most existing method our proposed algorithm is solved fuzzy bus terminal location problem directly. The parameters of our proposed algorithm are set by irace package to ensure fair space. To demonstrate the performance of our method, we make a comparison between our method and other existing algorithms. We make use of the non-parametric statistical test due to Wilcoxon's test and the Dolan–Moré performance profiles to assess the performance of the numerical algorithms.

Keywords Fuzzy bus terminal location problem · Variable neighborhood search · Modified Kerre's inequality

1 Introduction

Bus terminal location problem (BTLP) is one the special case of bus assignment with the objective of maximizing public transportation service (Ghanbari and Mahdavi-Amiri 2011). In this problem, bus stations, metro stations and etc with a known number are represented by n nodes. The main goal of BTLP is to maximized the public service by locating a number of locations among m terminals with definite neighborhoods and it is a specific case of facility location problem (FLP) (Ghanbari and Mahdavi-Amiri 2011).

According several studies, many types of FLPs were NP-hard problems. So, we can not solve large instances of them by using exact algorithms at a particular time (see Holmberg et al. (1999); Mirchandani and Francis (1990); Rahdar et al. (2022)). Because of this, other methods like approximation, heuristics and metaheuristics algorithms (see Ghanbari et al. (2020)) are used to solve FLPs problems.

Moreno Moreno Pérez et al. (2004) expressed that in real applications, the FLPs can be defined with fuzzy parameters or variables, that means if we considered a FLP problem as a network, the weights, some nodes, distances of edges and so on can be fuzzy numbers. Karagoz et al. (2021) presented a new model of FLP problems. They introduced a method with the name the additive ratio assessment (ARAS). Karagoz et al. (2021), presented their proposed method under the interval type-2 fuzzy environment. Also, they show the efficiency

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performance of theire proposed method on the real-life case study of Istanbul.

There are some true approximations methods for solving FLPs, which they were used local techniques. These methods had convergence problem. So, Grover et al. (2022) proposed an efficient algorithm as compared true approximation that used in inefficient ellipsoid method. Zhu et al. (2022) presented a FLP with drons (FLPD). They incorporated the demand uncertainty by using demand scenarios. The main goal of their model was to find location, allocation and assignment plane that has minimal cost. Also, Zhu et al. (2022) proposed three other models and compared FLP with FLPD and three proposed models. They used column and constraint generation and benders decomposition methods for solving proposed models.

The first corrugation of location models was crisp, and there may no little account for the case of uncertainty in location problems such as the p-median problem (p-MP) and maximal covering location problem (MCLP). Some examples of uncertainty in these problems are the uncertainty associated with the demands of nodes or the distances of edges in p-MP (Kutangila-Mayoya and Verdegay 2005) and the coverage radii of facilities or demands of nodes to be covered in MCLP (Batanovic et al. 2009). Apart from the above-mentioned papers, various FLPs were discussed by fuzzy logic methods (Bhattacharya et al. 1993; Wen and Iwamura 2008; Zhou and Liu 2007).

Here, we study a fuzzy bus terminal location problem (FBTLP), which is also considered a combination of two p-MP and MCLP problems (Djenić et al. 2016). The FBTLP has been addressed by authors such as Babaie-Kafaki et al. (2012, 2016), who suggested a BTLP in a network with uncertainty. In the previous studies on the FBTLP (Babaie-Kafaki et al. 2012, 2016), by defining a membership function, the neighborhood of each terminal is considered as a fuzzy set so that each node belongs to the neighborhood of each terminal with a grade of membership. Also, they defined a strictly decreasing function by using the distances between node and the terminal as a cost of service. Multiplying the cost of service and the value of membership for each node in the objective function is the specified attribute of each node for each terminal.

The definition of membership function means that the attribute of stations within the neighborhood radius of each terminal is proportional to the cost of receiving a service from each station (the grade of membership is considered equal to 1), and this attribute for stations outside the neighborhood radius reduces by defining a strictly decreasing for the grade of membership.

In the previous models, the attribute is distance dependent and also the small distances affected the model. While, in the real models, small distances between nodes do not affect the model, and their attributes for the terminal are the same. Therefore, we propose a new neighborhood for each terminal as a constant value, where there is no difference between stations that are close to each other. By developing the hypothesis, despite the proposed models in Babaie-Kafaki et al. (2012, 2016) also Djenić et al. (2016); Ghanbari and Mahdavi-Amiri (2011) in crisp, we introduce three fuzzy types of neighborhoods with new membership function for each terminal.

