S-duality in higher-derivative corrections of heterotic supergravity

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Abstract

This study investigates the consistency of heterotic supergravity, where the B-field gauge transformation becomes anomalous due to the Green-Schwarz mechanism, under T-duality transformations. We establish that preserving T-duality necessitates an infinite tower of higher-derivative couplings proportional to $e^{-2\Phi}$. For spacetimes featuring a Killing self-dual circle, these couplings remain immune to quantum corrections. Leveraging S-duality in heterotic/type I string theory, we then identify the corresponding couplings in type I theory. Notably, our analysis reveals that standard S-duality transformations do not involve higher-derivative corrections. Building on this insight, we derive the explicit form of the effective action for type I theory at the α' order, adhering to a scheme that omits derivatives of the dilaton.

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1 Introduction

As a robust framework for quantum gravity, string theory posits that the fundamental constituents of the universe are oscillations of a fundamental string. These oscillations manifest as various massless fields and an infinite hierarchy of massive fields within a 10-dimensional spacetime. Additionally, string theory incorporates non-perturbative, massive D_p-brane objects [1]. In the interacting theory, perturbative and non-perturbative objects interact to produce appropriate S-matrix elements, which can be effectively described by string field theory (see [2] and references therein). The massive perturbative fields, as well as non-perturbative objects, can be integrated out to yield a low-energy effective action in spacetime that includes only the massless fields and their derivatives at various orders of α'. This effective action encompasses perturbative couplings [3, 4, 5, 6, 7, 8, 9, 10], as well as non-perturbative couplings resulting from integrating out the non-perturbative objects [11, 12, 13, 14]. The derivation of this effective action involves imposing various global and local symmetries inherent to string theory.

The perturbative part of the low-energy effective action has the following genus expansion:

$$\mathbf{S}_{\text{per.}} \sim \sum_{g,b,c=0}^{\infty} \int d^{10}x\, e^{(2g+b+c-2)\Phi}\sqrt{-G}\,\mathcal{L}_{(g,b,c)}(\alpha')\,, \tag{1}$$

where g is the number of handles, b is the number of boundaries, and c is the number of cross-caps of the world-sheet. For oriented string theory, there are no boundaries or cross-caps. The Lagrangian $\mathcal{L}_{(g,b,c)}(\alpha')$ at each genus may include an α' expansion. In the classical theory, where $g = b = c = 0$, there are higher-derivative corrections at almost all orders of α'. However, in the quantum theory, where g, b, or c are non-zero, the classical couplings may be renormalized by some genus-correction. In chiral string theories, there are couplings at the first non-zero genus that are not present in the classical and higher genus levels. These couplings are necessary for anomaly cancellation [15]. Such couplings exist at order α'^3 in the chiral type I and heterotic string theories. Similar couplings are also found in the non-chiral type IIA theory, related to the presence of such couplings in 11-dimensional M-theory. This is the strong coupling limit of both chiral heterotic and non-chiral type IIA theories, depending on the compactification of this theory on an interval or circle, respectively [16]. The genus dependence of these specific couplings, as well as various couplings related to them by supersymmetry, is exact. For instance, the torus-level $\mathrm{Tr}(F^4)$ term in the 10-dimensional heterotic theory should not receive higher-genus corrections [17, 18].

The classical effective action, $\mathcal{L}_{(0,0,0)}(\alpha')$, at all orders in α', exhibits a global $O(d,d,\mathbb{R})$ symmetry, which emerges upon dimensional reduction of the effective action on a torus, $T^{(d)}$ [19, 20]. Using the observation that the classical effective action at the critical dimension is background independent [21], this symmetry, in principle, can serve as a constraint to determine the classical effective action at all orders in α'. It has been observed that imposing $O(1,1,\mathbb{R})$ symmetry, or its non-geometric discrete subgroup $\mathbb{Z}_2$—commonly referred to as the Buscher rule [22, 23]—on the circular reduction of the classical effective action allows one to derive all bosonic couplings in the heterotic string theory effective action at orders α' and α'^2 [24, 25]. Similarly, one can derive all classical NS-NS couplings in type II superstring theories at order α'^3, up to an overall factor proportional to $\zeta(3)$ [26]. Notably, the couplings in heterotic theory at orders α' and α'^2 are related via T-duality to the Lorentz Chern-Simons couplings at corresponding orders. These couplings arise by substituting the deformed B-field strength, as dictated by the anomalous B-field gauge transformation in the Green-Schwarz mechanism [15], into the leading two-derivative action.

These studies indicate that the Buscher rules receive higher-derivative corrections [27, 28]. Indeed, no covariant scheme exists in which T-duality transformations are free from corrections [26, 29]. Consequently, the couplings at orders α' and α'^2 derived in [24, 25] require additional couplings at orders α'^3, α'^4, and beyond, to ensure full consistency with T-duality. As a result, there exists an infinite number of higher-derivative covariant couplings in heterotic theory, which can, in principle, be determined through T-duality and are related to the two-derivative couplings by T-duality transformations[2].

On the other hand, while classical couplings are background independent, quantum corrections to the effective action depend on the background [30]. For backgrounds with one Killing self-dual circle, the aforementioned T-dual invariant couplings do not receive quantum corrections, as the two-derivative couplings are free from such corrections [8, 35]. Since these couplings are immune to quantum corrections in the presence of a Killing self-dual circle, their genus dependence, $e^{-2\Phi}$, is exact, enabling their analysis within the framework of S-duality.

