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PAPER

Systematic study of the isotopic dependence of heavy-ion fusion cross sections at below- and above-barrier energies

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Abstract

The present work provides a systematic study on the role of nuclear surface tension in the isotopic dependence of the fusion cross sections at below- and above-barrier energies over wide range of neutron content ($0.5 < N/Z < 1.7$). To realize our goal, we select three different versions of proximity-based potential, involving proximity potential 1977, 1988, and 2010, in order to calculate the nucleus-nucleus potential and ultimately the fusion barrier parameters. It is shown that the barrier positions, heights, and curvatures follow a (second-order) non-linear isotopic behavior with addition of neutrons which are dependent on the effect of variation in the nuclear surface tension. Our findings reveal that the sensitivity of isotopic dependence of the fusion barrier characteristics to the effect of surface energy coefficients γ increases by increasing the asymmetry of the colliding pair. In addition, we demonstrate the sensitivity toward the coefficient γ is seen more clearly from the more neutron-rich nuclei compared to the neutron-deficient ones. We discuss the isotopic dependence of the fusion cross sections at below- and above-barrier energies within the framework of the Wong model for a single potential barrier. For above-barrier energies, it is shown that the fusion cross sections follow an increasing (second-order) non-linear trend due to the addition of neutrons. While a decreasing (second-order) non-linear trend exists for the variation in the fusion cross sections at below-barrier energies. Simultaneous comparison the results obtained by the 3 versions of proximity potential for the isotopic dependence of fusion cross sections in the mentioned energy regions reveal the importance of the quantum tunneling and also nuclear structure effects.

1. Introduction

In recent decades, the study of heavy-ion fusion reactions has been recognized as one of the most widely used and attractive fields of research by nuclear physicists. If we want to provide a simple definition of this type of reaction, we must say that in general, the approaching process of two reacting nuclei, their overlapping and the formation of a composite nuclear system is called 'fusion reaction'. It should be noted that this physical process will be accompanied by the formation of a Coulomb potential barrier, so-called 'fusion barrier', due to the competition between the long-range repulsive Coulomb and short-range attractive nuclear forces. The barrier height V_B, position R_B and curvature $\hbar\omega_B$ are the parameters of the bare Coulomb barrier. To calculate the theoretical values of these parameters, we need to use the following conditions,

$$\left(\frac{dV_{\text{tot}}(r)}{dr}\right)_{r=R_B} = 0; \qquad \left(\frac{d^2V_{\text{tot}}(r)}{dr^2}\right)_{r=R_B} \leqslant 0; \qquad \hbar\omega_B = \hbar\left(\frac{1}{\mu}\frac{d^2V_{\text{tot}}(r)}{dr^2}\right)^{1/2}_{r=R_B}. \tag{1}$$

It is clearly visible from these conditions that the total interaction potential $V_{\text{tot}}(r)$ plays a key role in the evaluation of heavy-ion fusion reactions. Further, the nucleus-nucleus potential is essential for calculating the fundamental characteristics of this type of nuclear reactions, such as fusion cross sections [1–5]. The properties of the Coulomb component of the potential have already been studied quite well, whereas the situation turns to be more complicated for nuclear part of the nucleus-nucleus potential. Under these conditions, one can find that the starting point for performing the theoretical study of fusion dynamics is to define a realistic nuclear potential $V_N(r)$. With the passage of time, different theoretical models have been proposed to evaluate the strength of the potential energy of nuclear interaction during the fusion process [6–12]. The proximity potential formalism [13] is one of the most important and widely used approaches to obtain the nuclear part $V_N(r)$, see for example [3, 14–17]. As a result of the literature, this formalsim enables us to analyze the influence of various physical effects such as surface energy coefficients and nuclear surface diffuseness in the fusion of heavy-ions [18–20]. Recently, we systematically developed a theoretical framework to explore the role of nuclear surface tension coefficient γ in the isotopic behavior of the fusion dynamics [21]. In that study, we restricted ourselves to neutron-rich colliding systems with the condition of $1 \leqslant N/Z < 1.6$. The obtained results revealed that the isotopic dependence of the fusion barrier heights and positions as well as fusion cross sections in heavy-ion collisions above the Coulomb barrier energies are sensitive to the change in the nuclear surface tension between the reacting nuclei.

In the present study an effort is made to extend our previous study to the colliding systems with neutron- and proton-rich nuclei in the range $0.545 \leqslant N/Z \leqslant 1.678$ for their compound nuclei. We have studied three series of colliding nuclei (namely ^{A_1}O+ ^{A_2}Si, ^{A_1}Si+ ^{A_2}Si, and ^{A_1}Ca+ ^{A_2}Ca) involving different isotopes of $^{12-18}$O (with $N/Z = 0.5 - 1.25$), $^{22-40}$Si (with $N/Z = 0.571 - 1.857$), and $^{34-48}$Ca (with $N/Z = 0.70 - 1.40$). In order to analyze the impact of nuclear surface tension on the isotopic dependence of fusion barriers and cross sections, we use the original and modified forms of the proximity potential (namely proximity potentials 1977, 1988, and 2010) to calculate the nucleus-nucleus potential. In accordance with the different sets of γ parameters introduced in Ref. [20], one can find that the present proximity potentials provide an adequate range for the variation in the strength of nuclear surface tension. To study the isotopic dependence of fusion probabilities and thus fusion cross sections, our theoretical framework is based on the Wong formula for a one-dimensional potential barrier for spherical interacting nuclei. In the present work, we are interested in studying the isotopic variation in the calculated values of the fusion cross sections at both below- and above-barrier energies. The comparison of the obtained results in these two energy ranges gives us possibility to look for the role of nuclear structure effects in isotopic dependence of complete fusion cross sections.

This paper is organized as follows. The details of the calculations of the total nucleus-nucleus interaction potential are presented in section 2. The obtained numerical results and corresponding discussions are presented in section 3. Finally, the summary and conclusions of the present study are presented in section 4.

2. Theoretical formalism

As mentioned in section 1, one of the simple and applicable formalisms in describing the interaction of two colliding nuclei in the complete fusion channel is the proximity potential which was put forward for the first time by Blocki *et al* [13]. According to the original version of proximity potential 1977 (marked as Prox. 77), the nuclear component of the total interaction potential can be written as

$$V_N(r) = 4\pi\gamma b\overline{R}\Phi\left(\frac{r - C_1 - C_2}{b}\right). \quad (2)$$

As can be seen from this equation, the nuclear proximity potential is composed of several ingredients that depend on the shape and geometry of the reacting nuclei as well as the relative separation distance between surfaces of two interacting nuclei; $s = r - C_1 - C_2$. Notice that they allow us to increase the accuracy of the proximity potential formalism in describing the nucleus-nucleus interaction potential and thus heavy-ion fusion cross sections. For example in recent years, several modifications have been proposed over the original version Prox. 77 through the surface energy coefficients γ, nuclear radii R_i, and universal function $\Phi(\xi)$. Readers can find the more details about the proximity potentials Prox. 77, Prox. 88, and Prox. 2010 in Refs. [16–18, 21].