Similar to a crisp BTLP (Djenić et al. 2016), FBTLP is also an NP-hard problem. So, hybrid metaheuristic algorithms are proposed for solving it Babaie-Kafaki et al. (2012, 2016). Here, we suggest a metaheuristic algorithm based on the variable neighborhood search (VNS) Hansen et al. (2010) for solving an FBTLP; we named it FVNS (see different models and solving methods for fuzzy linear programming problems in Ghanbari et al. (2020)).

In the most studied methods, the common solution for solving FBTLP is to transform it into the crisp BTLP by using different ranking functions (Babaie-Kafaki et al. 2012, 2016; Mahdavi-Amiri and Nasseri 2006; Mahdavi-Amiri et al. 2009; Maleki et al. 2019). Using these defuzzification methods are not appropriate, since we don't solve a fuzzy problem in a fuzzy environment, but by using modified Kerre's inequality, we can do it. Also, the ranking functions are not accurate enough; see Ghanbari et al. (2019a).

In Babaie-Kafaki et al. (2012, 2016), the problem is transformed to the exact case by using some defuzzification methods like ranking functions, and next the exact model is solved. Here, we propose a new fuzzy model for BTLP (FMBTLP). In our model, we assume that the number of passengers corresponds to the fuzzy nodes. Using modified Kerre's inequality, we propose a new variable neighborhood search algorithm for solving a fuzzy bus terminal location problem. In our algorithm, we consider new types of neighborhoods to have a more realistic fuzzy model that there is no significant difference in the cost of service, when the difference between the distances is very small. To solve an FMBTL problem, by using Kerre's inequality (Ghanbari et al. 2019a, b), the fuzzy value of the objective function are compared to each other. In this situation, we don't leave the fuzzy environment. We propose a VNS algorithm with new types of neighborhoods to find a feasible solution.

The rest of paper is arranged as follows. In Sect. 2, we briefly review some notations and definitions. A new fuzzy model for a BTLP named $FMBTLP$ is presented in Sect. 3. In Sect. 4, our proposed idea and the idea of Babaie-Kafaki et al. (2012, 2016) implement on a illustrative example and we compare the result of them. To solve this fuzzy model, we present a VNS (FVNS) algorithm in Sect. 5. Then, we numerically compare our method with available methods, and for analyzing the performance of our proposed algorithm, we use the Dolan–Moré performance profiles and Wilcoxson's test in Sect. 6. In the last section, we make conclusions.

2 Basic definitions and concepts

Here, some concepts which are used in our paper are described.

2.1 Definitions and notation

Definition 1 (Zimmermann 2001) A fuzzy set is defined as follows:

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) \mid x \in \mathbb{R}\},$$

where, $\mu_{\tilde{A}}(x) \in [0, 1]$ is called the membership function.

Definition 2 (Nguyen and Walker 2000) A fuzzy number is a fuzzy quantity A satisfying the following conditions:

1. $\mu_{\tilde{A}}(x) = 1$, for exactly one x.
2. The support $\{x : \mu_{\tilde{A}}(x) > 0\}$ of A is bounded.
3. The α-cuts of A are closed intervals.

Definition 3 (Zimmermann 2001) A decreasing map $L : \mathbb{R}^+ \to [0, 1]$ is called a shape function if the following conditions hold:

$$\begin{cases} L(0) = 1, \\ L(1) = 0, \\ 0 < L(x) < 1, \quad x \neq 0, 1. \end{cases}$$

Definition 4 (Zimmermann 2001) A fuzzy number $\tilde{A}$ is of LR-type if there exist shape functions L (for left) and R (for right), and scalars $\alpha > 0$ and $\beta > 0$ such that

$$\mu_{\tilde{A}}(x) = \begin{cases} L\left(\frac{a-x}{\alpha}\right), & x \leq a, \\ R\left(\frac{x-a}{\beta}\right), & x > a. \end{cases}$$

The mean value of $\tilde{A}$, a, is a real number, and α and β are called the left and right spreads, respectively. Here, $\tilde{A}$ is denoted by $(a - \alpha/a/a + \beta)_{LR}$.

Remark 1 Based on Definition 4, another representation of an LR fuzzy number $\tilde{A}$ is $\tilde{A} = (A_L, A_R)$, where A_L is a shape function for the left arm and A_R is a shape function for the right arm.

