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Optimized multi-tower crane layout planning: determine height, location and type to improve operational safety

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Abstract

Purpose – In high-rise construction projects, the use of multiple tower cranes to transport materials has become common; however, optimizing their layout still poses a challenging problem. Key objectives such as minimizing costs related to crane operation (such as rental, installation, dismantling and operator wages) while reducing workdays, mitigating interruptions caused by crane overlapping and improving safety (such as preventing crane collisions and path blockages).

Design/methodology/approach – A mixed-integer linear programming (MILP) model is proposed to optimize the number, type and location of tower cranes as well as the location and number of supply points. The MILP incorporates crane height optimization and penalties for loading, crossing and unloading within overlapping areas to tackle interference issues. Additionally, a delay penalty is introduced into the objective function to minimize workdays and material delivery delays.

Findings – The proposed method was validated with a real-world case study. Results show that the introduced model can manage crane overlaps optimally by assigning tasks and ranking crane heights. Unlike similar works, the proposed method is able to find a path over other cranes by determining an optimum height. Applying the proposed method in the case study resulted in a cost reduction of up to 49%.

Originality/value – This study extends the previous approaches by addressing critical yet underexplored factors such as the number and capacity of supply points as well as considering safety issues like avoidance of path obstructions and crane collision(s) in the mathematical model.

Keywords Tower crane, Layout planning, Crane overlapping, Operational safety, Optimization, MILP model

Paper type Research paper

1. Introduction

Tower cranes are widely used in the construction of high-rise buildings (Huang *et al.*, 2021; Al-Hussein *et al.*, 2006; Kim *et al.*, 2016; Moselhi *et al.*, 2004). The main role of cranes is to provide efficient transportation of materials throughout the various levels of a construction site (Park *et al.*, 2013). Effective and optimal utilization of cranes can significantly reduce construction time, project costs and safety hazards associated with material transportation (Xiao Lin *et al.*, 2023; Tariq *et al.*, 2022). The performance of tower cranes is mainly affected by factors such as their type, number and installation location (Irizarry and Karan, 2012; Safouhi *et al.*, 2011) which are typically determined before the commencement of construction (Zhang and Pan, 2021). The operation schedule, which is set during the project implementation (Ji and Leite, 2020), also influences the crane's performance (Han *et al.*, 2015). Therefore, the tower crane problem can be decomposed into two sub-optimization problems: (1) layout (Ji and Leite, 2020; Zhang *et al.*, 1996; Tam *et al.*, 2001; Huang *et al.*, 2011; Lien and Cheng, 2014; Riga *et al.*, 2020; Dienstknecht, 2022) and (2) operation scheduling (Huang *et al.*, 2021; Zavichi and Behzadan, 2011; Zavichi *et al.*, 2014; Monghasemi *et al.*, 2016; Al Hattab *et al.*, 2017; Al Hattab *et al.*, 2018; Tarhini *et al.*, 2021).

Determining the best locations for cranes is one of the main steps in Tower Cranes Layout Planning (TCLP) which directly affects the cost and time of material transportation in a site (Han *et al.*, 2015; Zhang *et al.*, 2022; Xu *et al.*, 2020). Zhang *et al.* (1996), Tam *et al.* (2001), Tam and Tong (2003), Huang *et al.* (2011), Moussavi Nadoushani *et al.* (2017), Huang and

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Wong (2018) and Amiri *et al.* (2023) determined the location of a single crane to reduce the material transport time. Zhang *et al.* (1999), Lien and Cheng (2014), Wang *et al.* (2015) and Ji and Leite (2020) introduced methods to handle TCLP with multi-cranes. In these studies, the number of required cranes is assumed to be predetermined. Increasing the number of cranes can reduce the materials transport time; however, adding cranes more than necessary can increase the project costs (Tariq *et al.*, 2022; Tork, 2013; Yeo and Ning, 2006). Hence, Yeoh and Chua (2017), Marzouk and Abubakr (2016), Wu *et al.* (2020), Riga *et al.* (2020) and Dienstknecht (2022) tried to optimally determine the number and locations of required cranes in a site.

Tower cranes load materials from supply points and then unload them at demand points. Due to capacity limitations, they travel between these points several times, which extends construction time (Sugimoto *et al.*, 2016). Employing cranes with greater capacity and quicker movement speed is a possible solution (Farajmandi *et al.*, 2020). However, this approach can increase construction costs (Moussavi Nadoushani *et al.*, 2017; Chang *et al.*, 2012). Reducing the operating radius can be an affordable solution because the lifting capacity increases with a reduced operating radius (Amiri *et al.*, 2023). For these reasons, Moussavi Nadoushani *et al.* (2017), Huang and Wong (2018), Ji and Leite (2020) and Marzouk and Abubakr (2016) considered the types of tower cranes in TCLP.

In TCLP, the number and characteristics of supply points – such as capacity, location and material type – affect crane performance (Amiri *et al.*, 2023). Each of the supply points has a certain capacity to provide materials during a workday. Hence, Lien and Cheng (2014) considered the capacity limitation of supply points in TCLP. However, the location of the supply points was neglected by them. Reducing the distance between supply and demand points can reduce the materials transportation time (Riga *et al.*, 2020). Accordingly, Wang *et al.* (2015), Tam *et al.* (2001) (Tam and Tong, 2003) and Amiri *et al.* (2023) determined the location of crane(s) and the location of supply point(s) simultaneously. In these studies, the number of supply points is predetermined. On the one hand, increasing the number of supply points can reduce the distance between supply points and demand. On the other hand, considering a large number of supply points is not possible due to site space limitations. Hence, Huang *et al.* (2011), Moussavi Nadoushani *et al.* (2017), Ji and Leite (2020) and Riga *et al.* (2020) determined the number of supply points in the optimization process. However, in the mentioned studies, the capacity, number and location of supply points and types of materials have not been considered simultaneously.

The use of tower cranes in construction projects involves various safety risks, such as collision with other cranes or objects on the site (Kang and Miranda, 2006; Ali *et al.*, 2021; Gharaie *et al.*, 2015; U.S. Bureau of Labor, 2014). When multiple tower cranes operate in the same vicinity, the lower-height crane jib might collide with the higher-height crane mast or hook (Wu *et al.*, 2020). The overlapping of tower cranes in large construction projects seems inevitable (Al Hattab *et al.*, 2018). To mitigate overlapping risks, researchers have proposed different methods, such as setting a safe distance between tower cranes (Riga *et al.*, 2020; Zhang *et al.*, 1999), simulating tower crane motions (Irizarry and Karan, 2012; Wang *et al.*, 2015; Marzouk and Abubakr, 2016; Leite *et al.*, 2016), minimizing overlapping activities (Ji and Leite, 2020), choosing safe paths of movement (Xiao Lin *et al.*, 2023; Al Hattab *et al.*, 2018; Chang *et al.*, 2012; Olearczyk *et al.*, 2014) and adjusting of tower cranes height (Wu *et al.*, 2020). These approaches, however, have not considered all safety factors simultaneously and have paid less attention to the dangers of the movement path and the prohibited paths.

The installation location of tower cranes can have a great effect on both the project's safety and its cost. For instance, employing a crane near water pipelines may lead to the seepage of water through the pipes and affect the stability of the crane foundation (Almasabha *et al.*, 2024). At these locations, the cost of the tower crane foundation may also increase. Moreover, the installation of a tower crane on unstable slopes may lead to serious accidents, such as crane collapses, causing extensive damage. In order to mitigate these challenges, the use of slope detection systems (Shehadeh *et al.*, 2024a), high-strength concrete for crane foundations and predictive modeling of the crane foundation's structural behavior (Alshboul *et al.*, 2024) can help prevent damage caused by crane collapses. In addition, the crane foundation can be safeguarded and consolidated by shoring and reshoring methods (Shehadeh *et al.*, 2024b). Besides, equipment management and forecast of the crane accident safety hazards can be achieved with the assistance of expert systems (Shehadeh *et al.*, 2024c). Vehicle collision is also a kind of potential risk factor for tower cranes. As one of the most significant causes, driving at a high speed (Almadi *et al.*, 2023), therefore, should avoid setting up tower cranes near those high-speed limit roads.

Reducing project costs is one of the paramount objectives that have been considered in the utilization of tower cranes. Most research in the past has reduced the overall operating time of tower cranes to decrease construction costs. However, this idea may lead to an uneven distribution of lifting tasks among the tower cranes, potentially leading to an extension of workdays. An increase in workdays often increases costs such as renting construction equipment and labor wages. Therefore, through the balanced distribution of activities among tower cranes, working days and associated costs can be reduced. Moreover, expeditious project completion frequently yields economic benefits for investors.

Accurate calculation of tower crane operations time can be effective in their efficiency. The operating time of each tower crane depends on factors such as the speed of its movements, the type and weight of the transported load, the path of the crane, as well as the loading and unloading times for materials (Zhang *et al.*, 2022). Zhang *et al.* (1996) divided tower crane movements into radial, angular and vertical movements to calculate material transport time. This formula did not adequately consider the coordination between different movements and speed fluctuations (Hu *et al.*, 2021). Huang *et al.* (2011) and Huang and Wong (2018) added coordination between tower crane movements and material loading and unloading time to this formula, respectively. The loading and unloading time for each material can vary, but this aspect was not considered in the Huang and Wong (2018) formula.

In general, most of the common methods of tower crane deployment (e.g. Ji and Leite, 2020; Tam *et al.*, 2001; Huang *et al.*, 2011; Lien and Cheng, 2014; Tam and Tong, 2003; Huang and Wong, 2018; Moussavi Nadoushani *et al.*, 2017; Amiri *et al.*, 2023; Wang *et al.*, 2015; Yeoh and Chua, 2017; Younes and Marzouk, 2018) have often overlooked affecting factors, such as obstacles and hazards that might exist on the path of the move, the weight and volume of transported materials, the loading and unloading time for each material, and the visibility and skill of operators. This can lead to inaccurate estimates of the number of workdays and project costs. Despite the large number of introduced methods, all the main factors have not been covered adequately in a holistic method. This paper tries to propose a method that, in addition to determining the optimal solution for the number, type, location, and height of cranes, can determine the location of supply points in a complex construction site.

The proposed MILP considers more parameters for layout optimization of tower cranes and supply points, offering a more comprehensive and realistic approach than previous studies. The availability of required data is considered in the design of the proposed method. Some key data is directly received from the user and then the required parameters are calculated based on those. The efficiency and solvability of the proposed method are shown with a real-world case study. The proposed method effectively addresses key engineering challenges while remaining practical and implementable.

In the following section, most related works are first categorized and then reviewed.

2. Literature review

Table 1 tries to provide an overview of the existing body of knowledge in this area based on developed methods, operation time, overlapping and supply points. In Table 1, related works are first categorized based on the number of modeled cranes which is either single/multiple or a decision variable. All cells were marked by ✔ reflects that the corresponding paper considers the associated parameter mentioned in the corresponding column.