3. Results and discussion

In numerical calculations, we adopt 3 versions of the proximity potential formalisms including Prox. 77, Prox. 88 and Prox. 2010 to investigate the influence of nuclear surface tension upon the isotopic dependence of fusion dynamics for different colliding systems. Within the framework of these formalisms, in the first step, we calculate the parameters of the Coulomb barrier for 30 isotopic systems including ^{A_1}O+ ^{A_2}Si (9 fusion reactions with $0.545 \leqslant N/Z \leqslant 1.090$ and $Z_1Z_2 = 112$), ^{A_1}Si+ ^{A_2}Si (11 fusion reactions with $0.571 \leqslant N/Z \leqslant 1.678$ and $Z_1Z_2 = 192$), and ^{A_1}Ca+ ^{A_2}Ca (10 fusion reactions with $0.70 \leqslant N/Z \leqslant 1.40$ and $Z_1Z_2 = 400$). Herein, N and Z refer to the neutron and proton numbers of compound nuclei formed by different pairs of collision partners. On the base of the mentioned conditions for the neutron to proton ratio, one can find that the colliding systems studied presently consist of both proton-rich ($N/Z \leqslant 1$) and neutron-rich ($N/Z \geqslant 1$) participant nuclei. The

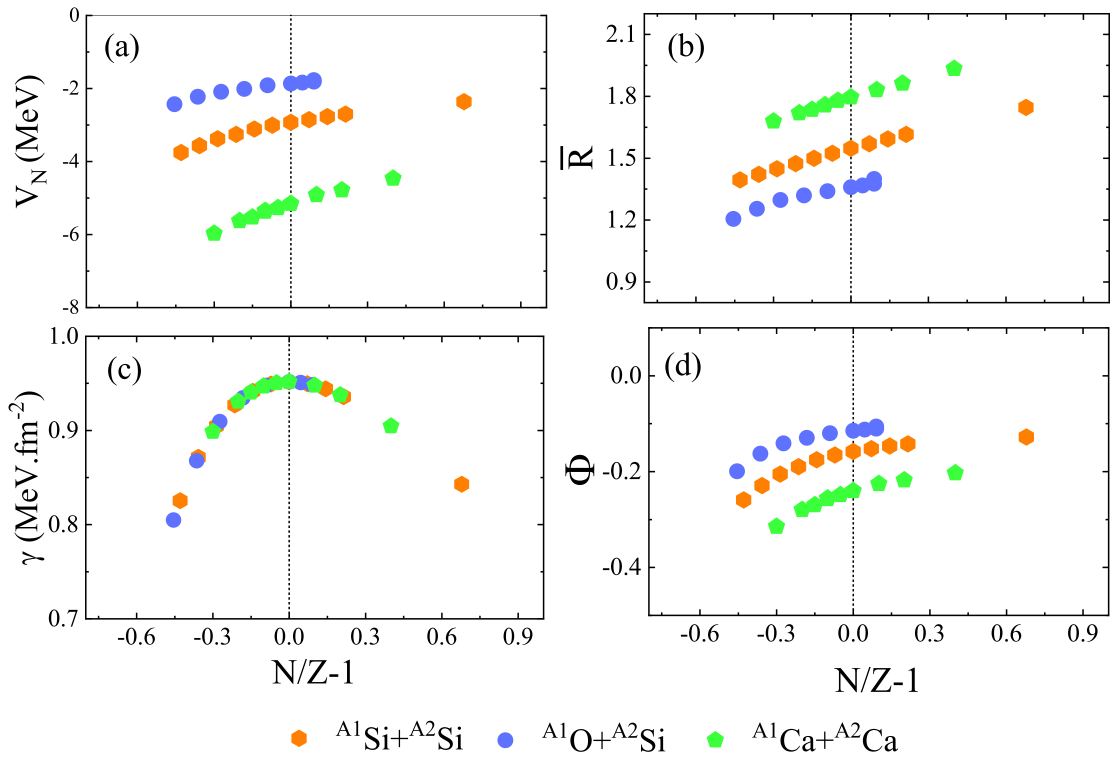

Figure 1. The variation trend of the nuclear potential V_N along with its different factors including mean curvature radius $\overline{R}$, surface energy coefficient γ, and universal function Φ as a function of $(\frac{N}{Z} - 1)$. Note that the calculations have been performed using Prox. 77 model at the radial distance equal to the fusion barrier position R_B, as an example.

theoretical values of the parameters R_B, V_B, and $\hbar\omega_B$ obtained from the three versions of Prox. 77, Prox. 88, and Prox. 2010 reveal the sensitivity of the fusion barriers to the variation of the surface energy coefficients.

It is of interest to explore the asymmetry dependence of different factors of nuclear potential, including the surface energy coefficient γ, mean curvature radius $\overline{R}$, and universal function $\Phi(\xi)$. To reach this goal we have plotted the variation trend of these quantities as a function of the N/Z ratio from neutron-deficient isotopic systems to neutron-rich ones, as shown in figure 1. The calculations of this figure have been performed using the original version Prox. 77. In addition, the strength of the nuclear potential and universal function have been calculated at a radial distance equal to the fusion barrier position R_B. From figure 1, we see a systematic linear decrease in the attractive strength of the nuclear potential with neutron-proton ratio. The universal function yields similar results, while the mean curvature radius $\overline{R}$ follows linear increase asymmetry dependence. Also from the figure 1, one notice that among different factors the surface energy coefficient has a strong dependence on the asymmetry of the reacting nuclei. Finally, it is clear that the obtained results for the isotopic system ^{A_1}Ca + ^{A_2}Ca have a stronger dependence toward the asymmetry content in comparison with the two other systems.

3.1. Analysis of the isotopic dependence of the fusion barrier characteristics

In order to show the effect of addition and/or removal of the neutrons on the calculated values of fusion barrier characteristics, including R_B, V_B, and $\hbar\omega_B$, one can analyze the variation in these values over the symmetric colliding nuclei ($N = Z$). To reach this goal, the percentage difference of the parameters of the Coulomb barrier are calculated for all isotopic systems using the following relations

$$\Delta X_B(\%) = \frac{X_B - X_B^0}{X_B^0} \times 100, \quad (\text{with } X = R_B, V_B, \hbar\omega_B). \tag{3}$$

Note that R_B^0, V_B^0, and $\hbar\omega_B^0$ are the barrier characteristics for the $N = Z$ case. Similar to previous works [22–24], one can find that the increase of barrier positions and decrease of barrier heights and barrier curvature are non-linear in the whole selected mass range from neutron-deficient ($N/Z \leqslant 1$) isotopic systems to neutron-rich ($N/Z \geqslant 1$) ones. It is well known that the nuclear potential becomes more attractive with the addition of neutrons [22–24]. Under these conditions, it would seem reasonable that the fusion barrier reduces in going from neutron-deficient to neutron-rich colliding systems. To parameterize the observed processes, we suggest the following equations