Definition 5 (Buckely and Jowers 2007) A triangular fuzzy number $\tilde{A}$ is defined by three real numbers $a < b < c$, where the base of the triangle is the interval $[a, c]$ and its vertex is at $x = b$.

Remark 2 Another representation of a triangular fuzzy number $\tilde{Q}$ is $\tilde{Q} = (Q_L, Q_R)$, where Q_L and Q_R are the functions for the left arm and the right arm of triangular fuzzy number $\tilde{Q} = (q_1/q_2/q_3)$, respectively.

Note 1 We show a triangular fuzzy number as $\tilde{N} = (n_1/n_2/n_3)$. So, $\tilde{N} = (n/n/n)$ represent a real number as N.

Theorem 1 (Zimmermann 2001) *Let* $\tilde{A} = \left(a^L/a/a^R\right)_{LR}$, *let* $\tilde{B} = \left(b^L/b/b^R\right)_{LR}$, *and let* $\alpha \in \mathbb{R}^+$. *Then,*

1. $\alpha\tilde{A} = \left(\alpha a^L/\alpha a/\alpha a^R\right)_{LR}$.
2. $-\tilde{A} = \left(-a^R/-a/-a^L\right)_{LR}$.
3. $\tilde{A} \bigoplus \tilde{B} = \left(a^L + b^L/a + b/a^R + b^R\right)_{LR}$.

2.1.1 Modified Kerre's inequality

Another well-known method for the comparison of fuzzy numbers is Kerre's inequality (Wang and Kerre 2001). We (Ghanbari et al. 2019a) presented modified Kerre's inequality and established some efficient formulas for comparison of fuzzy triangular numbers as the following theorem.

Theorem 2 (Ghanbari et al. 2019a) *If* $\tilde{A} = (A_1/A_2/A_3)$ *and* $\tilde{B} = (B_1/B_2/B_3)$ *are two triangular fuzzy numbers with* $A_2 \leq B_2$, *Then we have:*

1. *If* $A_3 \leq B_1$, *then*

$$r(\tilde{A}, \tilde{B}) = \frac{B_3 - B_1}{2} + \frac{A_3 - A_1}{2}.$$

2. *If* $A_2 = B_2$, *then*

$$r(\tilde{A}, \tilde{B}) = \frac{B_3 + B_1}{2} - \frac{A_3 + A_1}{2}.$$

3. *If* $A_2 < B_2$, *then*

$$r(\tilde{A}, \tilde{B}) = \frac{B_3 - B_1}{2} + \frac{A_3 - A1}{2} - \bar{u}(A_3 - B_1), \quad (1)$$

where $\bar{u} = A_R(\bar{x}) = B_L(\bar{x})$ *in which* $\bar{x}$ *is the length of the intersection point of* A_R *and* B_L *and defined as follows:*

$$\bar{x} = \frac{B_2 A_3 - A_2 B1}{(B_2 - B_1) + (A_3 - A_2)}.$$

Note 2 In this paper, the notations $\leq^K$, $\geq^K$, and $=^K$, are used for comparison two fuzzy numbers based on modified Kerre's inequality.

3 New proposed model for the FMBTLP

In the real models of FLP, there are some parameters like demands and distances, whose exact assumptions are not realistic (Moore 1997). So, to transform this model to a more realistic problem, it can be assumed that dates are uncertain and described them by using fuzzy variables.

Here, we propose an FMBTLP, with the assumptions that the number of passengers corresponding to each station and also that the number of the neighborhoods are fuzzy numbers.

To express an FMBTL problem, consider a network whose nodes are divided into two separate sets N and M, where $N = \{1, 2, \dots, n\}$ shows the index of the demand nodes (bus stations) and $M = \{1, 2, \dots, m\}$ shows the index set of the candidate centers for bus terminals. The objective is to select p terminals from the set M such that the service function is maximized.

The symbols used in this paper are listed in Table 1.

Remark 3 The neighborhood is defined for each terminal is a fuzzy set.

Remark 4 J_i^+ is a fuzzy set with the following membership function:

$$\mu_{J_i^+}(j) = \mu_{ij} = \begin{cases} 1, & c_{ij} < d_1, \\ K, & d_1 \le c_{ij} < d_2, \\ f(c_{ij}), & c_{ij} \ge d_2, \end{cases}$$

where, d_1, d_2, and K are given constants and $f(x) : [d_2, +\infty) \to [0, 1]$ is a strictly decreasing function.