The S-duality of type I/heterotic theory [33, 34] relates the low-energy effective action of unoriented type I theory at weak coupling to the low-energy effective action of oriented heterotic theory at strong coupling, and vice versa. If one considers (1) as the genus expansion of the type I effective action at weak coupling $e^{\Phi} < 1$, and the following genus expansion for the heterotic theory at weak coupling:

$$\mathbf{S}_{\rm per.} \sim \sum_{g=0}^{\infty} \int d^{10}x\, e^{2(g-1)\Phi'} \sqrt{-G'}\, \mathcal{L}_g(\alpha')\,, \tag{2}$$

where the prime over the fields indicates that they belong to the heterotic theory at weak coupling $e^{\Phi'} < 1$. The S-duality then relates the bosonic fields in the heterotic theory to the corresponding fields in type I theory as shown in [34]

$$\Phi' = -\Phi,\ G'_{\alpha\beta} = e^{-\Phi} G_{\alpha\beta},\ B'_{\alpha\beta} = C_{\alpha\beta} \equiv B_{\alpha\beta},\ A'_{\alpha i}{}^{j} = A_{\alpha i}{}^{j}. \tag{3}$$

The Yang-Mills (YM) gauge field is defined as $A_{\mu i}{}^{j} = A_{\mu}{}^{I}(\lambda^I)_i{}^j$, where the antisymmetric matrices $(\lambda^I)_i{}^j$ represent the adjoint representation of the gauge group $SO(32)$ with the normalization $(\lambda^I)_i{}^j(\lambda^J)_j{}^i = -\delta^{IJ}$. Note that the two-form in type I is an R-R field. However, for simplicity in notation, we denote it as $B_{\alpha\beta}$ and refer to the metric, dilaton, and the R-R field as the NS-NS sector, similar to the corresponding fields in the heterotic theory. Since the

[2] A specific set of an infinite number of non-covariant but locally Lorentz-invariant classical couplings has been proposed in [31, 32], based on the $\mathcal{N} = 1$ supersymmetry of the leading-order action (4). This action incorporates the Lorentz Chern-Simons term in H and modifies the spin connection to include torsion proportional to the deformed H. The infinite set of covariant T-dual classical couplings may correspond to the infinite set of locally Lorentz-invariant classical couplings through appropriate non-covariant field redefinitions.

two expansions (1) and (2) are at two different couplings, one cannot generally compare the couplings at each order of α' in the two theories unless the couplings at a specific order of α' are exact in the coupling constant [18]. Those terms transform from one theory to the other using the above transformation.

In fact, the above S-duality has been suggested in [34] by observing that the classical effective action of the heterotic theory at the two-derivative order, given as

$$\mathbf{S}^{(0)}_{\rm het.} = -\frac{2}{\kappa^2}\int d^{10}x\sqrt{-G'}e^{-2\Phi'}\Big[R' + 4\nabla_\alpha\Phi'\nabla^\alpha\Phi' - \frac{1}{12}H'_{\alpha\beta\gamma}H'^{\alpha\beta\gamma} + \frac{1}{4}F'{}_{\alpha\beta i}{}^{j}F'^{\alpha\beta}{}_{j}{}^{i}\Big], \quad (4)$$

transforms to the corresponding classical effective action of type I theory under the transformation (3), given as

$$\mathbf{S}^{(0)}_{\rm typeI} = -\frac{2}{\kappa^2}\int d^{10}x\sqrt{-G}\Big[e^{-2\Phi}(R + 4\nabla_\alpha\Phi\nabla^\alpha\Phi) - \frac{1}{12}H_{\alpha\beta\gamma}H^{\alpha\beta\gamma} + \frac{e^{-\Phi}}{4}F_{\alpha\beta i}{}^{j}F^{\alpha\beta}{}_{j}{}^{i}\Big]. \quad (5)$$

The YM gauge field strength and the B-field strength are given by

$$\begin{aligned} F_{\mu\nu i}{}^{j} &= \partial_\mu A_{\nu i}{}^{j} - \partial_\nu A_{\mu i}{}^{j} + \frac{1}{\sqrt{\alpha'}}[A_{\mu i}{}^{k}, A_{\nu k}{}^{j}]\,, \\ H_{\mu\nu\rho} &= 3\partial_{[\mu}B_{\nu\rho]} + \frac{3}{2}A_{[\mu i}{}^{j}F_{\nu\rho]j}{}^{i}\,. \end{aligned} \quad (6)$$

The NS-NS fields are dimensionless, and we have also normalized the YM gauge field to be dimensionless. Additionally, the Lorentz Chern-Simons three-form $\Omega_{\mu\nu\rho}$ appears in the B-field strength as a result of the Green-Schwarz mechanism [15]. This leads to terms with four and six derivatives , *i.e.,* $\alpha' H\Omega$ and $\alpha'^2\Omega\Omega$. There are no higher-genus corrections to the aforementioned two-derivative effective actions [8, 35]; hence, S-duality holds at the two-derivative level [34]. Note that by using the YM gauge symmetry in the internal space, one can adopt the local frame gauge in which the YM potential is zero, but its derivatives are not [24, 25]. Therefore, by rescaling the B field strength in type I theory as $H = e^{-\Phi}\tilde{H}$, one observes that the classical couplings in type I theory exhibit a genus of $e^{-\Phi}$ for massless open string fields and $e^{-2\Phi}$ for closed string fields.

The Lorentz Chern-Simons three-form in the B-field strength is defined as:

$$\Omega_{\mu\nu\alpha} = \omega_{[\mu\mu_1}{}^{\nu_1}\partial_\nu\omega_{\alpha]\nu_1}{}^{\mu_1} + \frac{2}{3}\omega_{[\mu\mu_1}{}^{\nu_1}\omega_{\nu\nu_1}{}^{\alpha_1}\omega_{\alpha]\alpha_1}{}^{\mu_1}\ ;\ \ \omega_{\mu\mu_1}{}^{\nu_1} = e^{\nu}{}_{\mu_1}\nabla_\mu e_\nu{}^{\nu_1}\,, \quad (7)$$

where $e_\mu{}^{\mu_1}e_\nu{}^{\nu_1}\eta_{\mu_1\nu_1} = G_{\mu\nu}$. The covariant derivative in the definition of the spin connection acts only on the curved indices of the frame $e_\mu{}^{\mu_1}$. The scale transformation of the metric $G_{\mu\nu}$ in (3) may result from the following scalar transformation of the flat-space metric:

$$\eta'_{\mu_1\nu_1} = e^{-\Phi}\eta_{\mu_1\nu_1}\,, \quad (8)$$

and no scalar transformation applied to the frame $e_\mu{}^{\mu_1}$. Consequently, the spin connection $\omega_{\mu\mu_1}{}^{\nu_1}$, like the YM connection $A_{\mu i}{}^{j}$, remains invariant. As a result, the Lorentz Chern-Simons three-form $\Omega_{\mu\nu\rho}$ also remains invariant under S-duality, i.e.,

$$\Omega'_{\mu\nu\rho} = \Omega_{\mu\nu\rho}\,. \quad (9)$$

This invariance ensures that the leading-order actions (4) and (5), which include the Lorentz Chern-Simons three-form, are invariant under S-duality.