Table 1. Comparison of tower crane layout planning methods

	Tower crane			Supply points			Overlapping		Operation time		Objective function					
											Costs				Penalties	
Method	Number	Type	Capacity	Number	Location	Capacity	Distance	Height	Load/ unload	Obstacles	Operation	Rental	Salary	Fix	Daily	Overlapping
Zhang *et al.* (1996)	Single										✓					
Zhang *et al.* (1999)	Multiple						✓				✓					
Tam *et al.* (2001)	Single				✓						✓					
Tam and Tong (2003)	Single				✓						✓					
Huang *et al.* (2011)	Single			✓	✓						✓					
Lien and Cheng (2014)	Multiple					✓					✓					
Moussavi Nadoushani *et al.* (2017)	Single		✓	✓	✓						✓	✓				
Yeoh and Chua (2017)	Variable	✓	✓								✓			✓		
Ji and Leite (2020)	Multiple	✓	✓	✓	✓						✓					
Wu *et al.* (2020)	Variable	✓					✓	✓	✓		✓	✓	✓	✓		
Riga *et al.* (2020)	Variable	✓	✓	✓	✓		✓				✓	✓	✓	✓		
Dienstknecht (2022)	Variable	✓	✓				✓	✓				✓	✓	✓		
Wang *et al.* (2015)	Multiple		✓		✓	✓					✓					

(continued)

Table 1. Continued

	Tower crane			Supply points			Overlapping		Operation time		Objective function					
									Load/		Costs				Penalties	
Method	Number	Type	Capacity	Number	Location	Capacity	Distance	Height	unload	Obstacles	Operation	Rental	Salary	Fix	Daily	Overlapping
Marzouk and Abubakr (2016)	Variable	✓	✓											✓		
Huang and Wong (2018)	Single		✓						✓		✓					
Dasović *et al.* (2019)	Single		✓		✓						✓					
Amiri *et al.* (2023)	Single	✓	✓		✓						✓	✓	✓	✓		
Huang *et al.* (2024)	Multiple	✓					✓					✓	✓	✓		
Fu *et al.* (2024)	Multiple	✓	✓				✓		✓		✓	✓		✓		
Liu *et al.* (2022)	Multiple		✓	✓	✓	✓					✓					
Li *et al.* (2023)	Single		✓						✓	✓	✓					
Li *et al.* (2024)	Single	✓	✓						✓	✓	✓					
Proposed model	Variable	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓

Source(s): Authors' own work

As can be seen in Table 1, all affecting parameters have not been adequality considered simultaneously. Now the most related works are reviewed to justify the novelty of the proposed method in this paper. Zhang *et al.* (1996) introduced a MILP model for sites with a single tower crane that led to up to 40% save of tower crane time. In their model, radial, angular and vertical movements were considered in travel time calculation. In a similar study, Zhang *et al.* (1999) improved Zhang *et al.* (1996) by simultaneously locating multiple tower cranes, creating a safe distance between them and distributing the workload among the tower cranes. Tam *et al.* (2001) introduced a TCLP model that could provide the optimum solution for locating a single tower crane as well as supply points. Tam and Tong (2003) proposed a combination of neural networks with a genetic algorithm to solve a single TCLP heuristically. Huang *et al.* (2011) introduced a similar work to Tam *et al.* (2001) and Tam and Tong (2003) but considered supply points as a decision variable. Lien and Cheng (2014) enhanced the previously reviewed works by covering operational costs (such as labor, rent and assembly) and the capacity of each supply point for only construction material. Moussavi Nadoushani *et al.* (2017) improved Huang *et al.* (2011) by considering tower cranes with different capacities.

Ji and Leite (2020) proposed a mixed integer programming (MIP) model to simultaneously locate tower cranes and supply points while reducing the possibility of tower cranes crashing. Riga *et al.* (2020) tried to reduce the overlapping problems of tower cranes by creating a safe distance between cranes. Wu *et al.* (2020) introduced a meta-heuristic algorithm to optimally adjust the height of tower cranes while maintaining a safe distance between them. Dienstknecht (2022) solved the problem similarly to Wu *et al.* (2020) but with a branch-and-bound approach. Huang and Wong (2018) developed a MILP model to determine the optimal location of a tower crane based on urgent material requests by considering loading and unloading time.

Using the building information model (BIM), the operation of tower cranes can be simulated before implementation (Astour and Franz, 2014). This approach can avoid the collision of tower cranes (Bryde *et al.*, 2013). Dasović *et al.* (2019) used BIM to optimize the location of supply points and the positioning of a single tower crane. By combining the firefly algorithm and BIM, Wang *et al.* (2015) tried to reduce safety risks in TCLP. Similarly by hiring genetic algorithm and BIM, Marzouk and Abubakr (2016) optimized the number and type of tower cranes while enhancing project safety. The demand for materials in different stages of construction is significantly different, which may require crane relocation during construction. Yeoh and Chua (2017) introduced a MILP model that could optimally determine the type and number of tower cranes for each phase of a construction project.

Recently, Li *et al.* (2023) applied deep learning methods in order to optimize the location of a single tower crane. In another similar work (Li *et al.*, 2024), they also used deep learning to determine the location and type of single tower crane, by taking into consideration safety factors such as site obstacles, operator visibility and the soil-bearing capacity at the crane installation site. Meanwhile, Amiri *et al.* (2023) introduced an ILP model aimed at simultaneously determining the location of the single tower crane and supply points to prevent the use of an over-capacity tower crane. Fu *et al.* (2024) applied a genetic algorithm to find the best locations for two tower cranes on a construction site. Inspired by the student competition, Shehadeh *et al.* (2024b) developed a new heuristic algorithm to identify the optimal locations for multiple tower cranes. Additionally, Liu *et al.* (2022) used the firefly algorithm to find the locations of the tower cranes and supply points simultaneously.

In summary, the TCLP problem seeks optimum decisions about the number, type and location of tower cranes as well as the number and location of supply points. The capacity of cranes, the overlap of cranes and costs such as operation, rental, maintenance, installation and uninstallation of tower cranes, and the operator's salary are the most effective parameters. Due to the high complexity of the TCLP problem, covering all the effective factors has not been simultaneously mathematically modeled. In addition, adjusting the height of tower cranes and optimizing the number of workdays have received less attention. This paper proposed a novel

MILP model to tackle TCLP while all aforementioned factors are considered. The proposed model also could optimally determine the number of workdays. Adjusting the height of tower cranes, creating a safe distance between them and removing blocked paths are other features of the proposed model. Another main novelty of this work is optimally determining safe transportation paths for each crane.

3. Proposed model

In this section, the proposed method is described in detail. This paper aims to propose a method that could supply the optimum solution for entire vertical transportation in a site (tower crane (s) layout and supply point(s)) while considering overlapping between tower cranes as well as obstacle avoidance. For this purpose, a MILP model is developed based on the following assumptions:

(1) Loading and unloading tasks take considerable time in practice for a wide range of reasons, so unlike some related works, it cannot be ignored.

(2) The speed of transportation materials is not equal between different tower cranes.

(3) Movement paths must be safe to avoid collisions.

(4) Candidate locations of supply points are predetermined. The proposed model selects the optimal among the candidate points.

(5) At each supply point, based on the available space and the type of materials, a limited amount of materials can be stored. It is possible to store several types of materials in one supply point.

(6) Candidate locations for installing tower cranes are predetermined. The proposed model selects the optimum locations among the candidates.

(7) A point can simultaneously be a candidate for creating a supply point and installing a tower crane.

(8) Tower cranes are divided into different types in terms of movement speed, lifting capacity, jib length, height, and costs of rental, maintenance and installation. The proposed model selects the optimum tower crane for each location.

(9) The operation of tower cranes by the same height in overlapping areas may result in their jib colliding. So, operating tower cranes with the same height in overlapping areas is prohibited.

(10) When a higher-height tower crane loads or unloads materials in overlapping areas, the movement path of the lower-height tower crane is blocked in that area.

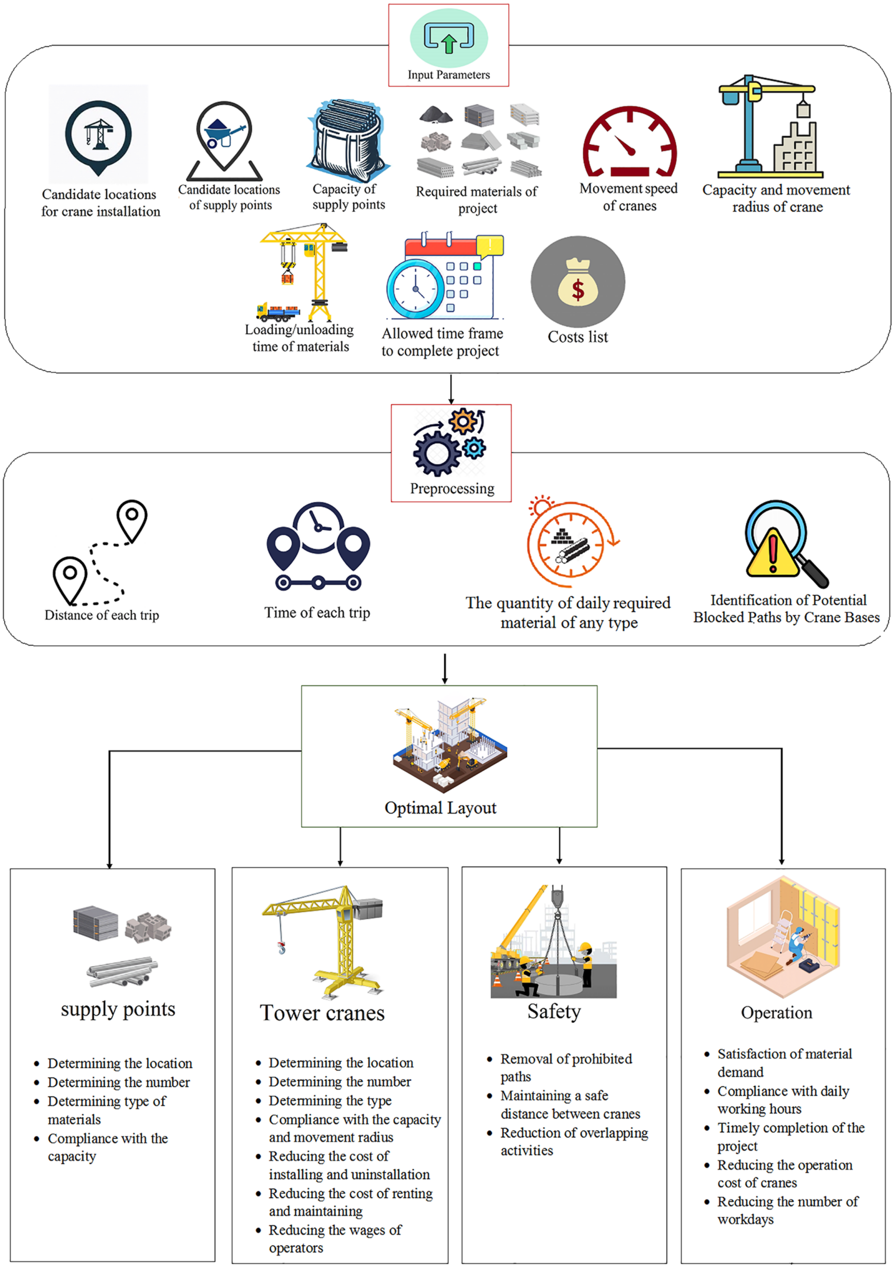
(11) The mast of a higher-height tower crane permanently blocks some of the movement paths of a lower-height tower crane.

(12) The location of demand points and the type and amount of materials required for each of these points are predetermined. The amount of daily required materials is determined based on the number of workdays of the project. Workdays are estimated based on the most time-consuming task.

(13) The activities assigned to each tower crane should be less than the daily working hour’s limit which can be different among the candidates.

(14) In case of a delay, the project will be postponed by applying a penalty function.

Figure 1 illustrates the flowchart of the proposed method.



Source(s): Author’s own work

Figure 1. Flowchart of the proposed method

3.1 Notation and symbols

The description of the parameters and variables of the proposed model are listed in Table 2.

Table 2. Description of the parameters and variables of the proposed model

Sets	Description
Indices	
I	Set of all candidate locations of supply points defined by their global coordinates (Indice i)
J	Set of users' predetermined locations of demand points defined by their global coordinates (Indice j)
M	Set of all required materials, where each element m represents a specific type of the materials to be transported
K	Set of all predetermined tower crane types with their characteristics (Indice k)
L	Set of all predetermined locations for placing tower cranes, where each element l corresponds to a potential installation site for a tower crane
Parameters	
$\overline{M}$	Big constant
T_m^{Load}	Loading time of material m in each travel
T_m^{Unload}	Unloading time of material m in each travel
$T_{i,j,m,k,l}^h$	Vertical movement time of crane k at location l to transfer material m from supply point i to demand point j
$T_{i,j,m,k,l}^r$	Radial movement time of crane k at location l to transfer material m from supply point i to demand point j
$T_{i,j,m,k,l}^{\omega}$	Tangential movement time of crane k at location l to transfer material m from supply point i to demand point j
$T_{i,j,m,k,l}^v$	Horizontal movement time of crane k at location l to transfer material m from supply point i to demand point j
$T_{i,j,m,k,l}^t$	The transfer time of material m from supply point i to demand point j by crane k at location l
$T_{k,l}^{Working}$	The allowed daily working hours of crane k at location l
$V_{i,j,m,k,l}^h$	Vertical movement speed of crane k at location l to transfer of material m from supply point i to demand point j
$V_{i,j,m,k,l}^r$	Radial movement speed of crane k at location l to transfer of material m from supply point i to demand point j
$V_{i,j,m,k,l}^{\omega}$	Tangential movement speed of crane k at location l to transfer of material m from supply point i to demand point j
$\Delta_{i,j,m,k,l}^h$	Vertical distance traveled by crane k at location l to transport material m from supply point i to demand point j
$\Delta_{i,j,m,k,l}^r$	Radial distance traveled by crane k at location l to transport material m from supply point i to demand point j
$\varphi_{i,j,l}$	Amount of angular changes of installed crane jib at location l to travel from supply point i to demand point j
S_i^z	Coordinates of supply point i on the Z axis (height)
D_j^z	Coordinates of demand point j on the Z axis (height)
$EM_{i,j,m,k,l}^h$	Extra vertical distance traveled by crane k at location l to transport material m from supply point i to demand point j due to the presence of obstacles and safety considerations
$EM_{i,j,m,k,l}^r$	Extra Radial distance traveled by crane k at location l to transport material m from supply point i to demand point j due to the presence of obstacles and safety considerations
$\rho(S_i, Cr_l)$	Horizontal distance of supply point i from candidate location l
$\rho(D_j, Cr_l)$	Horizontal distance of demand point j from candidate location l
$\rho(S_i, D_j)$	Horizontal distance of supply point i from demand point j
$\rho(l, l')$	The horizontal distance between two locations l and l'
$\alpha_{i,j,m,k,l}$	A continuous parameter between 0 and 1 that indicates the coordination degree of radial movement and tangential movement of crane k at location l in the transfer of material m from supply point i to demand point j (0 indicates simultaneous movement and 1 indicates sequential movement)
$\beta_{i,j,m,k,l}$	A continuous parameter between 0 and 1 that indicates the coordination degree of horizontal movement and vertical movement of crane k at location l in the transfer of material m from supply point i to demand point j (0 indicates simultaneous movement and 1 indicates sequential movement)
ND^{min}	Minimum number of required workdays to complete the project
ND^{max}	Maximum number of allowed workdays
$\overline{d}_{j,m}$	The total demand of point j to material m