Remark 5 We consider u_{ij} as follows:

$$u_{ij} = L.\mu_{ij}, \qquad \text{for all } i \in M,\ j \in N,$$

where L is a constant.

The variables (location and assignment) of the FMBTLP are described as follows:

$$s_i = \begin{cases} 1 & \text{if a bus terminal is located in the candidate center } i, \\ 0 & \text{O.W.} \end{cases}$$

$$x_{ij} = \begin{cases} 1 & \text{if the node} j \in N \text{receives service from} i \in M, \\ 0 & \text{O.W.} \end{cases}$$

We define one type of FMBTL problem as follows:

$$\text{FMBTLP} : \max \sum_{i \in M} \sum_{j \in N_i^+} u_{ij} \tilde{d}_j x_{ij}$$

$$s.t. \qquad (2)$$

$$\sum_{j \in N_i^+} x_{ij} \le |J| s_i, \qquad \text{for all } i \in M, \qquad (3)$$

$$\sum_{i \in M_j^+} x_{ij} = 1, \qquad \text{for all } j \in N, \qquad (4)$$

$$\sum_{i=1}^{m} s_i = p, \qquad (5)$$

$$x_{ij} \in \{0, 1\},\ s_i \in \{0, 1\}, \quad \forall i \in M,\ j \in N. \qquad (6)$$

The objective function determines the service amount of candidates node to all nodes. Constraints (3) show that when the node i is chosen as a bus terminal, it can give service to its neighborhood nodes. Also, constraints (4) show that each node $j \in J$ must receive service only from one terminal, and the constraint (5) controls the number of necessary terminals.

Definition 6 The public service function is defined as follows (Ghanbari and Mahdavi-Amiri 2011):

$$F(\tilde{S}) = \sum_{j=1}^{n} \tilde{d}_j \times (\max_{i \in S} u_{ij}), \quad \forall S \subseteq M. \qquad (7)$$

In the next section, we explain priorities of our proposed model by using an illustrative example.

4 Illustrative example

In this section, we discuss an illustrative example. In this example, we consider a bus network with six bus stations and three candidate terminals. The coordinates of stations and terminals are given in Tables 2 and 3, respectively. In Table 2 the numbers considered as triangular fuzzy numbers. According to FMBTLP, we want to determine two terminals between three candidates ones, such that maximized the public service function (7) and the chosen terminals according to our proposed model can give service to its neighborhood nodes. The goal of this example is that to compare the result of this example by using our proposed idea and the idea of Babaie-Kafaki et al. (2016).

The distance matrix is defined as follows:

$$Q = \begin{bmatrix} 0.3162 & 0.9434 & 0.9487 & 0.9849 & 2.2361 & 4.5607 \\ 0.3000 & 1.2166 & 1.3000 & 1.3038 & 2.5000 & 4.2720 \\ 5.3226 & 6.2241 & 6.2394 & 6.2769 & 7.5240 & 0.9434 \end{bmatrix}$$

First, we use the proposed idea by Babaie-Kafaki et al. (2016) for solving this problem. Suppose that $r = 0.5, d_r = 0.5$. We consider service function as defined in Ghanbari and Mahdavi-Amiri (2011) and $f(c_{ij}) = e^{-c_{ij}}$, so, the optimal value for objective function and optimal set for terminal are $\tilde{z}_1 = (7.4817e+03/7.5233e+03/7.5650e+03)$ and $S^* = \{1, 2\}$. Therefore, according to the optimal solution, stations with the numbers, 1, 2, 3 and 4 can receive service from terminals 1 and 2.

Now, we want to illuminate this issue by utilizing our proposed model for two radii $d_1 = 0.5$ and $d_2 = 1$. Let $L = 3.33$

Table 1 Description of symbols

Symbols	Decription
$M = \{1, \ldots, m\}$	Index set of the candidate terminals.
$N = \{1, \ldots, n\}$	Index set of the demand nodes.
$C = [c_{ij}]$	Distance matrix between the nodes $i \in M$ and $j \in N$
$\tilde{d}_j$	Number of passengers corresponding to the node $j \in N$.
N_i^+	Set of nodes in N that can receive service from the node $i \in M$.
M_j^+	Set of nodes in M that can service the node $j \in N$, $M_j^+ = \{i \in M;\ \mu_{ij} > 0\}$.
u_{ij}	Attribute of station $j \in N$ for the terminal $i \in M$.
x_{ij}	Vector of boolean variables for assignment.
s_i	Vector of boolean variables for location.
p	The number of required terminals.