The transformation (3) also connects certain higher-derivative couplings of the heterotic theory, specifically the anomaly cancellation terms [15] and the terms linked to them by supersymmetry, to the corresponding couplings in type I theory due to their precise genus dependence [18, 36].

The anomaly cancellation mechanism in heterotic theory introduces new couplings at both the classical level and at one-loop. While the S-duality of one-loop couplings, linked by supersymmetry to the anomaly cancellation term, has been thoroughly investigated in [18, 36], this paper centers its attention on the S-duality of classical couplings in heterotic theory. Indeed, as previously discussed, heterotic theory encompasses an infinite set of higher-derivative classical couplings that are interconnected through T-duality with classical couplings derived from the following substitution in the supergravity action (4), as well as its higher-order extensions dictated by T-duality [24, 25]:

$$H'_{\alpha\beta\gamma} \;\;\rightarrow\;\; H'_{\alpha\beta\gamma} + \frac{3}{2}\alpha'\Omega'_{\alpha\beta\gamma}. \tag{10}$$

This replacement is mandated by the Green-Schwarz mechanism [15], ensuring that T-duality consistently determines the coefficients of an infinite set of higher-derivative couplings at the classical level. These couplings extend the two-derivative terms (4) to T-dual couplings at orders α'^0, α', α'^2, and beyond, all governed by the same overall factor as the leading-order action (4). Since the two-derivative terms do not receive higher-genus corrections due to kinematic reasons [8, 35], T-duality ensures that the higher-derivative components of the T-dual couplings for the Killing self-dual circle also remain free from genus corrections[3]. It is an exact T-dual multiplet that can be studied under S-duality.

Furthermore, the classical theory includes another distinct set of T-dual couplings at orders α'^3, α'^4, and higher, characterized by an overall coefficient of $\zeta(3)$. Unlike the first set, this group is not directly associated with the leading-order action (4). As a result, T-duality does not prevent this collection of T-dual couplings from receiving higher-genus corrections. Similarly, there exist additional sets with coefficients $\zeta(5)$ and higher [38], all of which may also receive higher-genus corrections.

To determine the couplings via T-duality, it is essential to have a spacetime dimension compactified into a circle, where all fields remain independent of the Killing coordinate y. Consequently, the results are applicable to spacetimes with one Killing circle. While the classical effective action is background-independent—ensuring that the resulting T-duality invariant couplings are valid across all spacetimes—the quantum corrections to this action are influenced by the background. Thus, the quantum corrections governed by T-duality are applicable exclusively to spacetimes with a single Killing self-dual circle [30].

[3] A similar set of couplings arises in bosonic string theory. Specifically, bosonic theory includes an infinite number of higher-derivative couplings at orders α', α'^2, and beyond, governed by a single overall factor [37]. However, since this factor is not linked by T-duality to the two-derivative leading-order action, these couplings may not be exact and may receive higher-genus corrections.

A similar replacement to (10) exists in Type I theory. However, the corresponding $\alpha' H\Omega$ and $\alpha'^2\Omega\Omega$ terms do not appear in the usual untwisted sector. Instead, they emerge in the twisted sector of the type I effective action [39], which transforms under T-duality to the twisted sector of the type I' theory [40]. Unlike in heterotic theory, T-duality alone is insufficient to fully determine all couplings at orders α', α'^2, and higher in both type I and type I' theories due to inherent dimensional differences: the twisted sector in type I' theory is inherently 9-dimensional, whereas that in type I theory is 10-dimensional [39]. Nevertheless, S-duality applied to the exact T-dual multiplet of heterotic theory offers a framework to identify such couplings in type I theory.

The corresponding couplings in type I theory manifest at different genera. Using the S-duality transformation (3), it can be shown that the couplings at order α'^n within the exact T-dual multiplet in heterotic theory transform into couplings at genus $e^{(n+k-2)\Phi}$ in type I theory, where k represents half the number of YM field strengths in the couplings. In this paper, we demonstrate how these couplings can be systematically determined by imposing S-duality on the exact T-dual multiplet in heterotic theory. Specifically, we explicitly derive these couplings in type I theory at the α' order, adhering to a scheme that excludes dilaton derivatives. It is important to note that the exact T-dual multiplet in heterotic theory remains free from genus corrections only in spacetimes with a single Killing self-dual circle [30]. Consequently, the couplings derived through S-duality are expected to hold valid exclusively for such spacetimes.