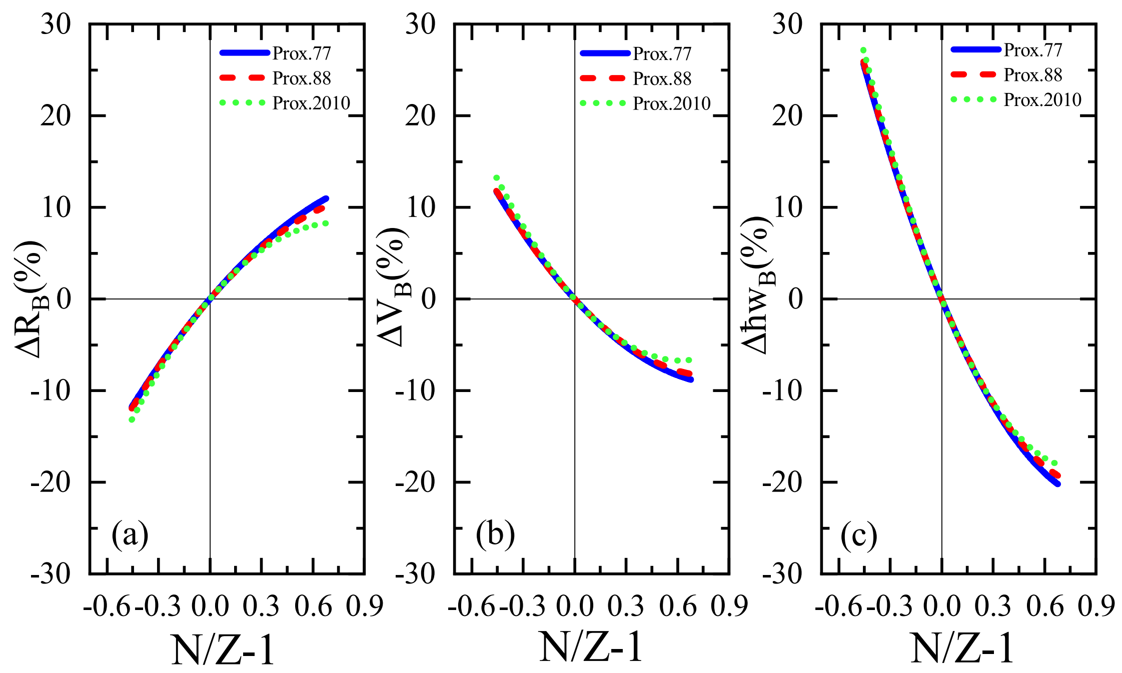

Figure 2. Comparison of the fitted lines of $\Delta R_B(\%)$, $\Delta V_B(\%)$, and $\Delta\hbar\omega_B(\%)$ due to the original version of the proximity potential with those obtained from the modified versions Prox. 88 and Prox. 2010.

Table 1. The values of constant coefficients α_i, β_i, and η_i extracting from the parametrization of isotopic variations in fusion barrier positions R_B, heights V_B, and curvatures $\hbar\omega_B$.

Proximity-model	α_1	α_2	β_1	β_2	η_1	η_2
Prox. 77	21.989	−8.548	-20.634	11.289	−45.671	23.385
Prox. 88	21.738	−10.051	-20.364	12.236	−45.456	25.117
Prox. 2010	22.205	−14.813	-21.384	17.030	−46.433	29.163

$$\Delta R_B(\%) = \alpha_1\left(\frac{N}{Z} - 1\right) + \alpha_2\left(\frac{N}{Z} - 1\right)^2, \tag{4}$$

$$\Delta V_B(\%) = \beta_1\left(\frac{N}{Z} - 1\right) + \beta_2\left(\frac{N}{Z} - 1\right)^2, \tag{5}$$

$$\Delta\hbar\omega_B(\%) = \eta_1\left(\frac{N}{Z} - 1\right) + \eta_2\left(\frac{N}{Z} - 1\right)^2, \tag{6}$$

where the extracted values of the constants α_i, β_i, and η_i using the Prox. 77, Prox. 88, and Prox. 2010 potential models are listed in table 1. The parameterized values of $\Delta R_B(\%)$, $\Delta V_B(\%)$, and $\Delta\hbar\omega_B$ using the Prox. 77 potential are simultaneously compared with those obtained by the other two versions of the potential in panels (a), (b) and (c) of figure 2, respectively. One can see that the results of different nuclear potentials are almost the same for the isotopic systems around $N = Z$. While the parameterized values of the barrier characteristics show clear differences when moving away from the symmetric colliding pairs.

From figure 2, it can be indicated that the observed differences increase with increasing the asymmetric effects in both ranges $N/Z \leqslant 1$ and $N/Z \geqslant 1$. This means that the role of nuclear surface tension between two approaching nuclei becomes less important when symmetry in the neutron/proton content is increased. However, it seems from figure 2 that the sensitivity of the isotopic behavior of the fusion barrier characterestics to the surface energy coefficient γ is more for colliding systems with neutron-rich participant nuclei than the neutron-deficient ones. This may be helpful in better understanding of the neutron-skin effects in the surface region of nucleus with a neutron excess [25]. In connection with the neutron skin formation, it should be noted that the preference of bulk nuclear matter for equality of the neutron and proton densities (and in fact for symmetry) leads to generate a driving force which tries to push the excess neutron out of the bulk region and into the surface of a neutron-rich nucleus ($N > Z$) [25]. Another point to note in figure 2 is that there is a direct link between the variation trend of the fusion barrier height and curvature. This subject clearly seen in figure 3. This

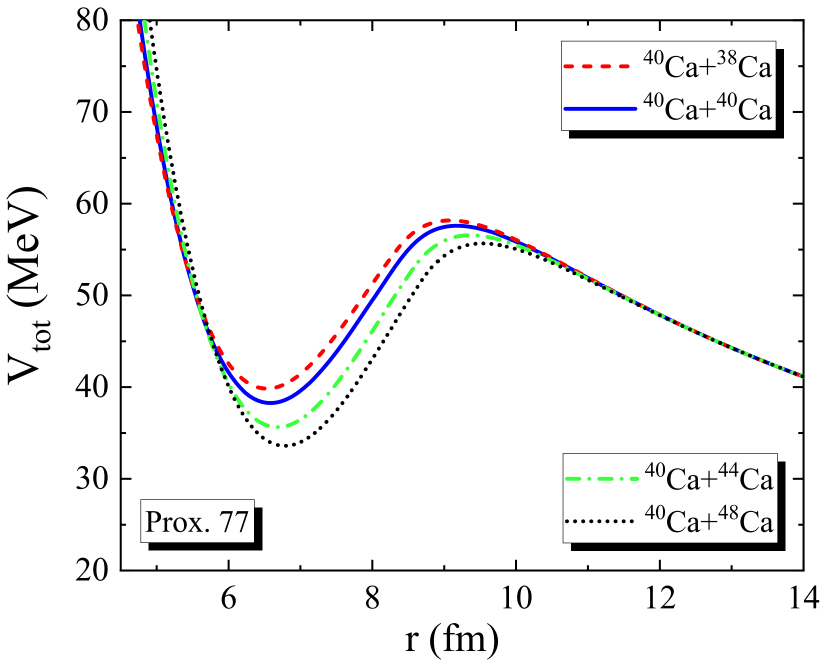

Figure 3. The distributions of the total interaction potential V_{tot} (in MeV) based on the original version of proximity potential 1977 for a few of isotopic systems ^{A_1}Ca+ ^{A_2}Ca as an example.

figure shows the distributions of the total interaction potential V_{tot} for a few of isotopic systems ^{A_1}Ca+ ^{A_2}Ca. Note that the calculations have been performed based on the proximity potential Prox. 77, as an example. From this figure, one can find out that the fusion barrier heights and therefore barrier curvatures follow a decreasing trend with increase in the neutrons by going from ^{40}Ca+^{38}Ca to ^{40}Ca+^{48}Ca