(*continued*)

Table 2. Continued

Sets	Description
$\overline{\overline{d}}_{j,m}$	The maximum daily consumption of material *m* at demand point *j*
$d_{j,m}$	The daily requirement of demand point *j* to material *m*
$\mu_{i,m}^{max}$	The maximum daily suppliable capacity of material *m* at supply points *i*
$U^{Supply\,point}$	Maximum allowed number of supply points
$\overline{U}^{Supply\,point}$	The maximum number of materials that can be supplied at each of the supply points
U^{crane}	The number of allowed cranes
$U_{i,j,m,k,l}$	The capacity of crane *k* at location *l* in transferring material *m* from supply point *i* to demand point *j*
$U_{k,l,k',l'}^{Overlap}$	The allowed overlap radius of crane *k* at location *l* and crane k' at location l'
$O_{k,l,k',l'}$	The overlap amount of cranes *k* and *k*' if installed in locations *l* and l'
R_k	Radius of crane *k*
$h_{k,l,k',l'}$	A binary parameter that is equal to 1 if the height of crane k' at location l' is less than crane *k* at location *l*, and is equal to 0 otherwise
$\tau_{i',j',k',l',j}$	A binary parameter that is equal to 1 if point *j* is in the movement path of crane k' at location l' from point i' to point j' and is equal to 0 otherwise
$\overline{\tau}_{i',j',k',l',i}$	A binary parameter that is equal to 1 if point *i* is in the movement path of crane k' at location l' from point i' to point j' and is equal to 0 otherwise
$\overline{\overline{\tau}}_{i,j,k,l,l'}$	A binary parameter that is equal to 1 if the jib of crane *k* at location *l* passes through the location l' while traveling from point *i* to point *j*, and is equal to 0 otherwise
Integer variables	
$\Lambda_{i,j,m,k,l}$	The number of times crane *k* at location *l* has transported material *m* from supply point *i* to demand point *j* ($\Lambda_{i,j,m,k,l} \in \mathbb{N} \cup \{0\}$)
Ω_i	A binary variable that is equal to 1 if the candidate location *i* is selected as the supply point and is equal to 0 otherwise ($\Omega_i \in \{0,1\}$)
$\overline{\Omega}_{i,m}$	A binary variable that is equal to 1 if material *m* is stored in supply point *i* and is equal to 0 otherwise ($\overline{\Omega}_{im} \in \{0,1\}$)
$\lambda_{k,l}$	A binary variable that is equal to 1 if the crane of type *k* is assigned to the candidate location *l*, otherwise is equal to 0 ($\lambda_{k,l} \in \{0,1\}$)
$y_{j,k,l}^{Unload}$	A binary variable that is equal to 1 if a crane with a height lower than the height of crane *k* unloads materials at demand point *j* and is equal to 0 otherwise ($y_{j,k,l}^{Unload} \in \{0,1\}$)
$y_{i,k,l}^{Load}$	A binary variable that is equal to 1 if a crane with a height less than the height of crane *k* loads material from supply point *i* and is equal to 0 otherwise ($y_{i,k,l}^{Load} \in \{0,1\}$)
$y_{i,j,k,l}^{cross}$	A binary variable that is equal to 1 if unloading or loading is done by a crane with a height higher than crane *k* at location *l* on the travel path from point *i* to point *j* and is equal to 0 otherwise ($y_{i,j,k,l}^{cross} \in \{0,1\}$)
$\theta_{i,j,k,l}$	A binary variable that is equal to 1 if higher-height cranes are in the movement path of crane *k* at location *l* from supply point *i* to demand point *j* and is equal to 0 otherwise ($\theta_{i,j,k,l} \in \{0,1\}$)
Continuous variables	
$T_{i,j,m,k,l}$	The total time spent by crane *k* at location *l* in transporting material *m* from supply point *i* to demand point *j* ($T_{i,j,m,k,l} \in \mathbb{R}^0$)
$T_{k,l}^{crane}$	The operation time of crane *k* at location *l* ($\overline{T}_{j,k,l}^{Unload} \in \mathbb{R}^0$)
$\overline{T}_{j,k,l}^{Unload}$	The total material unloading time at point *j* by crane *k* at location *l* ($\overline{T}_{j,k,l}^{Unload} \in \mathbb{R}^0$)
$\overline{T}_{i,k,l}^{Load}$	The total material loading time from point *i* by crane *k* at location *l* ($\overline{T}_{i,k,l}^{Load} \in \mathbb{R}^0$)
$x_{i,j,m,k,l}$	The amount of material *m* transported by crane *k* at location *l* from supply point *i* to demand point *j* ($x_{i,j,m,k,l} \in \mathbb{R}^0$)
ND	Number of workdays of the project ($ND^{min} \leq ND \leq ND^{max}$).
$\mu_{i,m}$	The amount of material *m* that is loaded from supply point *i* ($\mu_{i,m} \in \mathbb{R}^0$)

Source(s): Authors' own work

3.2 Tower crane operation time

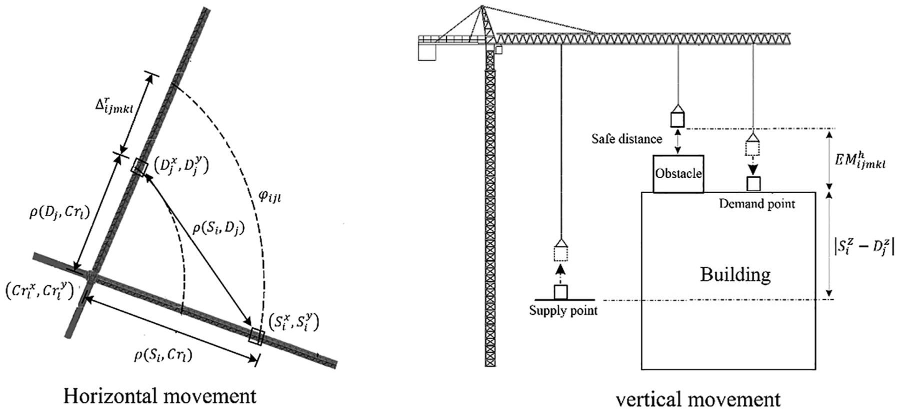
Tower crane operation mainly includes loading, transferring and unloading (Wu *et al.*, 2020). Based on this, Equation (1) calculates the tower crane operation time ($T_{i,j,m,k,l}$) (including the time of loading (T_m^{Load}), transporting ($T_{i,j,m,k,l}^t$), and unloading (T_m^{Unload}) materials and crane return time to supply point ($T_{i,j,k,l}^{Return}$)) for each of the tasks.

$$T_{i,j,m,k,l} = \left(T_m^{Load} + T_{i,j,m,k,l}^t + T_{i,j,k,l}^{Return} + T_m^{Unload}\right)\Lambda_{i,j,m,k,l}\ \forall i \in I, \forall j \in J, \forall m \in M, \forall k \in K, \forall l \in L \quad (1)$$

Loading and unloading are a time-consuming process in a construction project due to safety concerns (Wu and García de Soto, 2020). In practice, as it was observed in the field, the durations of loading and unloading depend on the site machinery as well as the type and packaging of materials (Huang *et al.*, 2021; Wu and García de Soto, 2020). So, might loading/unloading time between two sites with the exact same crane be different. As can be seen in Table 1, most similar studies have not taken loading/unloading time into account except a few studies (such as Huang and Wong, 2018; Wu *et al.*, 2020) that have considered loading/unloading time as a constant parameter. In this study, to cover this issue and provide a more practical solution for TCLP, loading and unloading times are calculated based on each site's features and materials specifications as reflected in Equation (1). It has been observed in a site that ignores loading/unloading time in the lift plan could lead to a compressed schedule or over-capacity problems.

The material transfer time ($T_{i,j,m,k,l}^t$) is calculated by dividing the distance of the tower crane's movement by its movement speed (Ji and Leite, 2020). Crane movements consist of vertical and horizontal movements (Huang *et al.*, 2021) (as shown in Figure 2). Another novelty of this paper is considering physical obstacles in the calculation of transfer time.

The tower crane's movement speed is affected by various factors, such as the tower crane type, the operator's skill the construction site characteristics (e.g. the operator's visibility and the potential obstacles and hazards in the movement path), the material weight and volume, and the distance between the crane and the supply and demand points. The crane's horizontal movement includes trolley radial movement and jib tangent movement (Figure 2). The amount of crane movements is determined based on the coordinates of the crane location and the coordinates of supply and demand points.



Source(s): Author's own work

Figure 2. Tower crane movements

However, cranes often have to perform extra movements due to obstacles in the movement path and safety precautions (Figure 2) which have not been adequately addressed in similar works. To tackle this issue, this paper proposed Equations (2) to (10) to calculate the crane movement time more realistically.

The vertical movement time ($T^h_{i.j.m.k.l}$) is calculated similarly to Ji and Leite (2020), Amiri *et al.* (2023), Wu *et al.* (2020) from Equation (2).

$$T^h_{i,j,m,k,l} = \frac{\Delta^h_{i,j,m,k,l}}{V^h_{i,j,m,k,l}} \tag{2}$$

Equation (3) is proposed to calculate the vertical movement distance ($\Delta^h_{i,j,m,k,l}$). This formula considers the extra vertical movement (EM^h_{ijmkl}) that the crane performs due to possible obstacles and safety concerns.

$$\Delta^h_{i,j,m,k,l} = \left|S^z_i - D^z_j\right| + EM^h_{i,j,m,k,l} \tag{3}$$

Similar to Ji and Leite (2020), Amiri *et al.* (2023), Wu *et al.* (2020), Equation (4) calculates radial movement time ($T^r_{i,j,m,k,l}$).

$$T^r_{i,j,m,k,l} = \frac{\Delta^r_{i,j,m,k,l}}{V^r_{i,j,m,k,l}} \tag{4}$$

Equation (5) is suggested to calculate the amount of radial movement ($\Delta^r_{i,j,m,k,l}$).

$$\Delta^r_{i,j,m,k,l} = \left|\rho(S_i, Cr_l) - \rho\left(D_j, Cr_l\right)\right| + EM^r_{i,j,m,k,l} \tag{5}$$

The tangential movement time ($T^\omega_{i,j,m,k,l}$) is calculated similarly to Ji and Leite (2020), Amiri *et al.* (2023), Wu *et al.* (2020) from Equation (6).

$$T^\omega_{i,j,m,k,l} = \frac{\varphi_{i,j,l}}{V^\omega_{i,j,m,k,l}} \tag{6}$$

According to the law of cosines (Ding, 2008) for the triangle of Figure 2 we have:

$$\rho\left(S_i, D_j\right)^2 = \rho(S_i, Cr_l)^2 + \rho\left(D_j, Cr_l\right)^2 - 2\rho(S_i, Cr_l)\rho\left(D_j, Cr_l\right)\cos\ (\varphi) \tag{7}$$

$\varphi_{i,j,l}$ is obtained from Equation (8), Ji and Leite (2020), Amiri *et al.* (2023), Wu *et al.* (2020).

$$\varphi_{i,j,l} = ArcCos\left(\frac{\rho(S_i, Cr_l)^2 + \rho\left(D_j, Cr_l\right)^2 - \rho\left(S_i, D_j\right)^2}{2\rho(S_i, Cr_l)\rho\left(D_j, Cr_l\right)}\right) 0 \le \varphi_{i,j,l} \le \pi \tag{8}$$