The observation that the exact T-dual multiplet in heterotic theory remains unaffected by quantum corrections in spacetimes with a single Killing self-dual circle has significant implications for the type IIA effective action. These implications arise from the application of S-duality between heterotic theory on the manifold T^4, which includes the circle, and type IIA theory on the manifold $K3$ [42, 16]. The S-duality correspondence between the 6-dimensional massless NS-NS fields in type IIA and their equivalents in heterotic theory is outlined in [43]:

$$\Phi' = -\Phi,\ G'_{\mu\nu} = e^{-2\Phi} G_{\mu\nu},\ H' = e^{-2\Phi} * H\,, \tag{11}$$

where $\Phi, G_{\mu\nu}, H_{\mu\nu\rho}$ are the six-dimensional NS-NS fields of type IIA theory. In this context, one can use the dimensional reduction of the exact T-dual multiplet on the torus T^4 to determine the corresponding six-dimensional couplings at all orders of α' with the exact genus dependence $e^{-2\Phi'}$. Using the aforementioned S-duality transformations, one can identify the corresponding higher-derivative six-dimensional couplings in type IIA theory, which appear at different genera. Applying the S-duality transformation to the exact T-dual multiplet, one finds that the couplings at order α'^n on T^4 appear at genus $e^{2(n-1)\Phi}$ in type IIA theory on $K3$. For example, the one-loop couplings in type IIA string theory on $K3$ arise at order α'. These couplings were first identified in [43] through S-duality transformations and subsequently confirmed by S-matrix calculations. Furthermore, they have been utilized in [44, 45] to investigate α' corrections to the dyonic string solution in 6-dimensional supergravity. The two-loop couplings in type IIA on $K3$ are at order α'^2, which may be found by imposing S-duality on the α'^2 couplings identified in [25] through T-duality.

On the other hand, the aforementioned S-duality can be utilized to establish a no-go theorem. As previously discussed, aside from the exact T-dual multiplet at orders α'^0, α', α'^2, and so forth, with an overall coefficient of 1, there exist T-dual multiplets at orders α'^3, α'^4, and beyond, characterized by an overall coefficient of $\zeta(3)$, along with an infinite number of T-dual multiplets at higher orders [38]. While the exact T-dual multiplet remains quantum-exact, all other T-dual multiplets are classical and subject to quantum corrections. Therefore, in heterotic theory, only the classical couplings at orders α'^0, α', and α'^2 remain free from quantum corrections. Since the T^4 reduction of these classical couplings preserves the number of derivatives in the couplings, it follows that no couplings arise at orders α' and α'^2 at the quantum level, i.e., at genus $e^{2(g-1)\Phi'}$ for $g > 0$, in the six-dimensional heterotic theory. Using the S-duality transformation, one finds no couplings at order α' at genus $e^{-2g\Phi}$ and no couplings at order α'^2 at genus $e^{-2(g-1)\Phi}$ in the six-dimensional type IIA theory. Based on the permitted genus expansion for the effective action in type IIA theory, it is determined that at order α', $g = 1$, and at order α'^2, $g = 1, 2$. Consequently, there must be no classical couplings at order α', and no classical or torus-level couplings at order α'^2 in the six-dimensional type IIA theory. This conclusion is reinforced by arguments derived from supersymmetry (see, for example, [46]). We will demonstrate that the $K3$ reduction of the classical couplings in type IIA theory at order α'^3, identified via T-duality [26], produces certain α' couplings that vanish upon applying field redefinition.

In the next section, we will demonstrate that S-duality transformations do not receive higher-derivative corrections. Utilizing this observation, we determine the couplings in type I theory at the α' order by dualizing the corresponding couplings in the heterotic theory, which have been recently identified in a minimal scheme through T-duality. In Section 3, we reduce the 8-derivative NS-NS couplings in type IIA theory on the manifold $K3$ and show that the resulting four-derivative couplings vanish upon field redefinition. In Section 4, we briefly discuss our results.

2 Heterotic/Type I duality

In this section, we will study the S-duality transformation of the exact couplings at order α' in the heterotic theory.

The study of leading-order two-derivative couplings (4), (5) under S-duality transformations (3) remains unaffected by both higher-derivative corrections to S-duality and field redefinition ambiguities [47, 3]. However, the analysis of higher-derivative couplings introduces two essential considerations: (i) the systematic implementation of field redefinitions, and (ii) the possible existence of higher-derivative modifications to the S-duality transformation rules. A crucial prerequisite for determining the S-duality transformation properties of these couplings involves establishing whether the fundamental S-duality relations (3) themselves admit higher-derivative corrections. This foundational question must be resolved before any consistent mapping between heterotic and Type I theories at higher orders in α' can be achieved.

Using the genus dependence of the couplings in type I (1) or heterotic theory (2), one realizes that S-duality does not receive higher-derivative corrections. To see this, note that the S-duality transformation (3) satisfies the $\mathbb{Z}_2$-group symmetry; hence, its higher-derivative extension should also satisfy this symmetry. If we consider the two-derivative corrections to the S-duality for NS-NS fields as

$$\begin{aligned}
\Phi' &= -\Phi + \alpha' \Delta\Phi(\Phi, G, B), \\
G'_{\alpha\beta} &= e^{-\Phi}\Big[G_{\alpha\beta} + \alpha' G_{\alpha\beta}\Delta\Phi(\Phi, G, B) + \alpha'\Delta G_{\alpha\beta}(\Phi, G, B)\Big], \\
B'_{\alpha\beta} &= B_{\alpha\beta} + \alpha'\Delta B_{\alpha\beta}(\Phi, G, B),
\end{aligned} \tag{12}$$

where $\Delta\Phi(\Phi, G, B)$, $\Delta G_{\alpha\beta}(\Phi, G, B)$, and $\Delta B_{\alpha\beta}(\Phi, G, B)$ present the two-derivative correction terms that depend on the NS-NS fields at order α'. Then, the $\mathbb{Z}_2$ symmetry implies that the corrections satisfy the following relations:

$$\begin{aligned}
\Delta\Phi(\Phi, G, B) - \Delta\Phi(-\Phi, e^{-\Phi}G, B) &= 0, \\
\Delta G_{\alpha\beta}(\Phi, G, B) + G_{\alpha\beta}\Delta\Phi(-\Phi, e^{-\Phi}G, B) + e^{\Phi}\Delta G_{\alpha\beta}(-\Phi, e^{-\Phi}G, B) &= 0, \\
\Delta B_{\alpha\beta}(\Phi, G, B) + \Delta B_{\alpha\beta}(-\Phi, e^{-\Phi}G, B) &= 0.
\end{aligned} \tag{13}$$

The corrections that satisfy these relations should have the dilaton factors $e^{\Phi/2}$ and $e^{3\Phi/2}$. For example, the dilaton correction $e^{\Phi/2}G^{\alpha\beta}\nabla_\alpha\Phi\nabla\Phi_\beta$ satisfies the first relation above. However, there are no half-integer numbers in the genus dependence of the effective action (1). Hence,

$$\Delta\Phi = 0,\ \Delta G_{\alpha\beta} = 0\,, \Delta B_{\alpha\beta} = 0\,. \tag{14}$$

This is unlike the corrections to the T-duality transformations, which must be non-zero to ensure T-duality invariant couplings at higher-derivative orders [28].