On analyzing the table 1, it is found that the extracted values of constant coefficients α_i, β_i, and η_i are sensitive to the cahnge in the strength of nuclear surface tension coefficient by going from the proximity potential Prox. 77 to Prox. 2010. In order to further understand the sensitivity of the isotopic behavior of fusion barrier characteristics to the surface tension effects in both ranges $N/Z \geqslant 1$ and $N/Z \leqslant 1$, in figure 4, we display the variations trend of three mentioned constants as a function of the coefficient γ_0. It is clear from the figure that these constants follow a linear decreasing and/or increasing trend as a function of the surface energy constant γ_0. We can formulate these trends as

$$\alpha_1 = 0.3397\gamma_0 + 21.5630, \qquad \alpha_2 = -11.8299\gamma_0 + 3.3030, \tag{7}$$

$$\beta_1 = -1.3158\gamma_0 - 19.1881, \qquad \beta_2 = 10.7450\gamma_0 + 0.4023, \tag{8}$$

$$\eta_1 = -1.3503\gamma_0 - 44.2056, \qquad \eta_2 = 10.9871\gamma_0 + 12.4770. \tag{9}$$

By substituting the present suggested formulas in equations (4), (5), and (6), the analytical parameterized forms for the percentage difference of the Coulomb barrier positions $\Delta R_B(\%)$, heights $\Delta V_B(\%)$, and curvatures $\Delta\hbar\omega_B(\%)$ become

$$\Delta R_B(\%) = [0.3397\gamma_0 + 21.5630]\left(\frac{N}{Z} - 1\right) + [-11.8299\gamma_0 + 3.3030]\left(\frac{N}{Z} - 1\right)^2, \tag{10}$$

$$\Delta V_B(\%) = [-1.3158\gamma_0 - 19.1881]\left(\frac{N}{Z} - 1\right) + [10.7450\gamma_0 + 0.4023]\left(\frac{N}{Z} - 1\right)^2, \tag{11}$$

$$\Delta\hbar\omega_B(\%) = [-1.3503\gamma_0 - 44.2056]\left(\frac{N}{Z} - 1\right) + [10.9871\gamma_0 + 12.4770]\left(\frac{N}{Z} - 1\right)^2. \tag{12}$$

The above equations confirm that the percentage difference of the Coulomb barrier positions, heights, and curvatures depend on not only the N/Z ratio but also the strength of nuclear surface tension coefficient in the whole isotopic region from proton-rich to neutron-rich colliding nuclei. In the previous work [21], we investigated such dependencies only for barrier heights and positions in the $N/Z \leqslant 1$ region.

3.2. Analysis of the isotopic dependence of the fusion cross sections

Here and in the following, we are interested in analyzing the isotopic behavior of the fusion cross sections over the present range of neutron content ($0.5 < N/Z < 1.7$). Note that the Wong formula [26] for the fusion cross

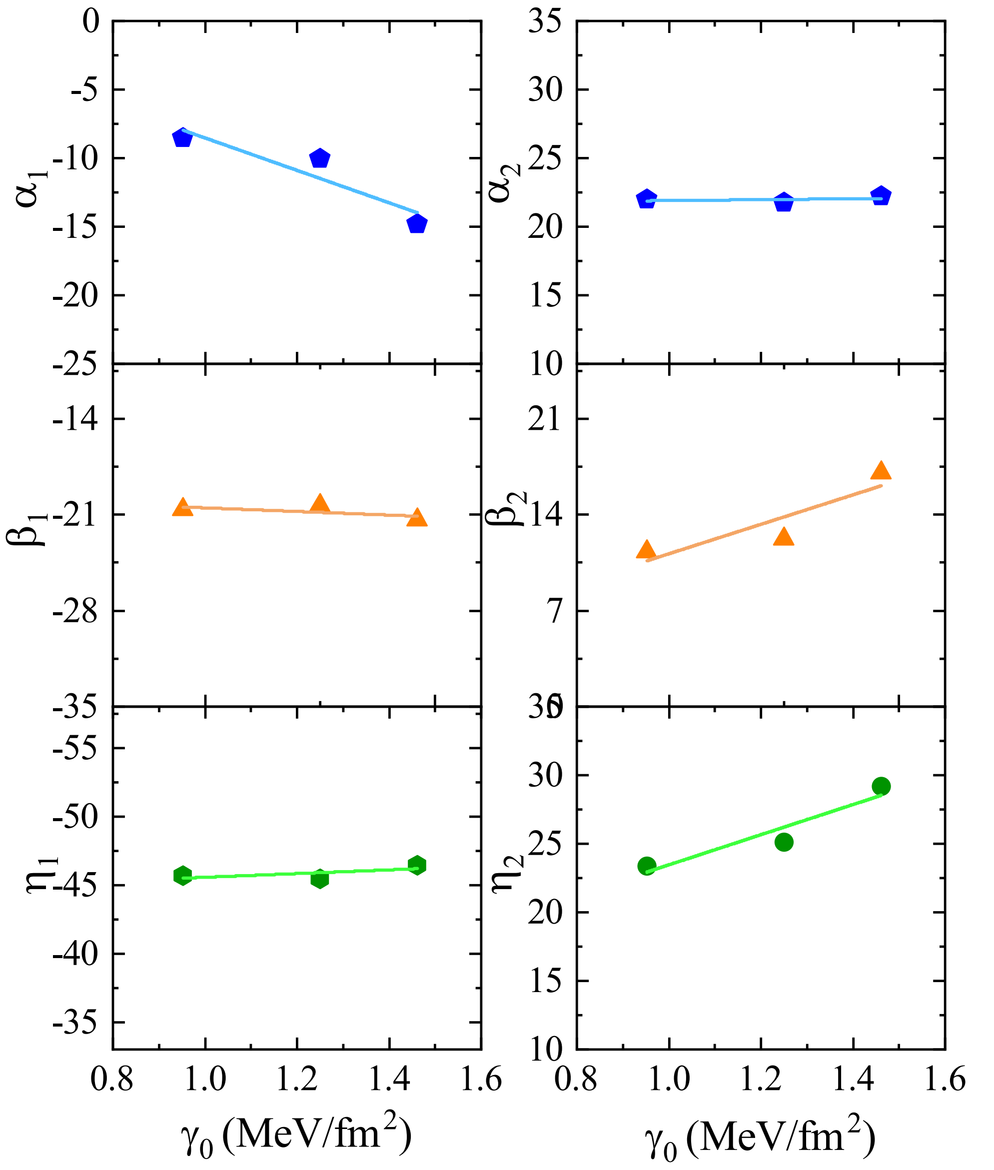

Figure 4. The variation of the coefficients α_i, β_i, and η_i appearing in the parameterization process of the isotopic behavior of fusion barrier position $\Delta R_B(\%)$, height $\Delta V_B(\%)$, and curvature $\Delta\hbar\omega_B(\%)$ as a function of the surface energy constant γ_0.

