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# Global nonexistence and stability of solutions of inverse problems for a class of Petrovsky systems

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**Abstract.** In this work, we find conditions on data guaranteeing the global nonexistence of solutions to inverse source problems for a class of Petrovsky systems. We also establish asymptotic stability results for the corresponding problems with the opposite sign of power-type nonlinearities and the integral constraint vanishing as time tends to infinity.

**Keywords.** Inverse problem, global nonexistence, asymptotic stability.

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## 1 Introduction and preliminaries

There are numerous papers devoted to the study of stability and global nonexistence results for direct problems and the existence, uniqueness of solutions of inverse problems for various evolutionary partial differential equations (see [2, 6, 7, 11, 13–15]). But less is known about the global nonexistence for solutions of hyperbolic and parabolic inverse problems. The interested reader is referred to the papers [4, 5].

One of the standard tools for establishing the global nonexistence of solutions is the concavity argument that was introduced by Levine [9, 10] and was generalized in [8]. In [5], Eden and Kalantarov applied the modified concavity method to the problem

$$\begin{gathered}
u_t - \Delta u - |u|^p u + b(x,t,u,\nabla u) = F(t)\omega(x), \quad x \in \Omega,\ t > 0,\\
u(x,t) = 0, \quad x \in \partial\Omega,\ t > 0,\\
u(x,0) = u_0(x), \quad x \in \Omega,\\
\int_\Omega u(x,t)\omega(x)dx = 1, \quad t > 0,
\end{gathered}$$

and established global nonexistence results as well as stability results depending on the sign of nonlinearity.

In this work, by modifying the methods in [5], we study the global in time behavior of solutions to an inverse problem for a class of Petrovsky systems. More-

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over, we establish an asymptotic stability result for the corresponding problem with the opposite sign of power-type nonlinearities.

We denote by $\theta = \theta(\Omega, n)$ the constant in the Poincaré–Steklov inequality

$$\|u\|^2 \le \theta \|\nabla u\|^2, \tag{1.1}$$

which is valid for each $u \in H_0^1(\Omega)$ for a bounded $\Omega$ in $R^n$. We denote by

$$C(\beta, q) = \frac{1}{q'(\beta q)^{q'/q}}$$

the constant in Young's inequality

$$ab \le \beta a^q + C(\beta, q) b^{q'}, \tag{1.2}$$

where $a, b \ge 0$, $\beta > 0$ and $\frac{1}{q} + \frac{1}{q'} = 1$.

Throughout this paper all the functions considered are real-valued. We denote by $\|\cdot\|_k$ the $L^k$ norm over $\Omega$. In particular, the $L^2$ norm is denoted by $\|\cdot\|$. The usual $L^2$ inner product is denoted by $(\cdot)$. We use the well-known function spaces $H_0^2$ and $H^4$.

We will use the following lemma.

**Lemma 1.1** ([8]). *Let $\alpha > 0, C_1, C_2 \ge 0$ and $C_1 + C_2 > 0$. Assume that $\psi(t)$ is a twice differentiable positive function such that*

$$\psi''\psi - (1+\alpha)[\psi']^2 \ge -2C_1\psi\psi' - C_2[\psi]^2$$

*for all $t \ge 0$. If*

$$\psi(0) > 0 \quad and \quad \psi'(0) + \gamma_2\alpha^{-1}\psi(0) > 0,$$

*then*

$$\psi(t) \to +\infty \quad as \quad t \to t_1 \le t_2 = \frac{1}{2\sqrt{C_1^2 + \alpha C_2}} \log \frac{\gamma_1\psi(0) + \alpha\psi'(0)}{\gamma_2\psi(0) + \alpha\psi'(0)}.$$

*Here*

$$\gamma_1 = -C_1 + \sqrt{C_1^2 + \alpha C_2} \quad and \quad \gamma_2 = -C_1 - \sqrt{C_1^2 + \alpha C_2}.$$

The content of this paper is organized as follows. In Section 2, we give some assumptions and touch upon the global nonexistence result (Theorem 2.2). Section 3 is devoted to proving the result for solutions when the integral constraint (3.2) vanishes at infinity which is given in Theorem 3.1.