Since S-duality (3) has no higher-derivative corrections, applying it to the heterotic couplings in a specific scheme allows one to derive the corresponding couplings in Type I theory. Imposing these resulting Type I couplings under S-duality reproduces the original heterotic couplings in the chosen scheme. To determine the Type I couplings in a specific scheme, one can begin with the maximal basis of couplings in Type I theory—where no field redefinitions have been applied and the coupling constants remain arbitrary. These couplings can then be transformed under S-duality (3) to their heterotic counterparts. The resulting expressions must match, up to total derivative terms and Bianchi identity constraints, the heterotic couplings in a fixed scheme where all coupling constants are predetermined. We are particularly interested in schemes where the heterotic effective action contains no dilaton derivatives. This matching condition fixes the arbitrary coupling constants in the Type I maximal basis. However, many couplings in this basis involve dilaton derivatives. To isolate the subset of Type I couplings without such terms, we must adopt a different scheme for the heterotic theory. This involves applying an arbitrary field redefinition to the heterotic couplings before equating them with the S-dual transformation of the Type I maximal basis. The resulting equality imposes constraints on the coupling constants in the maximal basis, though many remain undetermined. Each choice of these arbitrary parameters corresponds to a distinct scheme for the Type I effective action. By selecting appropriate values, one can derive a Type I action free of dilaton derivatives.

Let us consider the transformation of sphere-level couplings in the heterotic theory at order α'. These couplings in a specific scheme have been found in [4] by studying the S-matrix element of four vertex operators. Recently, these couplings in an arbitrary scheme have been found by T-duality [24]. The couplings in the minimal scheme, where the NS-NS part is the Metsaev-Tseytlin action [48], are as follows:

$$\begin{aligned}\mathbf{S}^{(1)}_{\rm het.} \;=\; & -\frac{2\alpha'}{8\kappa^2}\int d^{10}x\sqrt{-G'}e^{-2\Phi'}\Big[\frac{1}{4}F'_{\alpha}{}^{\gamma kl}F'^{\alpha\beta ij}F'_{\beta}{}^{\delta}{}_{kl}F'_{\gamma\delta ij}-\frac{1}{2}F'_{\alpha}{}^{\gamma}{}_{ij}F'^{\alpha\beta ij}F'_{\beta}{}^{\delta kl}F'_{\gamma\delta kl}\\ & -\frac{1}{8}F'_{\alpha\beta}{}^{kl}F'^{\alpha\beta ij}F'_{\gamma\delta kl}F'^{\gamma\delta}{}_{ij}+\frac{1}{4}F'^{\alpha\beta ij}F'^{\gamma\delta}{}_{ij}H'_{\alpha\gamma}{}^{\epsilon}H'_{\beta\delta\epsilon}-\frac{1}{8}F'^{\alpha\beta ij}F'^{\gamma\delta}{}_{ij}H'_{\alpha\beta}{}^{\epsilon}H'_{\gamma\delta\epsilon}\\ & -\frac{1}{2}F'_{\alpha}{}^{\gamma}{}_{ij}F'^{\alpha\beta ij}H'_{\beta}{}^{\delta\epsilon}H'_{\gamma\delta\epsilon}-\frac{1}{8}H'_{\alpha\beta}{}^{\delta}H'^{\alpha\beta\gamma}H'_{\gamma}{}^{\epsilon\varepsilon}H'_{\delta\epsilon\varepsilon}+\frac{1}{24}H'_{\alpha}{}^{\delta\epsilon}H'^{\alpha\beta\gamma}H'_{\beta\delta}{}^{\varepsilon}H'_{\gamma\epsilon\varepsilon}\\ & +R'_{\alpha\beta\gamma\delta}R'^{\alpha\beta\gamma\delta}-\frac{1}{2}H'_{\alpha}{}^{\delta\epsilon}H'^{\alpha\beta\gamma}R'_{\beta\gamma\delta\epsilon}+2H'^{\alpha\beta\gamma}\Omega'_{\alpha\beta\gamma}\Big]\,. \end{aligned} \tag{15}$$

The transformation of all couplings mentioned above—except those involving the Riemann curvature—under S-duality (see (3) and (9)) is straightforward and yields the corresponding couplings in Type I theory without dilaton derivatives. However, applying S-duality to the couplings containing the Riemann curvature generates, up to total derivative terms and Bianchi identities, the following dilaton-dependent couplings in type I theory:

$$\begin{aligned}&2R\nabla_\alpha\nabla^\alpha\Phi+\frac{1}{4}H_{\beta\gamma\delta}H^{\beta\gamma\delta}\nabla_\alpha\Phi\nabla^\alpha\Phi-3R\nabla_\alpha\Phi\nabla^\alpha\Phi+9\nabla_\alpha\nabla^\alpha\Phi\nabla_\beta\nabla^\beta\Phi-\frac{1}{2}H_\alpha{}^{\gamma\delta}H_{\beta\gamma\delta}\nabla^\alpha\Phi\nabla^\beta\Phi\\ &-2R_{\alpha\beta}\nabla^\alpha\Phi\nabla^\beta\Phi-7\nabla_\alpha\Phi\nabla^\alpha\Phi\nabla_\beta\Phi\nabla^\beta\Phi+48\nabla^\alpha\Phi\nabla_\beta\nabla_\alpha\Phi\nabla^\beta\Phi-H_\alpha{}^{\gamma\delta}H_{\beta\gamma\delta}\nabla^\beta\nabla^\alpha\Phi\,.\end{aligned} \tag{16}$$

We suppressed the overall dilaton factor of each coupling. If one imposes on the resulting couplings once again the S-duality (3), one would find the original couplings (15) in the heterotic theory up to total derivative terms and Bianchi identities. This reflects the fact that S-duality forms a $\mathbb{Z}_2$-group.