section ($\sigma_{\rm fus}$) for a single potential barrier is used to calculate the theoretical values of this quantity. It can be written in the following form

$$\sigma_{\rm fus}(E_{\rm c.m.}) = 10\frac{R_B^2\hbar\omega_B}{2E_{\rm c.m.}}\ln\left\{1 + \exp\left[\frac{2\pi}{\hbar\omega_B}(E_{\rm c.m.} - V_B)\right]\right\}, \tag{13}$$

where $E_{\rm c.m.}$ represents the energy of the collision in the center-of-mass frame. Notice that the calculations of the fusion cross sections are performed at near-barrier energies (including four below-barrier energies $E_{\rm c.m.} = 0.90V_B$, $E_{\rm c.m.} = 0.925V_B$, $E_{\rm c.m.} = 0.95V_B$, and $E_{\rm c.m.} = 0.975V_B$ as well as four above-barrier energies $E_{\rm c.m.} = 1.025V_B$, $E_{\rm c.m.} = 1.05V_B$, $E_{\rm c.m.} = 1.075V_B$, and $E_{\rm c.m.} = 1.10V_B$). For both energy regions, the theoretical values of $\sigma_{\rm fus}$ are calculated by equation (13). Let us examine the variation in the calculated values of the fusion cross sections with neutron content and also the strength of nuclear surface tension γ. In order to have a clearer picture about the isotopic behavior of the fusion cross sections in the near-barrier energies, in figure 5 (for above-barrier energies) and figure 6 (for below-barrier energies), we plot the percentage difference of $\Delta\sigma_{\rm fus}(\%)$ as a function of $(\frac{N}{Z} - 1)$ within various versions of proximity potential formalisms for all neutron- and proton-rich systems. We perform the calculations of these figures using the following relation

$$\Delta\sigma_{\rm fus}(\%) = \frac{\sigma_{\rm fus}(E^0_{\rm c.m.}) - \sigma^0_{\rm fus}(E^0_{\rm c.m.})}{\sigma^0_{\rm fus}(E^0_{\rm c.m.})} \times 100, \tag{14}$$

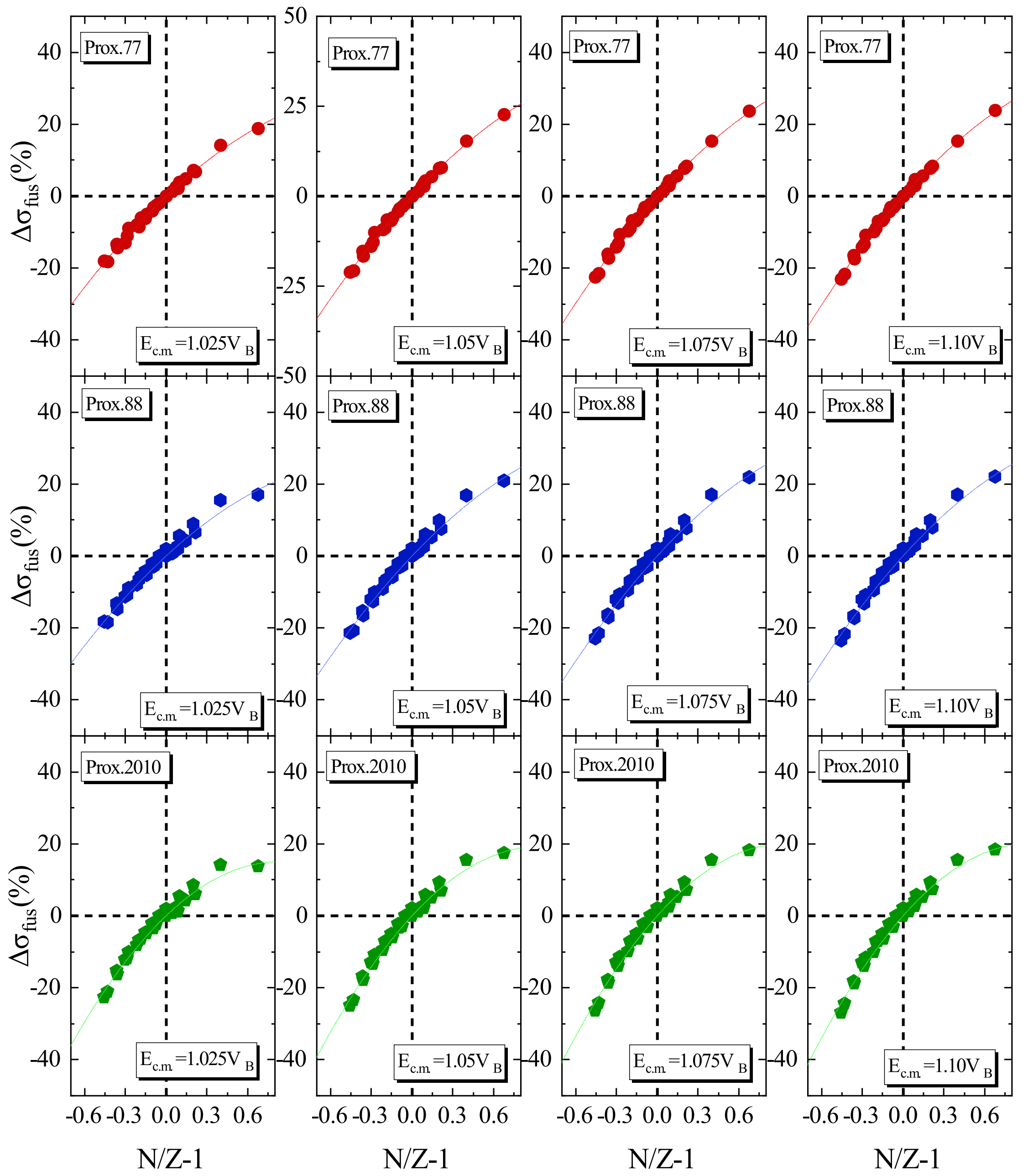

Figure 5. The variation trend of the calculated values of the $\Delta\sigma_{\text{fus}}(\%)$ as a function of $(\frac{N}{Z} - 1)$ using Prox. 77, Prox. 88, and Prox. 2010 models at above-barrier energies $E_{\text{c.m.}} = 1.025V_B$, $1.05V_B$, $1.075V_B$, and $1.10V_B$.

where $E^0_{\text{c.m.}}$ and σ^0_{fus} represent the center-of-mass energy and fusion cross section for a symmetric colliding pair, respectively. There are several important findings in the present calculations. (1) As can be seen from figures 5 and 6, the fusion cross sections follow a non-linear second-order behavior with the addition/removal of the neutron in both below- and above-barrier regimes. One can parameterize this behavior using following relations

$$\Delta\sigma_{\text{fus}}(\%) = \begin{cases} \xi_1\left(\dfrac{N}{Z} - 1\right) + \xi_2\left(\dfrac{N}{Z} - 1\right)^2, & \text{for } E_{\text{c.m.}} < V_B,\\ \lambda_1\left(\dfrac{N}{Z} - 1\right) + \lambda_2\left(\dfrac{N}{Z} - 1\right)^2, & \text{for } E_{\text{c.m.}} > V_B. \end{cases} \tag{15}$$

The extracted values of the coefficients ξ_i and λ_i appearing in the parametrization of isotopic variations in fusion cross sections have been listed in tables 2 and 3.