Table 2 Fuzzy coordinates of stations

station (j)	1	2	3
coordinate (j_1, j_2)	(3.3, 2.2)	(3.5, 1.3)	(3.3, 1.2)
potential (d_j)	$(10000 - \alpha/10000/10000 + \alpha)$	$(1000 - \alpha/1000/1000 + \alpha)$	$(1000 - \alpha/1000/1000 + \alpha)$
station (j)	4	5	6
coordinate (j_1, j_2)	(3.4, 1.2)	(4, 0.1)	(1.8, 6.5)
potential (d_j)	$(3000 - \alpha/3000/3000 + \alpha)$	$(500 - \alpha/500/500 + \alpha)$	$(1000 - \alpha/1000/1000 + \alpha)$

Table 3 Coordinates of terminals

Terminal (i)	1	2	3
coordinate (i_1, i_2)	(3, 2.1)	(3.3, 2.5)	(1, 7)

and let $K = 0.45$. By using (7), the optimal objective function and optimal terminals are $\tilde{z}_2 = (4.1985e{+}04/4.2469e{+}04/4.2953e + 04)$ and $S^* = \{1, 3\}$, respectively.

Due to Babaie-Kafaki et al.'s model, it is appeared that the terminal no.2 only gives service to station no.1. The distinction between the distance of station no.1 from terminal no.1 and terminal no.2 is exceptionally small (see the matrix Q). At that point, there's no critical distinction between station no.1 getting service from terminal no.1 or terminal no.2. In our proposed model, the neighborhood of the first-type affirms the same thing. The result of this assumption is compared with Babaie-Kafaki et al.'s model. So, terminal no.3 is built up rather than terminal no.2, and in expansion to stations no.1, no.2, no.3, and no.4, station no.6 get service, as well as objective function's value is expanded.

The interpretation of other neighborhoods in the proposed model is similar.

In this proposed model, all stations could receive service and objective function's value is increased to $\tilde{z}_2 = (4.1985e + 04/4.2469e + 04/4.2953e + 04)$.

5 Variable neighborhood search for solving *FMBTLP*

There are some methods with different ideas for escaping local optima (e.g., tabu search (Glover and Laguna 1997) and simulated annealing (Hendersom et al. 2003)), one of these methods is local search. Another way of escape from local optima is to change the neighborhoods during the search systematically; that is the basic idea of the VNS algorithm. This metaheuristic method has recently been proposed by Hansen and Mladenović (1997). Based on the advantages of VNS algorithm in solving optimization problems (Hansen et al. 2010), and also its effective performance of it for a BTLP (Babaie-Kafaki et al. 2016; Djenić et al. 2016), we propose the VNS for solving an $FMBTLP$. Based on the definition of the service function in (7), for each feasible solution, the value of an objective function is a fuzzy number. Thus, to compare the solutions in the proposed algorithm (e.g., Step 1-4), we use modified Kerre's inequality in Theorem 2. So, all fuzzy comparisons are performed in the fuzzy environment by using Kerre's inequality.

5.1 Initial solution

To construct the initial solution, we use a greedy method based on the defined potential objective function (POF) in

Algorithm 1 FVNS for solving FMBTLP

Step 0: $\{Intialization\}$

- Input MAX_{FVNS} and $MAX_{LS} \in \mathbb{Z}^+$ to determine the stopping conditions of VNS and local search (LS), respectively, $n_{\max} \in \mathbb{Z}^+$ to be the maximum number of neighborhoods in (8).
- Construct the initial solution S (see Subsection 5.1).
- Let $iter \leftarrow 1$.

Step 1: $\{Main\,steps\}$

1-1: Let $n \leftarrow 1$.
1-2: $\{Shaking\}$. Create a random solution $(S^{'})$ by using $N_k(S)$.
1-3: $\{Local\,search\}$. Use the LS method (see 5.3) on $S^{'}$. If MAX_{LS} iterations have been performed without improvement then stop, let $S^{''}$ be so the obtained local optima.
1-4: $\{move\,or\,not\}$. If $F(\tilde{S}^{''}) >^{K} F(\tilde{S})$, then let $S \leftarrow S^{''}$ and $n \leftarrow 1$; else, let $n \leftarrow n+1$.
1-5: If $n = n_{\max}$, then let $iter \leftarrow iter + 1$ and go to **Step 1-6**; else go to **Step 1-2**.
1-6: If $iter = MAX_{FVNS}$, then **Stop**; else, go to **Step 1-1**.