The horizontal movement, including radial movement and tangential movement, can be performed simultaneously according to path characteristics, load type, tower crane type and operator's skill. Accordingly, similar to Ji and Leite (2020), Amiri *et al.* (2023), Wu *et al.* (2020), Equation (9) calculates horizontal movement time ($T^v_{i,j,m,k,l}$).

$$T^v_{i,j,m,k,l} = max\left(T^r_{i,j,m,k,l}, T^\omega_{i,j,m,k,l}\right) + \alpha_{i,j,m,k,l}\, min\left(T^r_{i,j,m,k,l}, T^\omega_{i,j,m,k,l}\right) \tag{9}$$

Horizontal movement and vertical movement can also be done simultaneously. Hence, the materials transfer time ($T^{t}_{i,j,m,k,l}$) is calculated similarly to Ji and Leite (2020), Amiri *et al.* (2023), Wu *et al.* (2020) from Equation (10).

$$T^{t}_{i,j,m,k,l} = max\left(T^{h}_{i,j,m,k,l}, T^{v}_{i,j,m,k,l}\right) + \beta_{i,j,m,k,l}\, min\left(T^{h}_{i,j,m,k,l}, T^{v}_{i,j,m,k,l}\right) \quad (10)$$

Equation (11) calculates the operation time of each of the installed tower cranes ($T^{crane}_{k,l}$).

$$T^{crane}_{k,l} = \sum_{i\in I}\sum_{j\in J}\sum_{m\in M} T_{i,j,m,k,l}\ \forall k\in K, \forall l\in L \quad (11)$$

3.3 Materials required for demand points

The construction of a building requires a wide range of materials. The total demand for each material ($\overline{d}_{j,m}$) can be estimated before the construction commencement and distributed equally among the project workdays. Based on this, Equation (12) calculates the daily required materials of demand points ($d_{j,m}$).

$$d_{j,m} = \frac{\overline{d}_{jm}}{ND^{min}} \quad (12)$$

ND^{min} represents the minimum required workdays to complete the project. In estimating the shortest possible time (ND^{min}) to complete the project, it is assumed that the cranes will not add any delay to the construction. ND^{min} is calculated solely based on the construction velocity at the demand points, which can differ from each other due to factors such as the number of workers, site limitations and available equipment. The proposed Equation (13) calculates ND^{min} based on the most time-consuming project area.

$$ND^{min} = max\left\{\frac{\overline{d}_{j,m}}{\overline{\overline{d}}_{j,m}}\right\} \quad (13)$$

In Equation (13) the maximum daily consumption of materials ($\overline{\overline{d}}_{j,m}$) for each demand point can be considered different based on the effective factors mentioned above.

Similar to Ji and Leite (2020), Equation (14) ensures the satisfaction of the material demand.

$$\sum_{i\in I}\sum_{k\in K}\sum_{l\in L} x_{i,j,m,k,l} = d_{j,m}\ \forall j\in J, \forall m\in M \quad (14)$$

3.4 Number of workdays

The project can be completed in the shortest possible time (ND^{min}) if cranes fully satisfy the daily demand calculated from Equation (12). Due to the daily working hours ($T^{Working}_{k,l}$) limit, cranes may not be able to deliver $d_{j,m}$. To fulfill the promised d_{jm} can either increase the number of cranes or use cranes with more capacity which both are costly options. So, delaying the delivery of materials and increasing the number of workdays can be a more economical approach in some cases. To tackle this practical problem, this paper includes Equation (15) in the formulation which permits a slight delay in the delivery of some materials while respecting the daily working hours ($T^{Working}_{k,l}$) limit. Based on this, Equation (15) calculates workdays (ND) according to possible delays.

$$ND^{min} \times T_{k,l}^{crane} \leq ND \times T_{k,l}^{Working} \; \forall k \in K, \forall l \in L \tag{15}$$

In the calculation of allowed daily working hours ($T_{k,l}^{Working}$), it is possible to consider things such as unpredictable activities and possible blockage of some paths during the day due to the presence of temporary equipment.

The number of workdays (ND) cannot be less than ND^{min}. Moreover, contractors should respect the time stated in work contracts for the delivery of buildings. Constraint (16) is added to consider these two issues.

$$ND^{min} \leq ND \leq ND^{max} \tag{16}$$

The increase in workdays caused by the delay in delivery of materials by tower cranes ($ND - ND^{min}$) is penalized in the objective function (Equations 36 and 43).

3.5 Supply points

In this section, a novel mathematical model is presented which is in charge of optimally determining:

(1) Number of supply points

(2) Location of each supply point

(3) Type of material at each supply point

Equation (17) selects optimal supply points among candidates.

$$\sum_{m \in M} \frac{\mu_{i,m}}{\mu_{i,m}^{max}} \leq \Omega_i \leq \sum_{m \in M} \mu_{i,m} \; \forall i \in I \tag{17}$$

In Equation (17), if location i is selected as the supply point ($\Omega_i = 1$), then phrase $\sum_{m \in M} \frac{\mu_{i,m}}{\mu_{i,m}^{max}} \leq 1$ takes into account the capacity of supply points. Equation (18) determines the type and quantity of materials stored in each of the supply points ($\mu_{i,m}$).

$$\sum_{j \in J} \sum_{k \in K} \sum_{l \in L} x_{i,j,m,k,l} \leq \mu_{i,m} \; \forall i \in I, m \in M \tag{18}$$

Due to the limitations of the construction site, it is not possible to create a large number of supply points. Equation (19) considers this limitation.

$$\sum_{i \in I} \Omega_i \leq U^{Supply\,point} \tag{19}$$

Equation (20) specifies materials types stored in supply points.

$$\frac{\mu_{i,m}}{\mu_{i,m}^{max}} \leq \overline{\Omega}_{i,m} \leq \mu_{i,m} \; \forall i \in I, m \in M \tag{20}$$

A limited number of available materials can be stored at each supply point, a factor that is incorporated into Equation (21)

$$\sum_{i \in I} \sum_{m \in M} \overline{\Omega}_{i,m} \leq \overline{U}^{Supply\,point} \tag{21}$$

3.6 Type, location and number of tower cranes

Constraints (22) to (24) determine the location, type and number of required tower cranes. Equation (22) determines the best location from the possible options and assigns the suitable crane to each location.

$$\frac{1}{\overline{M}}\sum_{i\in I}\sum_{j\in J}\sum_{m\in M}x_{i,j,m,k,l}\leq\lambda_{k,l}\leq\sum_{i\in I}\sum_{j\in J}\sum_{m\in M}x_{i,j,m,k,l}\quad\forall k\in K,\forall l\in L \tag{22}$$

To avoid assigning more than one tower crane to a location, Equation (23) is added.

$$\sum_{k\in K}\lambda_{k,l}\leq 1\quad\forall l\in L \tag{23}$$

Equation (24) considers an upper bound for the number of allocated cranes based on the site budget (U^{crane}).

$$\sum_{k\in K}\sum_{l\in L}\lambda_{k,l}\leq U^{crane} \tag{24}$$

3.7 Capacity and movement radius of tower cranes

Equation (25) ensures that tower cranes do not exceed their capacity and movement radius during operation.

$$x_{i,j,m,k,l}\leq\Lambda_{i,j,m,k,l}\times\Upsilon_{i,j,m,k,l}\leq x_{i,j,m,k,l}+\Upsilon_{i,j,m,k,l}\,\forall i\in I,\forall j\in J,\forall m\in M,\forall k\in K,\forall l\in L \tag{25}$$

Tower crane capacity ($\Upsilon_{i,j,m,k,l}$) depends on the tower crane load range diagram, the tower crane height and the soil-bearing capacity (Ali *et al.*, 2021). $\Upsilon_{i,j,m,k,l}$ is chosen equal to zero for supply and demand points outside the tower crane radius and prohibited movement paths. Moreover, some places may be joint candidates for creating a supply point and installing a tower crane. Therefore, $\gamma_{i,j,m,k,l}$ for candidate locations i and l with equal coordinates is considered zero.

3.8 Overlap of tower cranes

Tower cranes often have to work in overlapping areas in a construction project. This can compromise the project's safety and disrupt the tower cranes' operation. The proposed model aims to reduce the possible dangers and stops of cranes working in overlapping areas by maintaining a safe distance and adjusting their height. Equation (26) ensures the safe distance ($U^{Overlap}_{k,l,k',l'}$) between two cranes.

$$\lambda_{k,l}\times\lambda_{k',l'}\times O_{k,l,k',l'}\leq U^{Overlap}_{k,l,k',l'}\quad\forall k\in K,\forall l\in L,\forall k'\in K,\forall l'\in L,l\neq l' \tag{26}$$

The amount of overlap of tower crane k at location l and tower crane k' at location l' is calculated by Equation (27).

$$O_{k,l,k',l'}=R_k+R_{k'}-\rho(l,l') \tag{27}$$

Figure 3 illustrates how tower cranes work in overlapping areas. Whenever a lower-height crane starts loading or unloading materials in overlapping areas, higher-height cranes can pass through that area by pulling back its hook (Figure 3a). But when the higher-height crane is loading or unloading materials in overlapping areas, lower-height cranes cannot pass through that area (Figure 3b). In this case, the lower-height crane has to wait outside overlapping areas until the higher-height crane exits (Su *et al.*, 2012). This can disrupt the schedule of cranes. δ in Figure 3 represents the allowed vertical distance for the crane to pass on overlapping paths.

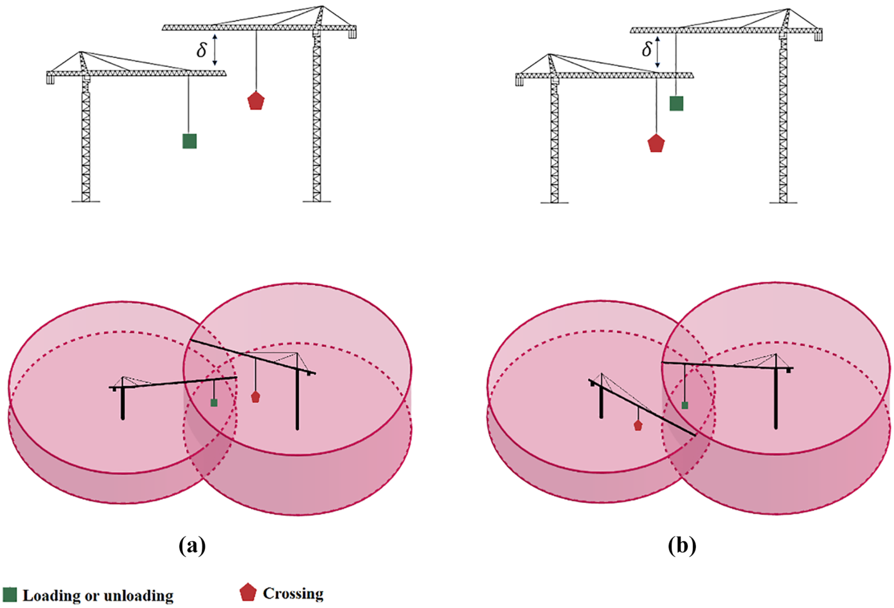


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Figure 3. Overlap of tower cranes

Equation (28) for each of the tower cranes specifies demand points where material unloading in them prevents the operation of other tower cranes.

$$y^{Unload}_{j,k,l} \geq \frac{\sum_{i'\in I}\sum_{j'\in J}\sum_{m'\in M}\sum_{k'\in K}\sum_{l'\in L} x_{i',j',m',k',l'} \times \tau_{i',j',k',l',j} \times h_{k,l,k',l'}}{\overline{M}} \qquad \forall j\in J, \forall k\in K, \forall l\in L \qquad (28)$$

where $y^{Unload}_{j,k,l}$ is a binary variable that is equal to 1 if a tower crane with a height less than the height of tower crane k at location l passes from point j and is equal to 0 otherwise. $h_{k,l,k',l'}$ is a binary parameter that is equal to 1 if the height of the tower crane k' at location l' is less than the tower crane k at location l, and is equal to 0 otherwise. $\tau_{i',j',k',l',j}$ is a binary parameter that is equal to 1 if point j is in the movement path of tower crane k' at location l' from point i' to point j' and in Otherwise, it is equal to 0.