(2) figure 5 shows an increasing trend occurs for fusion cross sections with the increase in N/Z ratio. To interpret this behavior in the whole range $0.5 < N/Z < 1.7$, it must be noted that the decreasing or increasing trend of the fusion cross sections at above-barrier energies mainly originate from the calculated values of the parameters R_B and V_B. In fact, equation (13) can be reduced to well-known sharp cut-off formula for these

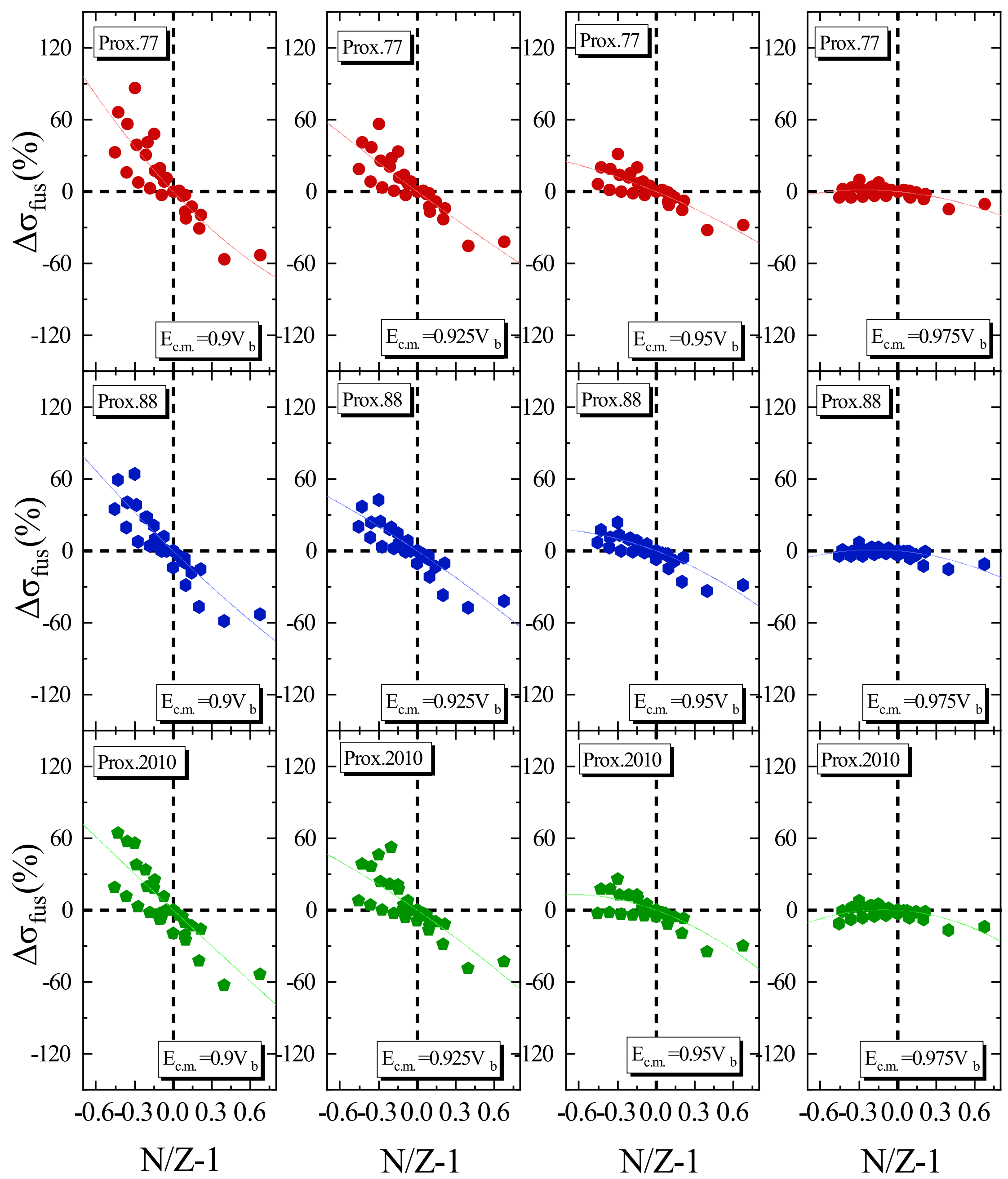

Figure 6. Same as figure 5, but for $\Delta\sigma_{\text{fus}}(\%)$ at below-barrier energies $E_{\text{c.m.}} = 0.90V_B$, $0.925V_B$, $0.95V_B$, and $0.975V_B$.

Table 2. The values of constant coefficients ξ_i extracting from the parametrization of isotopic variations in fusion cross sections σ_{fus} for the considered below-barrier energies.

	$E_{\text{c.m.}} = 0.9V_B$		$E_{\text{c.m.}} = 0.925V_B$		$E_{\text{c.m.}} = 0.95V_B$		$E_{\text{c.m.}} = 0.975V_B$	
Proximity-model	ξ_1	ξ_2	ξ_1	ξ_2	ξ_1	ξ_2	ξ_1	ξ_2
Prox. 77	−115.27	31.92	−78.93	4.93	−44.16	−11.96	−9.79	-18.96
Prox. 88	−104.28	12.10	−71.83	9.51	−40.38	−21.96	−8.72	−23.64
Prox. 2010	−100.48	2.16	−74.15	−10.65	−38.90	-28.49	−7.03	-21.68

energy regions as follows V_B [22–24]

$$\sigma_{\text{fus}}(E_{\text{c.m.}}) = 10\pi R_B^2\left(1 - \frac{V_B}{E_{\text{c.m.}}}\right). \tag{16}$$

Table 3. The values of constant coefficients λ_i extracting from the parametrization of isotopic variations in fusion cross sections σ_{fus} for the considered above-barrier energies.

	$E_{\text{c.m.}} = 1.025V_B$		$E_{\text{c.m.}} = 1.05V_B$		$E_{\text{c.m.}} = 1.075V_B$		$E_{\text{c.m.}} = 1.10V_B$	
Proximity-model	λ_1	λ_2	λ_1	λ_2	λ_1	λ_2	λ_1	λ_2
Prox. 77	34.77	10.77	40.92	−11.08	42.49	−11.92	43.02	-12.43
Prox. 88	34.84	−11.27	39.85	−11.35	41.44	−12.17	41.97	−12.68
Prox. 2010	36.20	−21.81	40.92	−21.68	42.42	−22.41	42.91	−22.87