Step 2: Return S.

Definition 7. In this method, p terminals with more POFs are established.

Definition 7 The fuzzy POF is defined as follows:

$$POF(i) = \sum_{j \in N_i^+} \tilde{d}_j \times u_{ij} \quad \text{for all } i \in M,$$

This formula has been extended in a fuzzy form by taking ideas from the formula of Ghanbari and Mahdavi-Amiri (2011) within the crisp case.

Here, since the potentials of stations are fuzzy numbers, the POF for each terminal is also a fuzzy number. So, to select p terminals among m candidate centers, based on Definition 7, we use the ranking function (Mahdavi-Amiri and Nasseri 2007) and map the value of each POF into a real number. Finally, p terminals are established that have more real values.

5.2 Neighborhood

Let that $S \subset M$ and that $|S| = p$. The k-th neighborhood of feasible solution S is establishing k currently closed terminals and closing k currently open terminals,

$$N_k(S) = \{T \mid T \subset M,\ |T| = p,\ |S - T| = l\}, \quad l = 1, 2, \ldots, l_{\max}, \tag{8}$$

where, $l_{\max}$ is a constant.

5.3 Local search method

The LS method used in our proposed FVNS is based on the first improvement approach. In each iteration of this proposed method, a random solution is computed from the occupant solution agreeing to the primary neighborhood in (8). In case the new solution is way better than the occupant solution, at that point the incumbent solution is upgraded; else, another solution is considered. The proposed LS is stopped when MAX_{LS} iterations have been performed without improvement.

5.4 Stopping condition

We stop the proposed FVNS algorithm when reach MAX_{FVNS} successive iterations without improvement.

6 Numerical results

Here, we want to show the performance of our proposed algorithm. We constructed many test problems of the $FMBTLP$ (we generate random test problem in MATLAB 7.0 programming environment on a notebook, Intel(R) Core(TM) i5-3210 M CPU 2.5 GHz, with 4.00 GB RAM). In each problem, the locations of candidate centers and stations are placed in a region depicted by $\{0 \le x_i \le 10, 0 \le y_i \le 10;\, i \in M\}$ and $\{0 \le x_j \le 10, 0 \le y_j \le 10;\, j \in N\}$, respectively. For each node $j \in N, \tilde{d}_j = (d_j - \alpha/d_j/d_j + \beta), j = 1, 2, \ldots, n$, is a triangular fuzzy number, where d_j is a random integer in the interval [1, 100], and α and β are equally selected from the interval [0, 10] (we generate our test problems similar to Babaie-Kafaki et al. (2012, 2016)). Also, in all executions, we set $d_1 = \frac{\max cij}{10}$ and $d_2 = \frac{\max cij}{5}$ in Definition 4, and the value of L in Remark 5 is considered equal to the maximum $\frac{1}{c_{ij}},\ i \in M,\ j \in N$.

We considered the membership function of $N_j^+,\ j = 1, 2, 3$, as follows:

$$\mu_{N_i^+}(j) = \mu_{ij} = \begin{cases} 1, & c_{ij} < d_1, \\ 1 + \frac{d_1 - d_2}{2C_{\max}}, & d_1 \le c_{ij} < d_2, \\ 1 + \frac{.5d_1 - c_{ij}}{C_{\max}}, & c_{ij} \ge d_2, \end{cases}$$

where $C_{\max}$ is the maximum $c_{ij},\ i \in M,\ j \in N$. In the test problems, we set

$$n = 2000, 1500, 1000, 750, 500,$$
$$m = \lfloor \frac{n}{2} \rfloor$$
$$p = \lfloor \frac{3m}{4} \rfloor,\ \lfloor \frac{m}{4} \rfloor,\ \lfloor \frac{m}{10} \rfloor,\ \lfloor \frac{m}{100} \rfloor.$$

6.1 Parameter settings

In this section, we want to use the irace package (Lopez-Ibanez et al. 2016) to ensure fair space for setting the

Table 4 Setting parameters of FVNS algorithm

Parameter	Explanation	Type	Interval	IRACE
$n_{\max}$	Number of neighborhood's structures	integer	[1, 10]	3
Max_{VNS}	Number of iterations	integer	[3, 30]	10
Max_{LS}	Number of iterations in Algorithm 1	integer	[100, 2000]	700

parameters of our proposed VNS algorithm. Table 4, shows the value of the parameters obtained on their defined intervals by using the irace package.