Equation (29) for each of the tower cranes specifies supply points where material loading from them prevents the operation of other tower cranes.

$$y^{Load}_{i,k,l} \geq \frac{\sum_{i'\in I}\sum_{j'\in J}\sum_{m'\in M}\sum_{k'\in K}\sum_{l'\in L} x_{i',j',m',k',l'} \times \overline{\tau}_{i',j',k',l',i} \times h_{k,l,k',l'}}{\overline{M}} \qquad \forall i\in I, \forall k\in K, \forall l\in L \qquad (29)$$

Equation (30) and Equation (31) for each of the tower cranes indicate paths in which the higher-height tower crane has loaded or unloaded materials.

$$y^{cross}_{i,j,k,l} \geq \frac{\sum_{j'\in J}\sum_{k'\in K}\sum_{l'\in L} \overset{=}{T}^{Unload}_{j',k',l'} \times \tau_{i,j,k,l,j'} \times h_{k',l',k,l}}{\overline{M}} \qquad \forall i\in I, \forall j\in J, \forall k\in K, \forall l\in L \qquad (30)$$

$$y^{cross}_{i,j,k,l} \geq \frac{\sum_{i'\in I}\sum_{k'\in K}\sum_{l'\in L} \overset{=}{T}^{Load}_{i',k',l'} \times \bar{\tau}_{i,j,k,l,i'} \times h_{k',l',k,l}}{\overline{M}} \quad \forall i\in I, \forall j\in J, \forall k\in K, \forall l\in L \tag{31}$$

where $y^{cross}_{i,j,k,l}$ is a binary variable that is equal to 1 if unloading or loading is done by a tower crane with a height higher than tower crane k at location l on the travel path from point i to point j and is equal to 0 otherwise. $\overset{=}{T}^{Unload}_{j',k',l'}$ denotes the total material unloading time at point j' by tower crane k' at location l' which is calculated from Equation (32). $\overset{=}{T}^{Load}_{i',k',l'}$ is equal to the total material loading time from point i' by tower crane k' at location l' which is calculated from Equation (33).

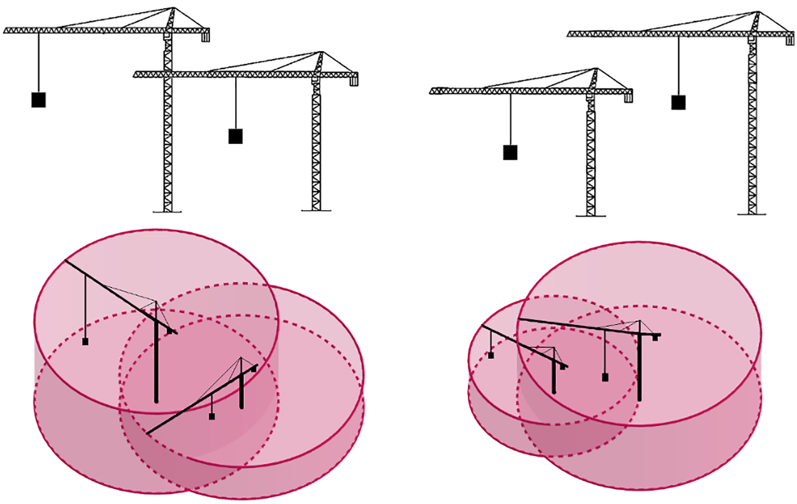
$$\overline{T}^{Unload}_{j,k,l} = \sum_{i\in I}\sum_{m\in M} T^{Unload}_{m} \times \Lambda_{i,j,m,k,l} \quad \forall j\in J, \forall k\in K, \forall l\in L \tag{32}$$

$$\overline{T}^{Load}_{i,k,l} = \sum_{j\in J}\sum_{m\in M} T^{Load}_{m} \times \Lambda_{i,j,m,k,l} \quad \forall i\in I, \forall k\in K, \forall l\in L \tag{33}$$

The mast of higher-height tower crane cans leads to the permanent blocking of some movement paths of lower-height tower cranes (Figure 4a), while mast of lower-height tower cranes does not disrupt the activity of other tower cranes (Figure 4b).

Equations (37) and (38) eliminate the prohibited paths created by the mast of higher-height tower cranes.

$$\lambda_{k',l'} \times \overset{=}{\tau}_{i,j,k,l,l'} \times h_{k',l',k,l} \leq \theta_{i,j,k,l} \tag{34}$$



(a) **(b)**

Source(s): Author's own work

Figure 4. Obstruction of movement paths by mast of higher-height tower cranes

$$\Lambda_{i,j,m,k,l} \leq \left(1 - \theta_{i,j,k,l}\right) \times \overline{M} \quad (35)$$

where $\overline{\overline{\tau}}_{i,j,k,l,l'}$ is a binary parameter that is equal to 1 if the jib of tower crane k at location l passes through the location l' and is equal to zero otherwise. $\theta_{i,j,k,l}$ is a binary variable that is equal to 1 if higher-height tower cranes are in the path of tower crane k at location l from i to j and is equal to 0 otherwise.

3.9 Objective function

The proposed model aims to reduce operational costs and safety risks. Hence, the objective function is defined as follows:

$$F = min(cost + safety) \quad (36)$$

where *cost* and *safety* denote operating costs and safety penalties, respectively. Operating costs are obtained from Equation (37).

$$cost = C^{Operation} + C^{Supply} + C^{Fix} + C^{Variable} + C^{wage} + C^{Delay} \quad (37)$$

where C^{Delay}, C^{wage}, $C^{Variable}$, C^{Fix}, C^{Supply} and $C^{Operation}$ are respectively equal to the delay penalty caused by cranes, the total operators wage, the total variable costs of cranes (such as rent and maintenance), the fixed cost of cranes (such as installation and uninstallation), the cost of creating supply points and the cost of operating cranes. Equation (38) calculates the operation cost of cranes based on material moving time.

$$C^{Operation} = ND^{min} \times \sum_{i \in I} \sum_{j \in J} \sum_{m \in M} \sum_{k \in K} \sum_{l \in L} \Lambda_{i,j,m,k,l} \times \left(T^{t}_{i,j,m,k,l} + T^{Returne}_{i,j,k,l}\right) \times \overline{C}^{Operation}_{k} \quad (38)$$

where $\overline{C}^{Operation}_{k}$ indicates the cost per minute of operation of crane k. Creating supply points cost is calculated from Equation (39).

$$C^{Supply} = \sum_{i \in I} \Omega_i \times \overline{C}^{Supply}_{i} \quad (39)$$

where $\overline{C}^{Supply}_{i}$ denotes the cost of creating supply point i. In Equation (39), $\overline{C}^{Supply}_{i}$ is mostly considered a small penalty to create as few supply points as possible to avoid unnecessary site congestion. Equation (40) calculates the fixed cost of tower cranes based on selected type and location.

$$C^{Fix} = \sum_{k \in K} \sum_{l \in L} \lambda_{k,l} \times \overline{C}^{Fix}_{k,l} \quad (40)$$

where $\overline{C}^{Fix}_{k,l}$ represents the fixed cost of tower crane k at location l. Costs such as the installation and uninstallation of tower cranes are considered fixed costs. Equation (41) calculates the variable cost of tower cranes according to the number of workdays.

$$C^{Variable} = \sum_{k \in K} \sum_{l \in L} \lambda_{k,l} \times ND \times \overline{C}^{Variable}_{k} \quad (41)$$

where $\overline{C}_k^{Variable}$ represents the daily variable cost of crane k. Costs such as renting and maintaining cranes are among the variable costs. Equation (42) calculates the wages of operators based on the number of workdays.

$$C^{wage} = \sum_{k\in K}\sum_{l\in L}\lambda_{k,l} \times ND \times \overline{C}_{k,l}^{wage} \tag{42}$$

where $\overline{C}_{k,l}^{wage}$ denotes the wage for working on the tower crane k at location l every workday. Equation (43) calculates the penalty for the delay in project exploitation.

$$C^{Delay} = \left(ND - ND^{min}\right) \times \overline{C}^{Delay} \tag{43}$$

where $\overline{C}^{Delay}$ indicates the penalty for each day of delay in project exploitation. Equation (43) and Equation (15) minimize possible delays through the balanced distribution of lifting activities between tower cranes.

For overlapping activities, safety penalties are considered in the objective function (Equation, 36). Safety penalties are obtained from Equation (44).

$$safety = C^{Unload} + C^{Load} + C^{cross} \tag{44}$$

where C^{Unload}, C^{Load} and C^{cross} are respectively equal to the penalty of material loading and unloading and crossing overlapping areas in case of path blockage. Safety penalties play a significant role in adjusting the height of tower cranes to prevent them from colliding and stopping in overlapping areas. The penalty for unloading and loading of materials by higher-height tower cranes in overlapping areas is calculated from Equations (45) and (46).

$$C^{Unload} = ND^{min} \times \sum_{j\in j}\sum_{k\in K}\sum_{l\in L}\overline{T}_{j,k,l}^{Unload} \times y_{j,k,l}^{Unload} \times \overline{C}^{Unload} \tag{45}$$

$$C^{Load} = ND^{min} \times \sum_{i\in j}\sum_{k\in K}\sum_{l\in L}\overline{\overline{T}}_{i,k,l}^{Load} \times y_{i,k,l}^{Load} \times \overline{C}^{Load} \tag{46}$$

where $\overline{C}^{Unload}$ represents the penalty for each minute of material unloading and $\overline{C}^{Load}$ represents the penalty for each minute of material loading from overlapping areas for higher-height tower cranes. If a higher-height tower crane hook blocks a path, Equation (47) considers a penalty for each time lower-height tower cranes pass through that path.

$$C^{cross} = ND^{min} \times \sum_{i\in I}\sum_{j\in J}\sum_{m\in M}\sum_{k\in K}\sum_{l\in L}\Lambda_{i,j,m,k,l} \times y_{i,j,k,l}^{cross} \times \overline{C}^{cross} \tag{47}$$

where $\overline{C}^{cross}$ is the penalty for each time a tower crane passes with a height lower than the overlapping areas with probability blockage.

3.10 Linearization of the proposed model

The proposed model is nonlinear due to Constraints (26), (41), (42), (45), (46) and (47). More and wider varieties of solvers are available for precisely solving linear models compared with nonlinear models. Thus, the authors have linearized nonlinear constraints without using any approximations in order to allow the use of a wide variety of precise solvers for solving the proposed model. This approach reinforces computational efficiency and also may provide many more accurate degrees in the solution process.

The term $\lambda_{k,l} \times \lambda_{k',l'}$ has led to the nonlinearity of Equation (26). Therefore, Equation (26) is rewritten as follows:

$$A^1_{k,l,k',l'} \times O_{k,l,k',l'} \leq U^{Overlap}_{k,l,k',l'} \; \forall k \in K, \forall l \in L, \forall k' \in K, \forall l' \in L, l \neq l' \tag{48}$$

where $A^1_{k,l,k',l'}$ is an auxiliary integer variable. Constraints (49) to (51) ensure the equality of $A^1_{k,l,k',l'}$ and $\lambda_{k,l} \times \lambda_{k',l'}$.

$$A^1_{k,l,k',l'} \geq \left(\lambda_{k,l} + \lambda_{k',l'} - 1\right) \quad \forall k \in K, \forall l \in L, \forall k' \in K, \forall l' \in L, l \neq l' \tag{49}$$

$$A^1_{k,l,k',l'} \leq \lambda_{k,l} \quad \forall k \in K, \forall l \in L, \forall k' \in K, \forall l' \in L, l \neq l' \tag{50}$$

$$A^1_{k,l,k',l'} \leq \lambda_{k',l'} \quad \forall k \in K, \forall l \in L, \forall k' \in K, \forall l' \in L, l \neq l' \tag{51}$$

Therefore, linear constraints (48) to (51) replace nonlinear constraint (26).

The term $\lambda_{k,l} \times ND$ has led to the nonlinearity of Equation (41). Therefore, Equation (41) is rewritten as follows:

$$C^{Variable} = \sum_{k \in K} \sum_{l \in L} A^2_{k,l} \times \overline{C}^{Variable}_k \tag{52}$$

where $A^2_{k,l}$ is an auxiliary integer variable. Constraints (53) to (55) ensure the equality of $A^2_{k,l}$ and $\lambda_{k,l} \times ND$.

$$A^2_{k,l} \leq \overline{M} \times \lambda_{k,l} \; \forall k \in K, \forall l \in L \tag{53}$$

$$A^2_{k,l} \leq ND \; \forall k \in K, \forall l \in L \tag{54}$$

$$A^2_{k,l} \geq ND - (1 - \lambda_{k,l}) \times \overline{M} \; \forall k \in K, \forall l \in L \tag{55}$$

Therefore, linear constraints (52) to (55) replace nonlinear constraint (41). Constraints (42) and (45) to (47) are linearized similarly.