According to this equation for incident energies $E_{\text{c.m.}} > V_B$, one can find that the variation in the calculated barrier positions R_B has a stronger effect on the isotopic behavior of the fusion cross section in comparison with fusion barrier heights. (3) From figure 6, it is evident that a decreasing dependence exists for the isotopic variation in fusion cross sections calculated at energies below the Coulomb barrier. From the physical point of view, it will be reasonable that the quantum tunneling phenomenon through the Coulomb barrier influences the calculations of heavy-ion fusion cross section at energies below the Coulomb barrier. In this situation, it is expected that one must take into account the calculated barrier curvatures $\hbar\omega_B$ to interpret the isotopic variations of the fusion cross sections. On the other hand according to the approximated formula for fusion cross section at these energy regions

$$\sigma_{\text{fus}}(E_{\text{c.m.}}) = 10\frac{R_B^2\,\hbar\omega_B}{2E_{\text{c.m.}}}\exp\left[\frac{2\pi}{\hbar\omega_B}(E_{\text{c.m.}} - V_B)\right], \tag{17}$$

it seems that barrier heights and curvatures play a more decisive role in specifying these variations compared to the calculated values of the fusion barrier position R_B. (4) Interestingly, the comparison between the results shown in figures 5 and 6 reveals that the fluctuations around the trend lines at below-barrier energies are relatively larger than the above-barrier ones. On the other hand, one can find that the observed fluctuations decrease by approaching the near-barrier energies ($E_{\text{c.m.}} \approx V_B$). Under these conditions, it can be concluded that the scattering around the trend lines may be the consequence of the importance of the nuclear structure effects (such as the coupling to the low-energy surface vibrational states as well as to the transfer of neurons with positive Q-values) at least for neutron-rich cases which are not included in the determination of the theoretical fusion cross sections within the framework of the Wong model for a one-dimensional potential barrier. However, there are many restrictions for imposing the coupled-channels effects in the calculations of the percentage difference of fusion cross sections $\Delta\sigma_{\text{fus}}(\%)$ for the isotopic systems of interest using the well-known computer codes such as CCFULL [27].

In figure 7, we display a comparison between the calculations results based on the Wong formula and the corresponding available experimental cross sections for the reactions involving O, Si, and Ca isotopes [28–31].

For better understanding, we have also displayed the results of the fusion cross sections as a function of energy $E_{\text{c.m.}}$ in linear scale. The role of surface energy coefficients in the sub-barrier fusion probabilitie is quite obvious. In fact, from this comparison we can conclude that the agreement with the experimental data enhances by increasing the strength of nuclear surface tension. In addition, one can see that the proximity potential 2010 well reproduces the experimental fusion cross sections for the lighter systems. While the experimental enhancement of fusion cross sections at sub-barrier energies for the heavier systems reveals the importance of the effects of nuclear structure of the colliding nuclei during fusion. In the following, we are interested in analyzing the energy-dependent behvaior of the coefficients ξ_i and λ_i extracted from the parametrization process of isotopic variations in fusion cross sections. To reach this goal, in figure 8, we plot the variation in the ratio of the calculated values of these coefficients to the surface energy constant γ_0 with the change in the center-of-mass energy $E_{\text{c.m.}}$ (in units of V_B) over the whole energy range from below- to above-barrier energies. In this figure, the results obtained from all 3 different proximity-based potentials Prox. 77, Prox. 88, and Prox. 2010 have been displayed. We notice that the energy-dependent behavior of the coefficients existing in the $(N/Z - 1)$ and $(N/Z - 1)^2$ terms of equation (15) are shown through δ_1 and δ_2 labels, respectively. Figure 8 allows us to analyze the sensitivity of the percentage difference of fusion cross sections $\Delta\sigma_{\text{fus}}(\%)$ to the change in the strength of the nuclear surface tension and also the the incident energy of the projectile. It is shown that the δ_1/γ_0 and δ_2/γ_0 ratios follow a second order non-linear dependence on the energy $E_{\text{c.m.}}$. Our detailed analysis on the below-barrier energies reveals that the coefficient δ_1 increases with the increase in the incident energy ofthe projectile. While the coefficient δ_2 shows the opposite behavior at $E_{\text{c.m.}} < V_B$ region. In fact, this coefficient decreases as the incident energy of the projectile increases. For above-barrier energies, one can see that the theoretical results obtained by different versions of the proximity potential formalisms tend to a constant value, especially for δ_2/γ_0 ratio. This means that the above-mentioned coefficients show a much weaker dependence on the variation in the incident energy at $E_{\text{c.m.}} > V_B$ region. We notice that the variation trend observed in figure 8 can be

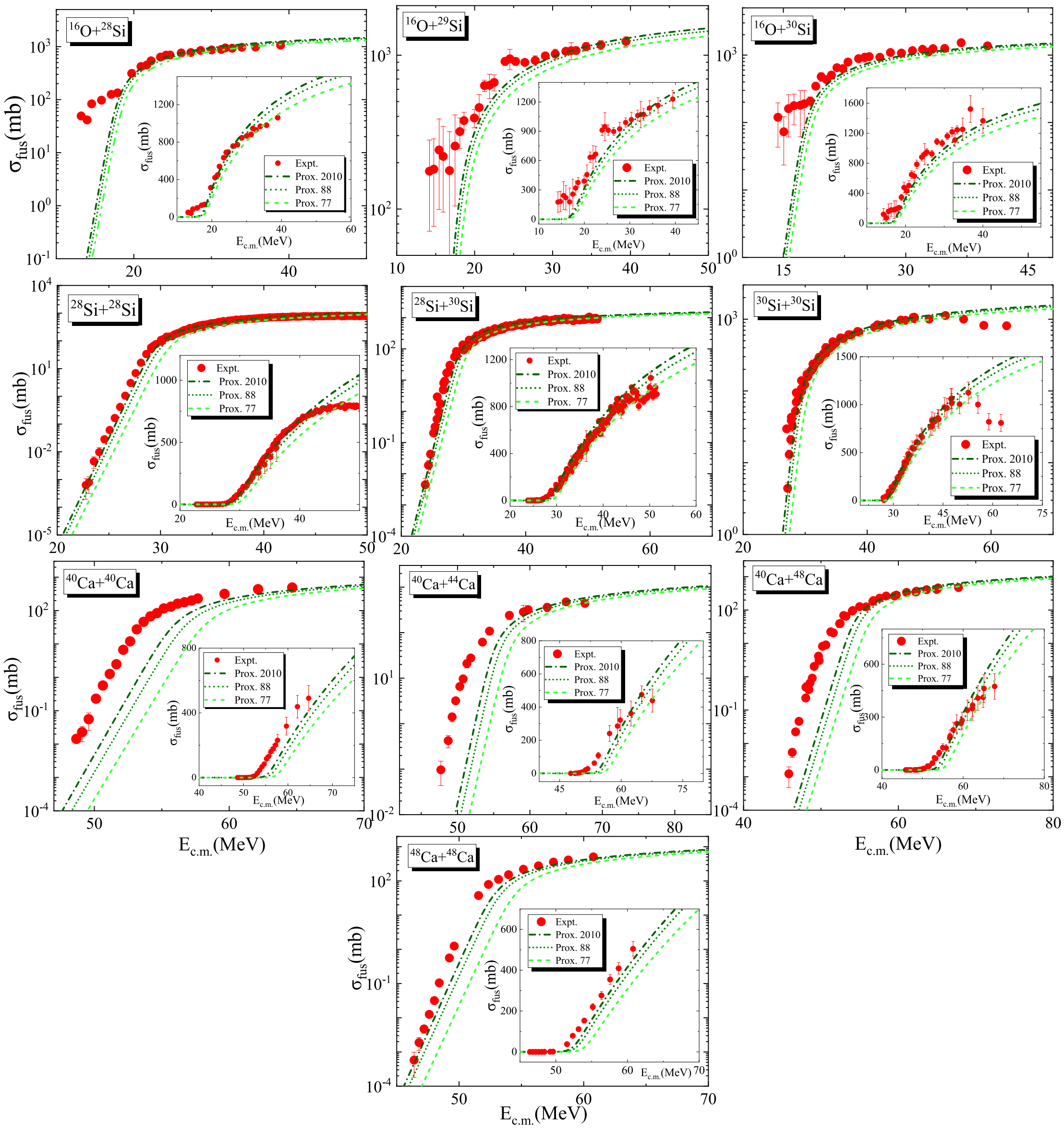

Figure 7. Comparison of experimental fusion cross sections [28–31] for different isotopic systems with results of the proximity potentials 77, 88, and 2010 calculations.