6.2 Implementation of FVNS

By using Table 4, we executed Algorithm 1 on all instances, and we summarized the obtained results in Table 5. Also, we solve the $FMBTLP$ by the α-cut method (Hendry Purba et al. 2017; Herrera and Verdegay 1995; Yi et al. 2016). Table 5 gives the obtained running times of the FVNS algorithm and α-cut. In this table, the first column shows the number of the demand nodes (n), the number of the candidate centers (m), and the number of the established terminals (p), respectively. The column entitled FVNS Objective function shows the objective function achieved by the FVNS algorithm, and the column entitled FVNS Time(s) shows the running time of the FVNS algorithm. The column entitled α-cut Objective function shows the objective function achieved by the α-cut method; the column entitled α-cut Time(s) shows the running time of the α-cut method; and finally, the last column shows the comparison of the objective function values corresponding to the solution FVNS algorithm (Sol_{FVNS}) and the one due to α-cut method ($Sol_{\alpha-\text{cut}}$) using (1).

It is observed that the FVNS algorithm presented here is to be faster than the α-cut. Moreover, by using the column entitled $r(Sol_{FVNS}, Sol_{\alpha-\text{cut}})$, the fuzzy objective function values obtained by our proposed algorithm for the numerical examples were higher than the ones obtained by α-cut method.

The results of Wilcoxon's test are shown in Table 6. We performed all statistical computations utilizing SPSS21 software. Consider the following test:

$$H_0 : \mu_{\text{FVNS}} = \mu_{\alpha-cut},$$
$$H_1 : \mu_{\text{FVNS}} > \mu_{\alpha-cut},$$

where, μ_{FVNS} represents the average speed of algorithm. From Table 6, the null hypothesis H_0 is rejected with the level of significance of 0.05. Since the p-value is lower than 0.05, FVNS algorithm appears to be more faster than $\alpha-$cut method.

Also, we use the performance profiles given by the Dolan–Moré diagrams (see details in Dolan and More (2002)). Performance profile gives, for every ω, the proportion $\rho(\omega)$ of the test problems that each considered algorithmic variant has a performance within a factor of ω of the best. Thus, based on the Dolan–Moré performance profile as shown in Fig. 1, we conclude that the FVNS algorithm performs significantly faster than $\alpha - cut$ method.

7 Conclusions and Future works

Inspired by a real-world problem, we have modeled BTLP and solved it. We proposed a new model as FBTLP and we presented a new model with the name FMBTLP. In our proposed model, we considered the number of passengers corresponding to each station and that the neighborhoods of each terminal are fuzzy numbers. For solving FBTLP, we proposed an FVNS algorithm, this algorithm used modified Kerre's inequality for solving FMBTLP. The parameters of our proposed algorithm were set by irace package to ensure fair space. We generated and solved some random test examples with triangular fuzzy coefficients, and we compared the performance of our proposed algorithm with the α-cut method. Finally, using the non-parametric statistical tests due to Wilcoxon's test and the Dolan–More's diagram for analyzing the performance of the algorithms, we demonstrated the efficiency of our proposed approach in comparison with the α-cut method.

The limitation of this paper is that our proposed algorithm is based on triangular fuzzy numbers. So, for the further research lines on fuzzy bus terminal location problem we can extend modified Kerre's inequality for comparison trapezoidal fuzzy numbers and propose algorithm based of trapezoidal fuzzy number also similar to some works in Akram (2011); Ghanbari et al. (2019), we can developed our model by using bipolar fuzy numbers.