4. Results

The proposed model is implemented in IBM® ILOG CPLEX 12.10 package and solved using OPL interface in a machine with Windows® 10, 32 GB RAM, and Core i7 CPU. In this section, the proposed MILP model is tested via a case study (Figure 5) that was borrowed from Riga *et al.* (2020). The site required information such as specifications of tower cranes, price of tower cranes and required materials were also borrowed from Ji and Leite (2020), Zhang *et al.* (1996), Riga *et al.* (2020), Moussavi Nadoushani *et al.* (2017), Wang *et al.* (2015). This case study consists of three independent zones.

The pilot construction site is a set of three five-story buildings in a U-shape arrangement and is located in Munich, Germany. It is situated within a larger development area in the city center. The proximity of existing buildings to the north and south, as well as ongoing construction sites to the northwest and southeast, results in spatial constraints in the construction area. The western area is left for parking, while the main access road to the site is a residential road to the east. The social and administrative facilities are accommodated in containers situated in the southern part of the construction area.

Figure 6 illustrates the zones and candidate locations of supply and demand points. On the site, there are nine candidate locations (O) for installing tower cranes and nine candidate

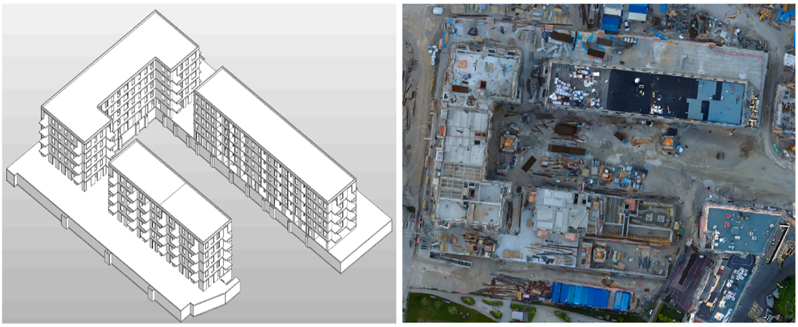


Figure 5. Case study (Riga *et al.*, 2020)

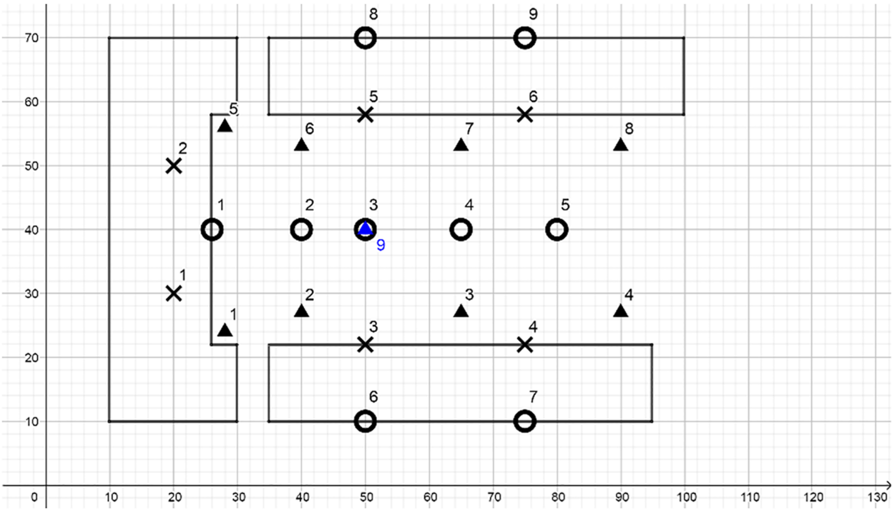


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Figure 6. Candidate locations of tower cranes and supply points

supply locations (▲). According to project geometry, there are six demand locations (×). As can be seen in Figure 7, O3 and ▲9 are located at exactly the same location.

It is assumed that the building needs four types of materials (*A*, *B*, *C* and *D*). The total request of demand points ($\bar{d}_{j,m}$) during this stage and the coordinates of demand points are shown in Table 3. The maximum amount that can be used from each of the materials ($\bar{\bar{d}}_{j,m}$) per day based on the speed of manpower and other effective factors is also given in Table 3. According to Equations (12) and (13), the total required materials for this stage are distributed among workdays in a balanced way. Based on this, the amount of materials that should be delivered to supply points on each workday ($d_{j,m}$) is calculated and reported in Table 3. According to Equation (13), construction will take at least 114 workdays ($ND^{min} = 114$). It is also assumed that tower cranes are allowed to operate for 8 h and 20 min ($T_{k,l}^{Working} = 500min$) on each workday. The construction should be completed in less than 300 workdays ($ND^{max} = 300$).

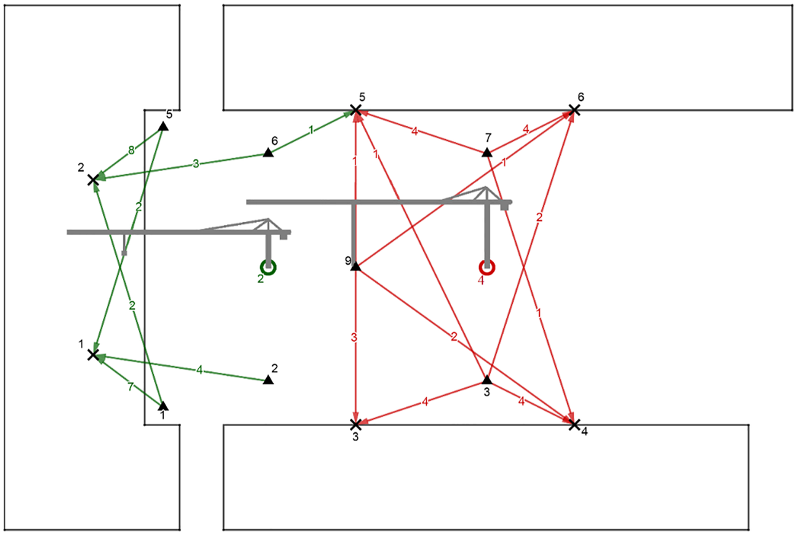

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Figure 7. Optimal layout

Table 4 shows the storage capacity of materials in candidate places for the supply of materials. In this project, it is assumed that a maximum of two types of materials can be stored at each point. The proposed model can handle other storage scenarios based on the project features. According to site limitations, a maximum of eight supply points can be created. It takes 12, 10, 9 and 8 min to load and unload materials A, B, C and D, respectively.

Nine tower cranes with varying heights, jib lengths and capacities are available. Information about Tower cranes (such as height, jib length, capacity and movement speed) is reported in Table 5 and Table 6. The cost per minute operation, the fixed cost (including installation and uninstallation costs) and the daily variable cost (including rental and maintenance) of tower cranes are listed in dollars in Table 7. The daily wage of operators is 200 $ and the penalty for each day of delay in exploitation is 1,500$. The penalty for each time passing through overlapping areas and the penalty for each minute of loading and unloading in overlapping areas are estimated at 0.2$ and 0.05$, respectively. To prevent collision of tower cranes with equal height in overlapping areas, $U^{Overlap}_{k,l,k',l'}$ is considered for tower cranes with equal height 0.

4.1 Optimal layout

The mathematical model for obtaining the optimum tower cranes layout has been developed and is available at: https://github.com/smartconstructiongroup/Locating-Tower-Cranes for those interested in adopting new cases. The obtained optimal solution for the case study is depicted in Figure 7 where locations 2 and 4 are selected as tower cranes locations. In detail, a Tower crane of type 1 must be installed at location 2 and a tower crane of type 2 must be mounted at location 4. Also, candidate locations 1 to 3, 5 to 7, and 9 have been selected as optimal supply points.

The travels made by the tower cranes installed at locations 2 and 4 are indicated by green and red arrows, respectively. The number of travels made in each path is written on arrows. The proposed model also can determine the amount of each material that must be stored at each supply point (Table 8).

Table 3. Demand points information

	Coordinates			$\bar{d}_{j,m}$				$\bar{\bar{d}}_{j,m}$				$d_{j,m}$			
Demand points	*x*	*y*	*z*	*A*	*B*	*C*	*D*	*A*	*B*	*C*	*D*	*A*	*B*	*C*	*D*
1	20	30	18	1,579	2,559	2,849	2,950	13.9	25	25	27.3	13.7	22.4	25	25.9
2	20	50	18	1,579	2,559	2,849	2,950	13.9	25	25	27.3	13.7	22.4	25	25.9
3	50	22	18	737	1,196	1,470	1,470	7.4	13.1	14.5	14.3	6.5	10.5	12.9	12.9
4	75	22	18	737	1,196	1,470	1,470	7.4	13.1	14.5	14.3	6.5	10.5	12.9	12.9
5	50	58	18	737	1,196	1,470	1,470	7.4	13.1	14.5	14.3	6.5	10.5	12.9	12.9
6	75	58	18	737	1,196	1,470	1,470	7.4	13.1	14.5	14.3	6.5	10.5	12.9	12.9

Source(s): Authors' own work

Table 4. Capacity of supply points

Supply points	Maximum material storage capacity A	B	C	D
1	36	63	61	67
2	36	63	61	67
3	36	63	61	67
4	36	63	61	67
5	36	63	61	67
6	36	63	61	67
7	36	63	61	67
8	36	63	61	67
9	36	63	61	67

Source(s): Authors' own work

Table 5. Tower crane information

Type	Height (m)	Jib length (m)	Capacity (ton) To 20 (m)	To 25 (m)	To 30 (m)	To 35 (m)	To 40 (m)	To 45 (m)	To 50 (m)	To 55 (m)
1	20	25	8	7.5	0	0	0	0	0	0
2	22	25	8	7.5	0	0	0	0	0	0
3	24	25	8	7.5	0	0	0	0	0	0
4	22	55	8	7.5	7	6.5	6	5.7	5.2	5
5	24	55	8	7.5	7	6.5	6	5.7	5.2	5
6	26	55	8	7.5	7	6.5	6	5.7	5.2	5
7	22	55	10	9	8	7.5	7	6.5	6	5.5
8	24	55	10	9	8	7.5	7	6.5	6	5.5
9	26	55	10	9	8	7.5	7	6.5	6	5.5

Source(s): Authors' own work

Table 6. Movement speed of tower cranes

Type	Movement speed (m/min) Vertical To 30 (m)	To 40 (m)	To 55 (m)	Radial To 30 (m)	To 40 (m)	To 55 (m)	Tangent To 30 (m)	To 40 (m)	To 55 (m)
1	66	–	–	59	–	–	8.3	–	–
2	66	–	–	59	–	–	8.3	–	–
3	66	–	–	59	–	–	8.3	–	–
4	66	60	54	59	53	48	8.3	7.6	6.8
5	66	60	54	59	53	48	8.3	7.6	6.8
6	66	60	54	59	53	48	8.3	7.6	6.8
7	66	60	54	59	53	48	8.3	7.6	6.8
8	66	60	54	59	53	48	8.3	7.6	6.8
9	66	60	54	59	53	48	8.3	7.6	6.8

Source(s): Authors' own work

Figure 8 shows the cost components related to the optimal layout. The objective function has a total cost of 162,437$, which consists of the operating cost of tower cranes 7,843$, fixed cost 36,500$, variable cost 60,704$, operator wages 48,082$ and delay penalty 9,308$ (for 6 days late). Moreover, the total safety penalty is 352$ (including 306.4$ penalty to unload materials

Table 7. The cost of tower cranes

Type	Operation cost	Fix cost	Variable cost
1	1.92	18,000	250
2	1.92	18,500	255
3	1.92	19,000	260
4	1.92	21,000	270
5	1.92	22,000	275
6	1.92	23,000	280
7	1.92	28,000	340
8	1.92	29,000	345
9	1.92	30,000	350

Source(s): Authors' own work

Table 8. Materials stored at supply points

Candidate point 1		Candidate point 2		Candidate point 3		Candidate point 5		Candidate point 6		Candidate point 7		Candidate point 9	
A	*D*	*A*	*C*	*B*	*C*	*C*	*D*	*A*	*B*	*C*	*D*	*A*	*D*
37.3	25.9	13.7	15	34.5	25.8	35	25.9	13.7	14.9	25.8	33.3	26	18.3

Source(s): Authors' own work

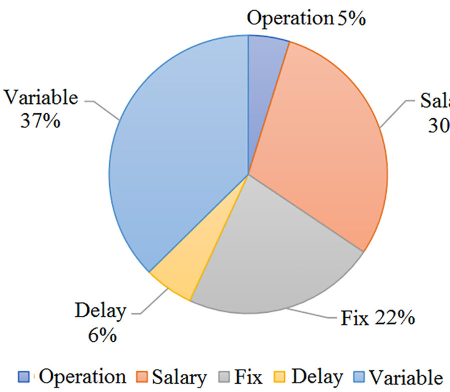

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Figure 8. Costs related to optimal layout

at demand point 5 from tower crane at location 4 and 45.6$ penalty to pass the tower crane at location 2 from demand point 5).