However, to identify the Type I couplings without dilaton derivatives, we must perform field redefinitions on the couplings in (15) to alter their scheme. As noted earlier, we must use the maximal basis at order α' in Type I theory, which comprises 42 couplings with arbitrary coefficients, as explicitly constructed in [24]. By applying S-duality (3) to this basis, we transform the couplings and equate the resulting expressions with those in (15)—modified by the inclusion of arbitrary field redefinition terms. To derive the relationships among the 42 coefficients, we employ total derivative terms and exploit Bianchi identities in the heterotic theory. For a thorough and detailed treatment of this procedure, we refer the reader to [24].

Our findings show that the above equality produces 24 relations, leaving 18 arbitrary parameters. A specific choice of these parameters fixes the couplings in type I theory in a particular scheme. We choose these parameters such that there is no derivative of the dilaton in the couplings, as in the heterotic theory (15). There remain three parameters, and a specific choice for them produces the following couplings in type I theory:

$$\mathbf{S}^{(1)}_{\rm typeI} \;=\; -\frac{2\alpha'}{8\kappa^2}\int d^{10}x\sqrt{-G}\Big(e^{\Phi}\Big[\frac{1}{8}F_{\alpha}{}^{\gamma kl}F^{\alpha\beta ij}F_{\beta}{}^{\delta}{}_{kl}F_{\gamma\delta ij}-\frac{1}{16}F_{\alpha\beta}{}^{kl}F^{\alpha\beta ij}F_{\gamma\delta kl}F^{\gamma\delta}{}_{ij}$$

$$+\frac{1}{1024}F_{\alpha\beta ij}F^{\alpha\beta ij}F_{\gamma\delta kl}F^{\gamma\delta kl}\Big]+\Big[\frac{1}{4}F^{\alpha\beta ij}F^{\gamma\delta}{}_{ij}\tilde{H}_{\alpha\gamma}{}^{\epsilon}\tilde{H}_{\beta\delta\epsilon}-\frac{1}{8}F_{\alpha}{}^{\gamma}{}_{ij}F^{\alpha\beta ij}\tilde{H}_{\beta}{}^{\delta\epsilon}\tilde{H}_{\gamma\delta\epsilon}$$
$$+\frac{1}{48}F_{\alpha\beta ij}F^{\alpha\beta ij}\tilde{H}_{\gamma\delta\epsilon}\tilde{H}^{\gamma\delta\epsilon}-\frac{1}{4}F^{\alpha\beta ij}\tilde{H}_{\beta\gamma\delta}\nabla_{\alpha}F^{\gamma\delta}{}_{ij}+\frac{3}{64}F_{\alpha\beta ij}F^{\alpha\beta ij}R-\frac{1}{2}F_{\alpha}{}^{\gamma ij}F_{\beta\gamma ij}R^{\alpha\beta}\Big]$$
$$+e^{-\Phi}\Big[\frac{1}{16}R^2-\frac{1}{16}\tilde{H}_{\alpha\beta}{}^{\delta}\tilde{H}^{\alpha\beta\gamma}\tilde{H}_{\gamma}{}^{\epsilon\varepsilon}\tilde{H}_{\delta\epsilon\varepsilon}+\frac{1}{96}\tilde{H}_{\alpha\beta\gamma}\tilde{H}^{\alpha\beta\gamma}\tilde{H}_{\delta\epsilon\varepsilon}\tilde{H}^{\delta\epsilon\varepsilon}-R_{\alpha\beta}R^{\alpha\beta}$$
$$+\frac{1}{24}\tilde{H}_{\alpha}{}^{\delta\epsilon}\tilde{H}^{\alpha\beta\gamma}\tilde{H}_{\beta\delta}{}^{\varepsilon}\tilde{H}_{\gamma\epsilon\varepsilon}+R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}-\frac{1}{2}\tilde{H}_{\alpha}{}^{\delta\epsilon}\tilde{H}^{\alpha\beta\gamma}R_{\beta\gamma\delta\epsilon}+2\tilde{H}^{\alpha\beta\gamma}\Omega_{\alpha\beta\gamma}\Big]\Big), \tag{17}$$

where $\tilde{H}=e^{\Phi}H$. Note that the YM couplings in the first bracket are double traces. The couplings in the last bracket occur at the disk level, while the others appear at higher genus levels. As mentioned in the Introduction section, these couplings are valid for spacetimes with a single Killing self-dual circle because the original couplings in the heterotic theory (15) are exact only in such spacetimes.

At the four-derivative order, the disk-level effective action of type I theory incorporates the single-trace coupling $\mathrm{Tr}(F^4)$. In heterotic theory, this coupling emerges as a one-loop contribution and is related, via a supersymmetry transformation, to the anomaly cancellation mechanism in 10-dimensional spacetime. Consequently, it represents an exact coupling in 10 dimensions for both type I and heterotic theories [18]. However, in spacetimes with a single Killing circle—effectively reducing the theory to 9 dimensions—[30] argues that the anomaly cancellation term vanishes in the heterotic theory, along with the coupling $\mathrm{Tr}(F^4)$. As a result, in such backgrounds, the coupling $\mathrm{Tr}(F^4)$ in type I theory can no longer be considered exact and, consequently, does not appear in the 9-dimensional heterotic theory.

A similar approach can be applied to the sphere-level heterotic couplings at order α'^2, as identified in [25], to derive the corresponding six-derivative couplings in type I theory.