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## 2 A global nonexistence result

In this section, we study the global in time behavior of solutions for an inverse problem of determining a pair of functions $\{u, f\}$ satisfying the equation

$$u_{tt} + \Delta^2 u - |u|^p u + a(x,t,u,\nabla u,\Delta u) = f(t)\omega(x), \quad x \in \Omega,\ t > 0, \quad (2.1)$$

the initial conditions

$$u(x,0) = u_0(x), \quad u_t(x,0) = u_1(x), \quad x \in \Omega, \quad (2.2)$$

the boundary conditions

$$u(x,t) = \partial_\nu u(x,t) = 0, \quad x \in \partial\Omega,\ t > 0, \quad (2.3)$$

and the over-determination condition

$$\int_\Omega u(x,t)\omega(x)dx = 1, \quad t > 0, \quad (2.4)$$

where $\Omega$ is a bounded domain in $R^n$ with smooth boundary $\partial\Omega$ and a unit outer normal $\nu$. The functions $a, \omega, u_0, u_1$ and the positive number $p$ are given, while $\{u, f\}$ is unknown.

We assume

$$\omega \in H^4(\Omega) \cap H_0^3(\Omega) \cap L^{p+2}(\Omega), \quad \int_\Omega \omega^2(x)dx = 1, \quad (A1)$$

$$|a(x,t,u,\mathbf{p},\mathbf{q})| \le M_1|\mathbf{q}| + M_2|\mathbf{p}| + M_3|u| \quad (A2)$$

for all $x \in \Omega$, $t > 0$ and $M_i > 0$ $(i = 1,2,3)$. The initial functions satisfy the conditions

$$u_0 \in H_0^2(\Omega) \cap L^{p+2}(\Omega), \quad u_1 \in L^2(\Omega), \quad \int_\Omega u_0(x)\omega(x)dx = 1. \quad (A3)$$

We consider the following problem by substituting $u(x,t) = e^{\lambda t}v(x,t)$ in (2.1)–(2.4):

$$v_{tt} + \Delta^2 v + \lambda^2 v + 2\lambda v_t - e^{\lambda p t}|v|^p v + e^{-\lambda t}\tilde{a}(t,v) = e^{-\lambda t} f(t)\omega(x),$$
$$x \in \Omega,\ t > 0, \quad (2.5)$$

$$v(x,t) = \partial_\nu v(x,t) = 0, \quad x \in \partial\Omega,\ t > 0, \quad (2.6)$$

$$v(x,0) = u_0(x), \quad v_t(x,0) = u_1(x) - \lambda u_0(x), \quad x \in \Omega, \quad (2.7)$$

$$\int_\Omega v(x,t)\omega(x)dx = e^{-\lambda t}, \quad t > 0, \quad (2.8)$$

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where

$$\tilde{a}(t,v) = a(x,t,e^{\lambda t}v, e^{\lambda t}\nabla v, e^{\lambda t}\Delta v).$$

Multiplying the equation (2.5) by $\omega(x)$ and using (A1), (2.8) we obtain

$$f(t) = e^{\lambda t}(v, \Delta^2\omega) - e^{\lambda(p+1)t}(|v|^p v, \omega) + (\tilde{a}(t,v), \omega). \tag{2.9}$$

Adapting the idea of Prilepko et al. [13], the key observation is that the problem (2.5)–(2.8) is equivalent to the problem (2.5)–(2.7) in which the unknown function $f(t)$ in (2.5) is replaced by (2.9). (The value of the parameter $\lambda$ will be given later.)

Once the unknown function $f(t)$ is eliminated, the standard theory of nonlinear hyperbolic equations also becomes applicable to deduce the local in time existence of solutions (see [1, 3, 12]).