Table 5 Comparison of results

n	m	p	FVNS Objective function	Time(s)	α-cut Objective function	Time(s)	$r(Sol_{FVNS}, Sol_{\alpha-\text{cut}})$
2000	1000	750	(3.2814e+007/3.8603e+007/4.2847e+007)	12.81	(3.1524e+007/3.0314e+007/ 3.9865e+007)	50.56	$5.9460e+06$
		250	(3.4620e+007/3.8567e+007/4.3009e+007)	24.70	(2.5103e+007/3.2631e+007/ 3.8523e+007)	83.63	$9.3562e+06$
		100	(3.4650e+007/3.8280e+007/3.3750e+007)	26.70	(3.1365e+007/3.0254e+007/ 4.3274e+007)	96.32	$1.0376e+06$
		10	(3.3698e+007/3.5770e+007/3.6160e+007)	47.78	(2.9231e+007/3.2301e+007/4.3265e+007)	111.2	$1.2269e+06$
1500	750	562	(0.8561e+007/1.0233e+007/1.0914e+007)	9.40	(0.5432e+007/0.9236e+007/1.2410e+007)	67.32	$1.6084e+06$
		187	(0.9727e+007/1.0220e+007/1.1209e+007)	14.44	(0.6321e+007/0.9632e+007/1.0154e+007)	73.12	$2.4779e+06$
		75	(0.8695e+007/1.0111e+007/1.1209e+007)	13.10	(0.6124e+007/0.9012e+007/1.0254e+007)	76.25	$2.4076e+06$
		7	(0.7824e+007/0.9281e+007/1.0409e+007)	47.78	(0.5978e+007/0.8652e+007/0.9922e+007)	93.62	$1.6504e+06$
1000	500	375	(4.3037e+006/4.8776e+006/5.8437e+006)	5.44	(4.0251e+006/4.3651e+006/ 5.5544e+006)	188.23	$5.5070e+04$
		125	(4.5236e+006/4.8629e+006/5.3049e+006)	7.70	(4.6103e+006/4.8800e+006/5.3285e+006)	1200.21	$6.4748e+05$
		50	(4.3174e+006/4.8829e+006/5.3828e+006)	8.68	(4.0659e+006/4.6521e+006/ 5.0254e+006)	1602.5	$4.7851e+05$
		5	(4.0182e+006/4.41116e+006/4.8775e+006)	29.15	(3.9856e+006/4.0136e+006/4.4365e+006)	1812.61	$4.4064e+05$
700	350	262	(6.7574e+006/7.5593e+006/8.8015e+006)	6.42	(6.3205e+006/7.1024e+006/ 8.6111e+006)	86.14	$6.8020e+05$
		87	(6.8209e+006/7.7617e+006/8.7029e+006)	4.87	(6.2501e+006/7.0254e+006/8.3012e+006)	1432.01	$9.7797e+05$
		35	(7.1272e+006/7.5260e+006/7.6741e+006)	6.37	(6.9825e+006/7.0325e+006/7.4825e+006)	1500.47	$2.5495e+05$
		3	(5.7065e+006/6.5194e+006/7.2636e+006)	24.62	(5.8125e+006/6.6325e+006/7.3100e+006)	1612.3	$3.7472e+05$
500	250	187	(0.9657e+006/0.9578e+006/1.0584e+006)	2.29	(0.7533e+006/0.8944e+006/1.1794e+006)	72.13	$9.4594e+04$
		62	(0.8385e+006/0.9643e+006/1.0338e+006)	2.90	(0.6044e+006/0.6357e+006/0.9666e+006)	93.25	$2.4282e+05$
		25	(0.8636e+006/0.9286e+006/0.9785e+006)	4.24	(0.7236+006/0.8132e+006/1.1547e+006)	124.3	$4.2634e+05$
		2	(0.7272e+006/0.7699e+006/0.8889e+006)	13.27	(0.5231e+006/0.6981e+006/0.9364e+006)	158.3	$1.3175e+05$

Table 6 Results of Wilcoxon's test on times of the pair $(FVNS(Time(s)), \alpha - \text{cut}(Time(s)))$ for all test problems

Wilcoxon	−3.308
level of significance	0.005

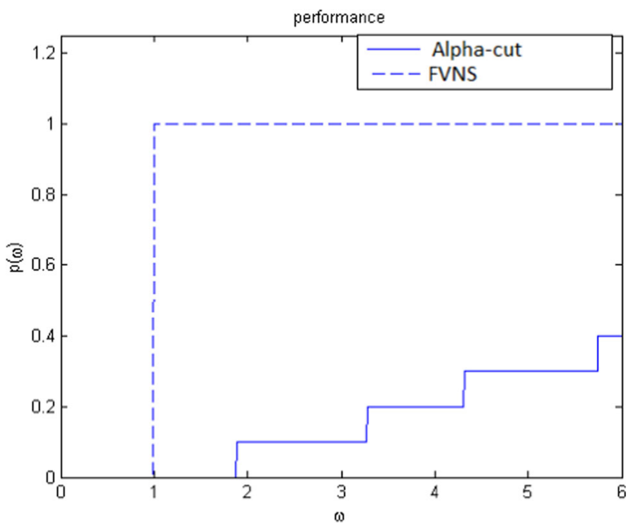

Fig. 1 The Dolan–Moré diagram for comparison of FVNS (Time(s)) algorithm and $\alpha - $ cut method (Time(s))

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Declarations

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