3 Six-dimensional Heterotic/Type IIA duality

In this section, we will employ the no-go theorem, which asserts that there are no sphere-level NS-NS couplings at order α' in the reduction of the type IIA effective action on a $K3$ manifold, to verify the sphere-level NS-NS couplings of the 10-dimensional type II theory recently derived through T-duality [26]. The R^3H^2 structure of these couplings has already been identified in [46] using the S-matrix method, and it has been shown that their $K3$ reduction yields couplings that can be transformed into six-field and higher-order couplings through appropriate field redefinition. Since the complete NS-NS couplings are now known from T-duality, we can verify them by applying the aforementioned no-go theorem.

Assuming the dilaton and B-field are independent of the $K3$ manifold and the metric is block-diagonal as

$$ds^2=G_{\mu\nu}(x)dx^{\mu}dx^{\nu}+g_{ab}(y)dy^a dy^b, \tag{18}$$

the reduction of the leading two-derivative NS-NS couplings is

$$\mathbf{S}^{(0)}_{\rm typeII}=-\frac{2V}{\kappa^2}\int d^6x\sqrt{-G}e^{-2\Phi}\left[R+4\nabla_{\alpha}\Phi\nabla^{\alpha}\Phi-\frac{1}{12}H_{\alpha\beta\gamma}H^{\alpha\beta\gamma}\right], \tag{19}$$

where V is the volume of the $K3$ manifold,

$$\int_{K3} d^4y\sqrt{g} = V. \tag{20}$$

We have kept only the six-dimensional NS-NS fields and ignored all moduli fields.

The sphere-level NS-NS couplings of type II have been identified in [26] by imposing T-duality on the minimal basis of NS-NS fields, which includes 872 couplings. T-duality uniquely determines all coupling constants up to one overall factor [26]. These couplings can be expressed in various schemes. In particular, the scheme in which the dilaton appears only through the overall factor $e^{-2\Phi}$ [49] includes the following terms, which contain the Riemann-squared term:

$$\begin{aligned}\mathbf{S}^{(3)}_{\rm typeII} &= \frac{2\alpha'^3\zeta(3)}{2^6\kappa^2}\int d^{10}x\sqrt{-G}e^{-2\Phi}\Big[R_{\mu\nu\rho\tau}R^{\mu\nu\rho\tau}\Big(-\frac{5}{32}H_{\alpha\beta}{}^{\delta}H^{\alpha\beta\gamma}H_{\gamma}{}^{\epsilon\varepsilon}H_{\delta\epsilon\varepsilon}+\frac{5}{4}H_{\alpha}{}^{\delta\epsilon}H^{\alpha\beta\gamma}R_{\beta\delta\gamma\epsilon}\\ &\qquad -\frac{5}{24}\nabla_{\delta}H_{\alpha\beta\gamma}\nabla^{\delta}H^{\alpha\beta\gamma}\Big)+\cdots\Big],\end{aligned} \tag{21}$$

where the dots refer to all other terms that do not include the Riemann-squared term $R_{\mu\nu\rho\tau}R^{\mu\nu\rho\tau}$. Up to a factor of $-2/3$, the couplings in the scheme [26] also include the aforementioned terms. Since $K3$ surfaces are Ricci-flat, the Ricci tensor vanishes. However, the Riemann curvature tensor does not. The integral of the Riemann-squared term on the $K3$ manifold is a constant, which is given by [46]

$$\frac{1}{32\pi^2}\int_{K3} d^4y\sqrt{g}R_{abcd}R^{abcd} = 24\,. \tag{22}$$

Using the above result, one finds that the $K3$ reduction of the 10-dimensional action (21) produces the following four-derivative couplings in six dimensions, as well as the same eight-derivative couplings as in (21), but in six dimensions, which we are not interested in:

$$\begin{aligned}\mathbf{S}^{(1)}_{\rm typeII} &= \frac{48\pi^2\alpha'^3\zeta(3)}{\kappa^2}\int d^6x\sqrt{-G}e^{-2\Phi}\Big[\Big(-\frac{5}{32}H_{\alpha\beta}{}^{\delta}H^{\alpha\beta\gamma}H_{\gamma}{}^{\epsilon\varepsilon}H_{\delta\epsilon\varepsilon}+\frac{5}{4}H_{\alpha}{}^{\delta\epsilon}H^{\alpha\beta\gamma}R_{\beta\delta\gamma\epsilon}\\ &\qquad -\frac{5}{24}\nabla_{\delta}H_{\alpha\beta\gamma}\nabla^{\delta}H^{\alpha\beta\gamma}\Big)\Big].\end{aligned} \tag{23}$$

Up to some total derivative terms, the above couplings can be written as

$$\delta\Phi D\Phi + \delta G_{\alpha\beta}DG^{\alpha\beta} + \delta B_{\alpha\beta}DB^{\alpha\beta}, \tag{24}$$

where the six-dimensional equations of motion $D\Phi, DG^{\alpha\beta}, DB^{\alpha\beta}$ are

$$\begin{aligned} D\Phi &\equiv -2V\sqrt{-G}e^{-2\Phi}\Big(R+4\nabla_\alpha\nabla^\alpha\Phi-4\nabla_\alpha\Phi\nabla^\alpha\Phi-\frac{1}{12}H^2\Big)=0,\\ DB^{\alpha\beta} &\equiv \frac{V}{2}\sqrt{-G}\nabla_\gamma(e^{-2\Phi}H^{\alpha\beta\gamma})=0,\\ DG^{\alpha\beta} &\equiv V\sqrt{-G}\Big[\frac{1}{4}e^{-2\Phi}H^{\alpha\gamma\delta}H^{\beta}{}_{\gamma\delta}-e^{-2\Phi}\Big(R^{\alpha\beta}+2\nabla^\alpha\nabla^\beta\Phi\Big)\Big]-\frac{1}{4}D\Phi^{\alpha\beta}=0,\end{aligned} \tag{25}$$

and the field redefinitions defined as $G_{\alpha\beta}+\alpha'\delta G_{\alpha\beta}$, $B_{\alpha\beta}+\alpha'\delta B_{\alpha\beta}$ and $\Phi+\alpha'\delta\Phi$ are:

$$\begin{aligned}
\delta B_{\alpha\beta} &= \frac{5c}{4}\nabla_\gamma H_{\alpha\beta}{}^\gamma,\\
\delta\Phi &= \frac{5c}{32}H^2+\frac{5c}{2}\nabla_\alpha\Phi\nabla^\alpha\Phi,\\
\delta G_{\alpha\beta} &= \frac{5c}{8}H_{\alpha\gamma\delta}H_\beta{}^{\gamma\delta}+5c\nabla_\beta\nabla_\alpha\Phi,
\end{aligned}\tag{26}$$

where $c=-24\pi^2\zeta(3)\alpha'^2/V$. Hence, the NS-NS couplings (21) satisfy the no-go theorem in six dimensions. These couplings have already been confirmed by demonstrating that their cosmological reduction satisfies the $O(d,d,\mathbb{R})$ symmetry [50], and their R^3H^2 component aligns with the results from S-matrix calculations [51].