parameterized by the following second order non-linear forms

$$\delta_1(E_{\text{c.m.}}) = \gamma_0[-4045.387E_{\text{c.m.}}^2 + 8745.438E_{\text{c.m.}} - 4688.787], \tag{18}$$

and

$$\delta_2(E_{\text{c.m.}}) = \gamma_0[1901.900E_{\text{c.m.}}^2 - 3894.972E_{\text{c.m.}} + 1975.560], \tag{19}$$

By substituting the present suggested formulas in equation (15), we can introduce a new energy-dependent form for the calculated values of $\Delta\sigma_{\text{fus}}(\%)$ over the whole energy range as follows

$$\begin{aligned} \Delta\sigma_{\text{fus}}(\%) &= \gamma_0[-4045.387E_{\text{c.m.}}^2 + 8745.438E_{\text{c.m.}} - 4688.787]\left(\frac{N}{Z} - 1\right) \\ &+\gamma_0[1901.900E_{\text{c.m.}}^2 - 3894.972E_{\text{c.m.}} + 1975.560]\left(\frac{N}{Z} - 1\right)^2, \end{aligned} \tag{20}$$

This relation introduces a simultaneous dependence for the percentage difference of fusion cross sections upon the incident energy, nuclear surface tension, and N/Z ratio. It is clearly visible from figure 8 that the sensitivity to the change in the strength of the nuclear surface tension decreases by going from the below-barrier energies to above-barrier ones. These observations may provide a useful means of clarifying the importance of the nuclear structure effects at low incident energies.

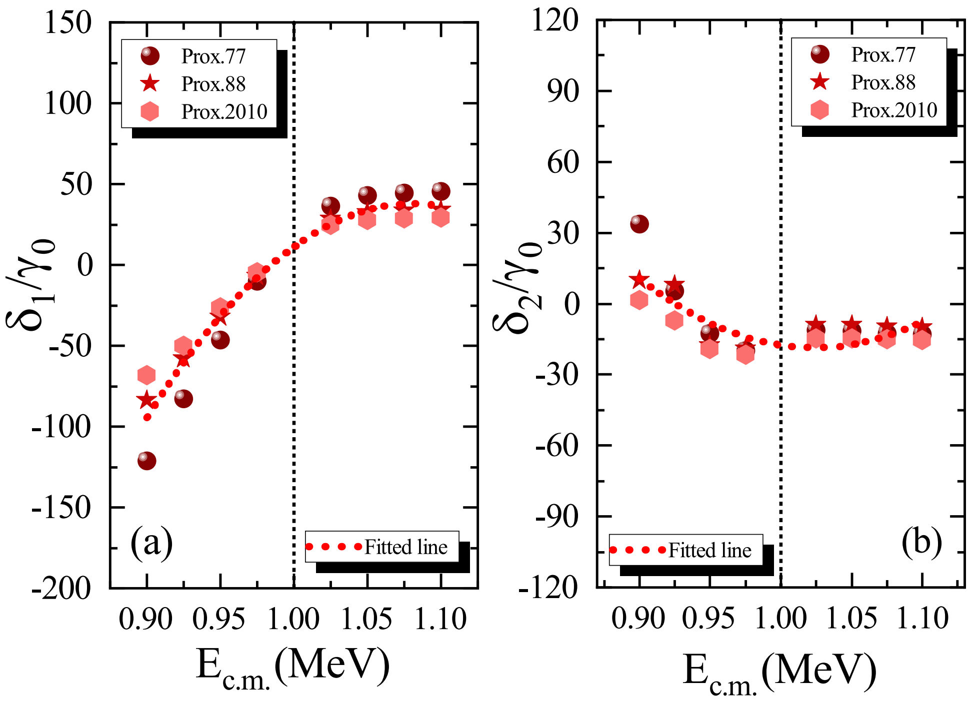

Figure 8. The variation of the δ_1/γ_0 and δ_2/γ_0 ratios appearing in the parameterization process of the isotopic dependence of fusion cross section as a function of the center-of-mass energy $E_{c.m.}$ (in unit of V_B) using the proximity potentials Prox. 77, Prox. 88, and Prox. 2010.

4. Conclusions

With an aim to extend the findings of the previous study, in the present work attempt is made to analyze systematically the importance of the effect of the surface tension coefficnet on the isotopic dependence of various characteristics of fusion process over a wide range of isotopic systems from proton-rich region to neutron-rich one. To reach this goal we have performed the calculations of the fusion barriers and cross sections using the proximity potential formalisms and Wong model, respectively. The present isotopic systems consist of colliding pairs O, Si and Ca with $112 \leqslant Z_1Z_2 \leqslant 400$. Our findings for analyzing the isotopic behavior of the fusion barrier characteristics demonstrate that the calculated values of $\Delta R_B(\%)$, $\Delta V_B(\%)$, and $\Delta\hbar\omega_B(\%)$ follow a non-linear behavior by adding neutrons gradually to either of the colliding pairs. Comparison of the parameterized values of the $\Delta R_B(\%)$, $\Delta V_B(\%)$, and $\Delta\hbar\omega_B(\%)$ resulting from the original version of the proximity potential with those obtained by the Prox. 88, and Prox. 2010 models shows that the results of Prox. 77 is less sensitive toward the asymmetry of colliding nuclei. In addition, our systematic observations and calculations well reveal that the nuclear surface tension effects have significant dependence of the asymmetry of reaction. In the present study, we have systematically analyzed the isotopic dependence of the fusion cross sections for all proton- and neutron-rich systems at both below- and above-barrier energies. The obtained results at energies $E_{c.m.} > V_B$ confirm a non-linear increasing trend for the percentage difference of $\Delta\sigma_{fus}(\%)$ in the whole range of $0.5 < N/Z < 1.7$. For below-barrier energies, it is shown that the calculated values of $\Delta\sigma_{fus}(\%)$ follow a non-linear decreasing trend. This dependence can be explained in terms of the quantum tunneling phenomenon and in fact originates from the isotopic dependence of the effective fusion barrier heights and curvatures. Our findings confirm that the isotopic dependence of the fusion cross sections can be simultaneously dependent on the incident energy $E_{c.m.}$ and nuclear surface tension coefficient γ. This result can provide a new path for studying the role of the coefficient γ in the fusion of heavy-ions. It is shown that the sensitivity to the nuclear surface tension decreases by increasing the incident energy of the projectile and going from below-barrier energies to above-barrier ones. Eventually, simultaneous comparison the results obtained by the three versions Prox. 77, Prox. 88, and Prox. 2010 for the isotopic dependence of fusion cross sections in the $E_{c.m.} > V_B$ and $E_{c.m.} < V_B$ regions indicates the signature of the importance of the coupled-channels effects at low energies.

Data availability statement

The data cannot be made publicly available upon publication because no suitable repository exists for hosting data in this field of study. The data that support the findings of this study are available upon reasonable request from the authors.

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