4.2 Overlap of tower cranes

In an optimal layout, stops and safety risks caused by overlapping are minimized by adjusting the height of tower cranes and the appropriate distribution of lifting activities between the active cranes (as seen in Figure 7). In the case study, only one overlap occurs at demand point 5, where the crane at location 4 unloads materials six times and the crane at location 2 does it once.

As seen in Figure 7, when the tower crane at location 4 loads materials from supply point 9, there is a possibility that the jib of this crane will collide with the mast of the crane at location 2. The proposed model avoids this collision by adjusting a higher height for the tower crane at location 4 (Figure 7). If safety constraints and penalties are ignored, the proposed model selects the tower crane of *type 1* to be installed at locations 2 and 4, which leads to the mentioned collision. So, neglecting the safety constraints and penalties can result in some movement paths being blocked. Most of the methods available in the literature review (Table 1) do not consider the height adjustment of cranes in the TCLP. Therefore, unlike the proposed method, these methods cannot guarantee the prevention of collisions between the jibs and masts of adjacent cranes.

To emphasize the impact of safety constraints and penalties, fixed costs, variable costs, and operators' wages were excluded from the objective function. This scenario is likely to lead to a rise in the number of tower cranes at the construction site. Consequently, this increase results in a higher probability of tower cranes overlap. The obtained layout and tower crane movement paths in this scenario are illustrated in Figure 9. As can be seen, four tower cranes are used to transport materials. Type 7 Tower Crane at Location 1, Type 8 Tower Crane at Location 2, Type 9 Tower Crane at Location 4, and Type 1 Tower Crane at Location 5. Figure 9 demonstrates that despite the rise in the number of tower cranes, the overlap of lifting activities has not changed significantly.

Figure 9 illustrates that the overlap between cranes has been minimized through the proper distribution of tasks among the cranes. This could be due to the overlap penalties in the proposed model's objective function (loading-crossing-unloading). As shown in Table 1, none of the methods reviewed in the literature account for these penalties. Therefore, as the number of cranes on the site increases, the proposed method becomes more practical and effective compared to previous approaches.

To investigate the changes in the overlap Figure 10 is generated. This Figure confirms that the rise in the number of tower cranes has not led to an increase in the overlap of tower cranes. The only overlap in this scenario occurs when tower cranes at locations 2 and 5 load materials from supply point 3 (Figure 9), which may even be avoided in practice with a suitable

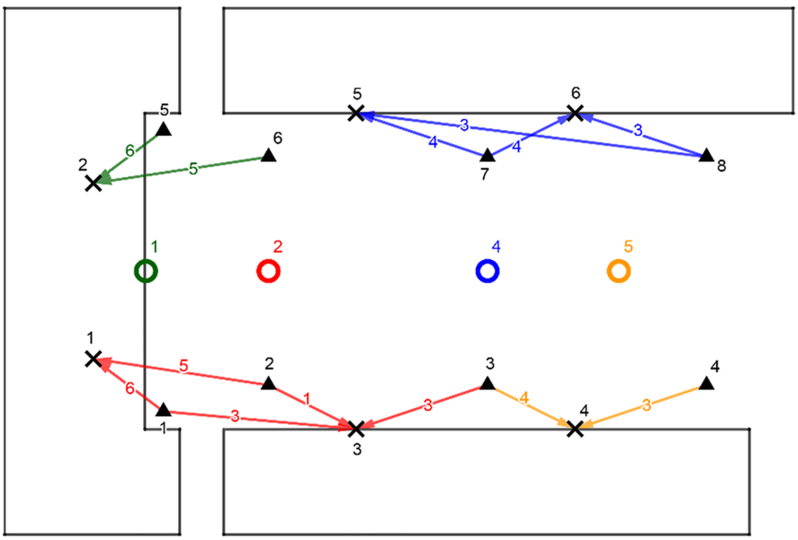

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Figure 9. Layout without considering tower crane costs

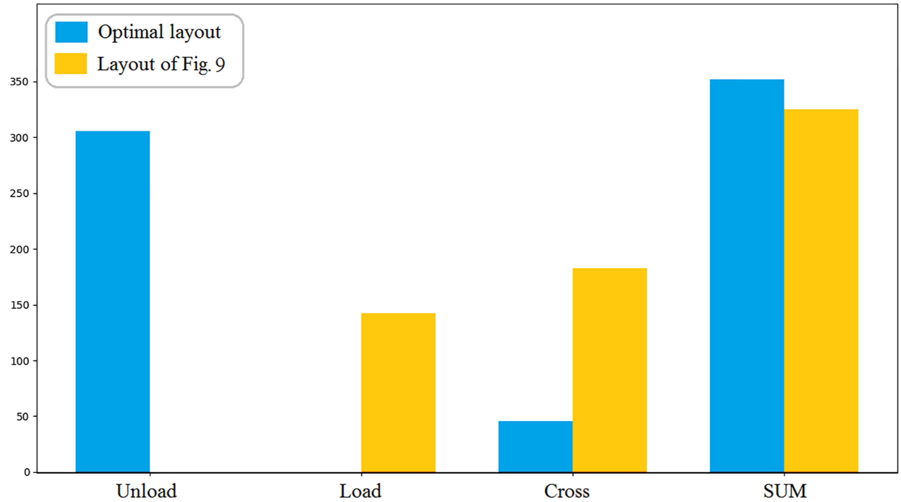

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Figure 10. The effect of increasing the number of tower cranes on overlap

schedule. This issue reveals that even with the increase in the number of cranes on site, safety constraints and penalties can minimize the overlap of lifting activities through a layout.

4.3 Tower crane costs

If crane costs (such as installation, dismantling, rent, and operator’s salary) are ignored from the objective function, the layout of Figure 9 is obtained. In this scenario, more cranes are used to reduce the material transfer time. However, this leads to an increase of about 115% in costs compared to the optimal layout (Figure 11). Therefore, the proposed method, compared to

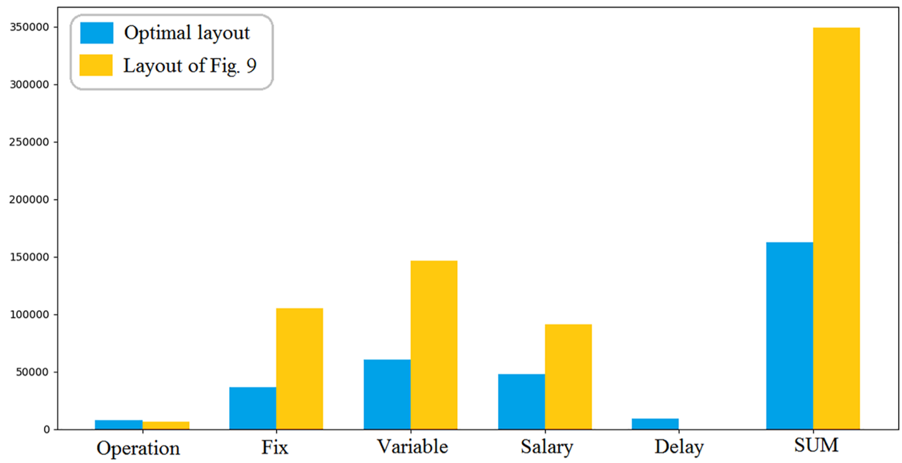

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Figure 11. The effect of ignoring fixed and variable costs

some existing approaches in the literature (Table 1), such as Ji and Leite (2020), Huang and Wong (2018), Dasović *et al.* (2019), Liu *et al.* (2022) and Li *et al.* (2023, 2024) which ignore these costs, can provide a more cost-effective layout.

4.4 Completion time

The cranes at locations 2 and 4 have almost equal operation time which are 31,477 seconds and 31,633 seconds, respectively. The obtained optimum results show that both selected cranes operate slightly longer than their daily limit (3,000 seconds), which results in a minor delay in serving a number of requests. This delay pushes the initial expected completion time from 114 days to 120 days. This time extension happened due to the provided flexibility by Equation (15). If this equation is relaxed ($T_{k,l}^{crane} \leq T_{k,l}^{Working}$), then the proposed model employs cranes with more capacity and speed (type 7 in location 2 and type 5 in location 4) to satisfy the demand within the project duration time. This leads to a 6% increase in total cost. This sensitive analysis shows that in practice, permitting a delay might lead to a more cost-effective solution.

Unlike the abovementioned issue, if the delay penalty ($\overline{C}^{Delay}$) is relaxed, the model obtains a cheaper solution including lower-cost cranes but a longer project duration. For instance, if Equation (43) is relaxed, the optimum solution consists of only one crane (type 4 at location 3). This leads to a significant time extension from 120 days (optimum duration) to 261 days. As shown in Table 1, none of the methods reviewed in the literature account for delay penalties. Therefore, the proposed method, in comparison to other similar approaches, has the potential to generate greater cost savings by reducing the number of workdays and minimizing project completion delays.

4.5 Type and number of tower crane

In the proposed model the type and number of tower cranes are considered as decision variables. To investigate the importance of this issue the following scenarios are defined:

(1) Fixed Number of Cranes

(2) Fixed Type of Cranes

(3) Increase in construction rates

Table 9 operates under the assumption that the number of cranes is fixed while the crane type remains as a decision variable.

(1) If only one crane is used project costs and the number of workdays increase by 97% and 86% respectively than the optimal solution.

(2) If two cranes are used, the same optimal layout is obtained.

(3) If the number of cranes is set to 3, the project costs will increase by about 34%, while workdays will decrease by around 5%.

Table 9. Layout in case the number of tower cranes is considered fixed

Crane(s) number	Crane(s) type	Location	Workdays	Safety penalties	Total costs
1	7	2	224	0	320,232
2	1, 2	2, 4	120	352	162,437
3	1, 2, 1	1, 2, 5	114	649	216,871

Source(s): Authors' own work

This highlights that determining the optimal number of cranes during the optimization process plays a critical role in reducing both costs and workdays. Consequently, the proposed method is more practical in real-world applications compared to approaches that predefine the number of cranes (e.g. Ji and Leite, 2020, Moussavi Nadoushani *et al.*, 2017, Huang and Wong, 2018, Amiri *et al.*, 2023, Dasović *et al.*, 2019, Huang *et al.*, 2024, Fu *et al.*, 2024, Liu *et al.*, 2022 and Li *et al.*, 2023, 2024).

In the case study, there are three tower crane models (other models differ only in height). Model 1 includes cranes of *type 1 to 3*, model 2 includes cranes of *type 4 to 6*, and model 3 includes cranes of *type 7 to 9*. Table 10 shows the importance of determining the crane model in the optimization process. In this table, the crane model is predetermined, while the number and height of cranes are decision variables.

(1) If the crane of *model 1* is used, the same optimal layout is obtained.

(2) The crane of *model 2* has a larger jib than the crane of *model 1*. If the crane of *model 2* is used, the project costs will increase by 6% without reducing workdays. This shows that determining the crane jib length in the optimization process can prevent unnecessary costs.

(3) The tower crane of *model 3* has a larger radius and capacity compared to the tower crane of *model 1*. But using it leads to a 15% increase in costs while reducing working days by only 6 days (Table 10). Therefore, considering the crane type as a decision variable has a prominent impact on optimizing project costs.

(4) The crane of *model 2* has a larger jib length compared to the crane of *model 1*. For this reason, its use can increase the overlapping and thus the safety penalties (Table 10). However, the crane *model 3* also has a longer jib length, but due to the higher lifting capacity, less number of times loading and unloading are done at overlapping points. For this reason, safety penalties have been reduced in the case of using the Model 3 crane (Table 10). This shows that the crane model (jib length and lifting capacity) affects the overlap of cranes and possible stoppages.

The above discussion highlights that determining the type of crane (jib length and lifting capacity) in the optimization process significantly impacts costs and crane overlap. Therefore, the proposed method, compared to approaches such as Huang and Wong (2018), Dasović *et al.* (2019), Liu *et al.* (2022) and Li *et al.* (2023), where the crane type is not treated as a decision variable, can produce safer and more cost-effective layouts.

By increasing the facilities available on the site, such as the number of workers and construction equipment, more materials can be spent on construction during the day so that the project can be completed in a shorter time. This issue increases the need to use crane(s). In Table 11, two scenarios are included to examine the effect of the amount of daily required materials on the optimal determination of the number and type of cranes.