4 Discussion

In this paper, we have demonstrated that the consistency of heterotic supergravity (4) with T-duality requires an infinite number of new covariant couplings at all orders of α'. The couplings at orders α' and α'^2 have already been identified in [24, 25]. This T-dual multiplet may correspond, through suitable non-covariant field redefinitions, to the locally Lorentz-invariant supersymmetric multiplet proposed in [31, 32]. Such field redefinitions at order α' for NS-NS fields have been explicitly derived in [52]. Because the T-dual multiplet incorporates the leading-order two-derivative couplings, its genus dependence is exact and remains free from quantum corrections in spacetimes with a single Killing self-dual circle. However, at the classical level, there exist infinitely many additional T-dual multiplets at orders α'^3 and higher that do not contain the leading-order couplings [38]. Consequently, in such spacetimes, all of these additional multiplets may be subject to quantum corrections.

To clarify the final point, we note that classical couplings are background-independent. As a result, the radius of the circle in the T-duality constraint can be arbitrary, and classical T-dual multiplets in heterotic theory remain valid for any radius—even in globally flat Minkowski spacetime. In contrast, quantum corrections are background-dependent. The constraints imposed by T-duality at the quantum level generally depend on the radius of the circle. However, at the self-dual radius, the quantum T-duality constraints align with their classical counterparts [30]. Therefore, classical couplings can be extended to include quantum corrections at the self-dual radius by promoting the overall factor of the classical T-dual multiplet to one that incorporates quantum effects. This factor can be computed using the S-matrix method in backgrounds with a single self-dual circle. Importantly, the quantum T-dual multiplet that includes the leading two-derivative action receives no quantum corrections, as dictated by S-matrix calculations, which show that the leading-order effective action remains uncorrected [8, 35]. On the other hand, classical T-dual multiplets with coefficients such as $\zeta(3)$ do not contain two-derivative terms and thus do receive quantum corrections. Consequently, only the classical T-dual multiplet that includes the two-derivative couplings is exact and unaffected by quantum corrections at the self-dual radius. All other multiplets, especially those with coefficients involving $\zeta(3)$ and higher ζ-functions, are subject to quantum corrections and therefore do not constitute exact T-dual multiplets. As a result, they cannot be reliably analyzed within the framework of S-duality.

We then examine the exact T-dual multiplet under the S-duality of 10-dimensional heterotic/type I theory to explore its manifestation in type I theory. These multiplets appear at various genera in type I theory. Since the T-dual multiplet involves higher-derivative couplings, their transformations under S-duality require understanding whether S-duality receives higher-derivative corrections. We observe that S-duality does not receive higher-derivative corrections. Using this insight, we identify the order α' couplings in Type I theory (17), which contain no dilaton derivatives. Additionally, we utilize the S-duality of six-dimensional heterotic/type IIA theory to verify the 10-dimensional NS-NS couplings of type IIA theory at order α'^3, which were recently identified through T-duality.

The couplings at order α' in type I theory (17) encompass disk-level couplings as well as couplings at genus with Euler numbers 0 and -1. While calculating the three- and four-graviton, R-R, and open string YM vertex operators on the world-sheet with Euler numbers 0 and -1 presents significant challenges, the corresponding disk-level calculation remains feasible. Conducting this calculation to confirm the couplings in the last bracket of (17) would be a valuable endeavor. Furthermore, utilizing the T-duality relationship between type I theory compactified on a circle and type IIA theory on the corresponding orbifold [40], known as type I' theory, presents exciting opportunities. Applying T-duality to the couplings (17) in type I theory has the potential to uncover corresponding couplings in type I' theory.

If one could calculate all genus and all α' couplings in the effective action (2), then this action could generate any S-matrix element in quantum gravity that is UV finite. It would produce the result for quantum gravity that ordinary heterotic string theory produces. The higher α' expansion dictates the stringy nature of quantum gravity, and the genus expansion dictates the quantum corrections required for higher-derivative couplings to be consistent with quantum theory. However, the exact T-dual multiplet has no genus expansion in spacetimes with a single Killing self-dual circle. It is

$$\mathbf{S}_{\rm exact} \;\sim\; \int d^{10}x\, e^{-2\Phi}\sqrt{-G}\,\mathcal{L}_{\rm exact}(\alpha')\,, \qquad (27)$$

where $\mathcal{L}_{\rm exact}(\alpha')$ represents the supergravity (4) and all higher-derivative exact-corrections that can, in principle, be calculated via T-duality. The higher-derivative terms highlight the stringy nature of gravity; however, each higher-derivative term lacks quantum correction. The above action represents heterotic supergravity, extended to maintain consistency with T-duality. While it is known that heterotic supergravity is not UV finite, the inclusion of an infinite number of higher-derivative couplings, akin to ordinary string theory, suggests it may achieve UV finiteness. If so, this T-dual heterotic supergravity could be proposed as a definition of quantum gravity. Given its consistency with T-duality, one might identify a particular chiral string theory, such as those proposed in [53, 54], whose low-energy effective action is (27). While this effective action has only classical couplings in one setting, its type I setting includes both classical and quantum couplings. Investigating this proposal for quantum gravity would be intriguing.

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