(1) If the daily required materials are increased by 1.5 times compared to the study case, three cranes are used to complete the construction on time (80 workdays).

Table 10. Layout in case the type of tower crane is considered fixed

Crane model	Cranes number	Cranes type	Location	Workdays	Safety penalties	Total costs
1	2	1, 2	2, 4	120	352	162,437
2	2	4, 5	2, 4	120	595	172,053
3	2	7, 8	2, 4	114	323	187,578

Source(s): Authors' own work

(2) If the daily required materials are increased by 2 times compared to the study case, four cranes are used to complete the construction on time (58 workdays).

Therefore, in determining the number and type of required crane(s) for the project, the proposed model takes into account other facilities available on the site and the rate of material consumption.

4.6 Loading and unloading time

Loading/unloading time takes a significant amount of crane operation time. In the existing literature, only a few works have included this issue in their model (such as Huang and Wong, 2018; Wu *et al.*, 2020). In the case study, if the material loading and unloading time is ignored, the proposed model places only a type 4 crane at location 2 which results in 137 and 122% increase the project costs and the number of workdays, respectively. The other important issue is about the practicality of the obtained optimum solution because ignoring the duration of loading/unloading time leads to an incorrect capacity calculation of a crane. Similarly, if number of cranes is set to 2, then the proposed model places type 1 crane at location 2 and type 2 crane at location 5 (Figure 12). In this case, the mathematical model only optimizes the material transportation time without considering loading/unloading time. The solution is unbalanced, where operational times of cranes at locations 2 and 5 are respectively 13 and 4.5 hours. Furthermore, the project completion time and project cost are facing 47 and 84% increase.

Table 11. Layout in case of daily required material increase

	Consumption rate	ND^{min}	Cranes number	Cranes type	Location	Workdays	Total costs
Case study	1	114	2	1, 2	2, 4	120	162,437
Scenario 1	1.5	76	3	1, 2, 1	1, 2, 4	80	177,846
Scenario 2	2	57	4	1, 2, 1, 1	1, 2, 3, 5	58	185,107

Source(s): Authors' own work

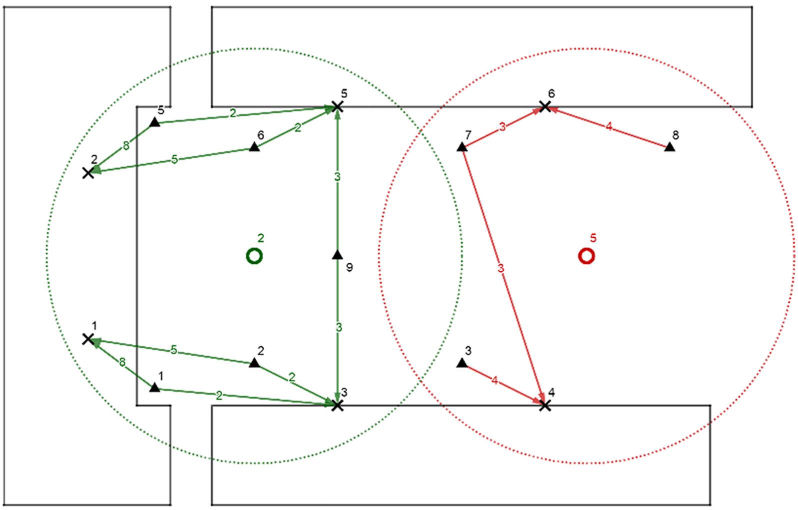

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Figure 12. Layout of tower cranes without considering loading and unloading time

4.7 Supply points

In the proposed mathematical model, a crane installation location can be a supply point as well. This model flexibility in the following scenario is examined.

In this scenario, it is assumed that it is not possible to overlap a supply and installation location. Then the obtained solution (Figure 13) includes 2 cranes but with larger jib arms which leads to a slight (around 2%) cost increase.

As can be perceived from Figure 13, tower cranes overlaps are increased particularly at locations 2 and 6. Also, the loading of the crane at location 4 from supply point 2 blocks the movement path from supply point 1 to demand point 3. This issue leads to triple safety penalties.

The final scenario is about ignoring the practical capacity of supply points. Not taking into account the limited capacity of supply points (Figure 14) reduces the project costs by only 0.7%, but the solution is not practical because as shown in Table 12, around half of the operational time, there is no available material at the supply points which could lift. Therefore, the proposed method is more practical compared to most approaches reviewed in the literature (Table 1).

In summary, in this section, it was tried to demonstrate that the proposed model has enhanced the related works by proposing a novel mathematical model to provide a practical and optimum solution.

5. Discussion

As described in the previous section, the proposed method was tested and validated using a real-world case study. The proposed method successfully determined the number, type and location of required tower cranes for the project. Moreover, it optimized the location and number of supply points as well as the quantities of materials stored at each point.

One of this study's primary objectives and innovations is to address the issue of tower crane overlaps. In the case study, the proposed method effectively prevented blockages in movement paths and collisions between crane jibs and bases by ranking the heights of overlapping cranes (Figure 7). Previous methods (Table 1) largely neglected this issue in their optimization

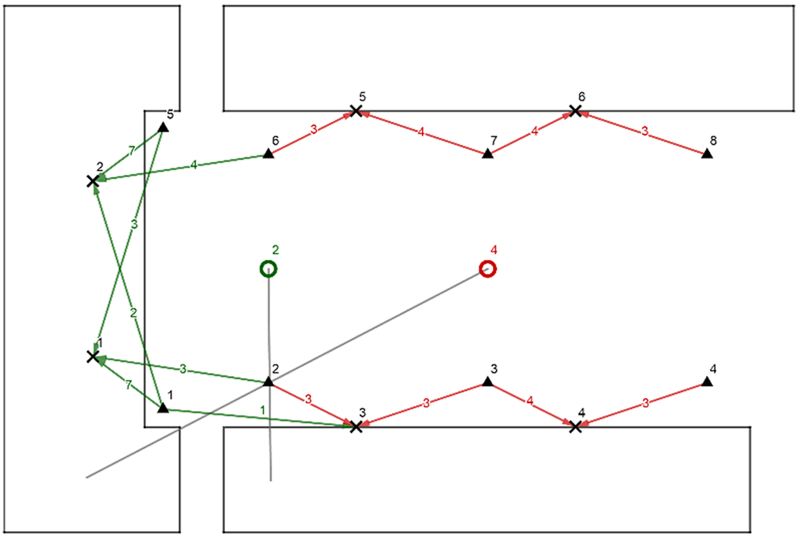

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Figure 13. Investigating the impact of selecting supply points in the optimization process

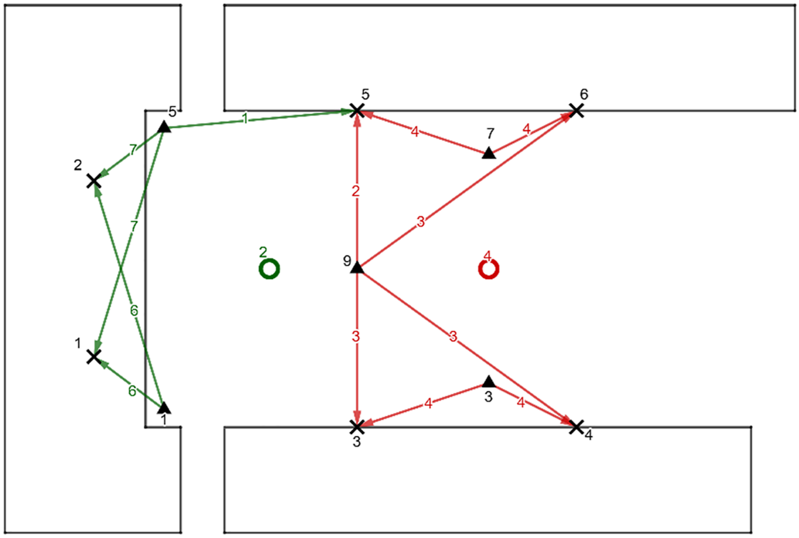

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Figure 14. Layout of tower cranes without considering the capacity of supply points

Table 12. The materials loaded from supply points and the shortage percentage of materials in them for layout Figure 14

Material type	Supply point 1 A	C	Supply point 3 C	D	Supply point 5 B	C	Supply point 7 C	D	Supply point 9 A	B
Required amount	27.4	50	25.8	25.8	47.8	51.8	25.8	25.8	26	39
Deficiency percentage	58		0		53.2		0		34.1	

Source(s): Authors' own work

processes. Therefore, the proposed method offers a more effective solution to reduce movement blockages by appropriately selecting the heights of overlapping cranes.

One of the major concerns in tower crane operations is the interruptions caused by activity conflicts in overlapping zones due to loading or unloading by another crane (Figure 3). Such interruptions can substantially delay material delivery, extend project workdays, and increase costs. The proposed method minimizes crane overlap activities by ranking crane heights and introducing penalties for loading, crossing and unloading in overlapping zones (Figure 7). In the case study, only one instance of overlap was observed, which can typically be avoided with proper on-site planning during execution. To further validate the proposed method, the number of cranes was increased from 2 to 4 (Figure 9). Even with this increase, the method successfully prevented additional overlaps, movement blockages, and crane collisions through effective task allocation among the cranes (Figures 9 and 10). Thus, the proposed approach can efficiently manage the interruptions and risks associated with crane overlaps.

The proposed method considers all important cost factors: the installation and dismantling cost of cranes, the rental and maintenance cost of cranes, and the cost of operators' wages. Results show that ignoring these costs might lead to wrong decisions regarding the number and type of used cranes, resulting in cost increases of up to 115% in the case study (Figure 11). By considering these costs, the number and types of cranes can be optimally determined by the proposed method. The results show that optimization in the number of cranes can reduce costs by 49% and workdays by 46% (Table 9). Furthermore, optimization in the type of crane can result in a 13% cost reduction in the case study (Table 10). Therefore, compared to existing methods (Table 1), which often neglect these considerations, the proposed approach is more practical and effective.

The consideration of the number of working days as a decision variable makes the proposed method unique from other approaches. This aspect of the model gives a more realistic estimation of the crane rental cost and the operator's wages. Therefore, the proposed method presents better decisions on the selection of cranes. The inclusion of the penalty for project delays in the objective function decreases the project's workdays by 54% in the case study. To further validate this, two additional scenarios were designed by increasing the construction speed. In both scenarios, the proposed method effectively managed working days and delays by adjusting the number of tower cranes (Table 11). Existing methods reviewed in the literature (Table 1) do not account for penalties related to material delivery delays. The proposed method can thus provide more accurate selections of crane numbers and types with huge savings in practice.

In the case study, the number, locations, and storage capacities of supply points were successfully determined in the optimization process (Table 8). The findings demonstrated that pre-selecting supply points outside the optimization process increased material transportation time and raised costs by 2% (Figure 13). Furthermore, this study showed that neglecting the capacity of supply points could lead to material shortages at certain points (Figure 14 and Table 12). This would increase transportation times and compromise schedule predictability. By incorporating supply point capacities, the proposed method effectively addresses these challenges (Table 8).

6. Conclusion and future research directions

This paper proposes a MILP model for optimizing the layout of tower cranes in construction projects. The proposed method successfully identifies the number, type, and locations of required tower cranes, as well as supply points and maintained materials at each of these supply points. Some of the major innovations developed herein manage crane overlap and optimize crane heights to provide a more efficient and safer operational environment than similar work. Furthermore, the proposed approach accounts for costs like crane rental, operator wages, and delays in project completion, leading to a more accurate determination of the number and type of required tower cranes. Additionally, to enhance practical efficiency, other relevant costs, such as installation and dismantling of cranes, and crane operational costs, are also taken into consideration in the proposed method.

Like other existing methods in the literature, however, the proposed method is prone to a few limitations. This limitation indeed opens avenues for further research and development in this area. The following suggestions are made for improvement in future studies:

(1) Incorporating the Costs of Re-layout During Construction: Future work could consider the costs associated with adjusting tower crane layouts during the construction process.

(2) Sequencing and Prioritization of Material Deliveries: Future models could also explore the sequencing and prioritization of material deliveries when planning tower crane locations, ensuring that delivery schedules are efficiently integrated with crane positioning.

(3) Incorporating Mobile and Rail Cranes in TCLP: The current model focuses on traditional tower cranes. Future research could extend this approach by integrating mobile and rail cranes into TCLP, accounting for their unique operational dynamics.

(4) Continuous Space Allocation for Crane Placement: Instead of using discrete candidate points for tower crane placement, future models could consider allocating continuous space, providing greater flexibility and realism in crane positioning.

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