We define the energy function for the solution $v(x,t)$ of the direct problem (2.5)–(2.7) by

$$E(t) = \frac{e^{\lambda p t}}{p+2}\|v\|_{p+2}^{p+2} - \frac{1}{2}(\|v_t\|^2 + \lambda^2\|v\|^2 + \|\Delta v\|^2). \tag{2.10}$$

**Lemma 2.1.** *Assume that* (A1)–(A3) *hold. Let $v(x,t)$ be a solution of the direct problem* (2.5)–(2.7). *Then*

$$E(t) \geq E(0) - D_1,$$

*where*

$$\begin{aligned} E(0) &= -\frac{1}{2}\|u_1\|^2 - \frac{\lambda^2}{2}\|u_0\|^2 - \frac{1}{2}\|\Delta u_0\|^2 + \frac{1}{p+2}\|u_0\|_{p+2}^{p+2}, \\ D_1 &= \left(2\lambda + \sum_{i=1}^{3} M_i\right)^{-1}\left[\frac{\|\Delta^2\omega\|^2}{2M_1} + \frac{\lambda^{p+2}}{(p+2)[\frac{p+2}{p+1}]^{p+1}}\|\omega\|_{p+2}^{p+2} \right. \\ &\quad \left. + \frac{\lambda^2 M_1^2 + 2\theta\lambda^2 M_2 M_3 + 2M_3^2}{2M_3}\|\omega\|^2\right], \end{aligned} \tag{2.11}$$

*with some positive constant*

$$\lambda = \max\left\{\mu, p^{-1}\left(p + 2 + \sum_{i=1}^{3} M_i\right)\right\},$$

*where $\mu$ is the maximum root of* $2\lambda^3 - \theta M_2\lambda^2 - M_3 = 0$.

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*Proof.* Multiplication of equation (2.5) by $v_t$ and integration on $\Omega$ give

$$
\begin{aligned}
-\frac{d}{dt}\Big[\frac{e^{\lambda pt}}{p+2}\|v\|_{p+2}^{p+2}-\frac{1}{2}\big(\|v_t\|^2+\lambda^2\|v\|^2+\|\Delta v\|^2\big)\Big]&\\
+\frac{\lambda p}{p+2}e^{\lambda pt}\|v\|_{p+2}^{p+2}+2\lambda\|v_t\|^2&\\
=-e^{-\lambda t}(\tilde{a}(t,v),v_t)-\lambda e^{-\lambda t}(v,\Delta^2\omega)&\\
+\lambda e^{\lambda(p-1)t}(|v|^p v,\omega)-\lambda e^{-2\lambda t}(\tilde{a}(t,v),\omega),& \qquad (2.12)
\end{aligned}
$$

Inserting (2.10) into (2.12), we obtain

$$
\begin{aligned}
-\frac{d}{dt}E(t)+\frac{\lambda p}{p+2}e^{\lambda pt}\|v\|_{p+2}^{p+2}+2\lambda\|v_t\|^2 &= -e^{-\lambda t}(\tilde{a}(t,v),v_t)\\
-\lambda e^{-\lambda t}(v,\Delta^2\omega)+\lambda e^{\lambda(p-1)t}(|v|^p v,\omega)&-\lambda e^{-2\lambda t}(\tilde{a}(t,v),\omega). \qquad (2.13)
\end{aligned}
$$

Let us recall the condition (A2) and Poincaré and Young's inequalities (1.1), (1.2). Taking some suitable values for $a, b, q, q'$ and $\beta$ in (1.2) to estimate the terms on the right side of (2.13), we get the following estimations.

**Estimations (1):**

$$
\begin{aligned}
e^{-\lambda t}|(\tilde{a},v_t)| &\le M_1\|\Delta v\|\|v_t\|+M_2\|\nabla v\|\|v_t\|+M_3\|v\|\|v_t\|\\
&\le \frac{\lambda^2 M_3}{4}\|v\|^2+\Big(\frac{M_1}{2}+\frac{M_2}{4}\Big)\|\Delta v\|^2\\
&\quad+\Big(\frac{M_1}{2}+\theta M_2+\frac{M_3}{\lambda^2}\Big)\|v_t\|^2,\\
\big|\lambda e^{-\lambda t}(v,\Delta^2\omega)\big| &\le \frac{M_2\lambda^2}{4}\|v\|^2+\frac{e^{-2\lambda t}}{M_2}\|\Delta^2\omega\|^2,\\
\big|\lambda e^{\lambda(p-1)t}(|v|^p v,\omega)\big| &\le e^{\lambda pt}\|v\|_{p+2}^{p+2}+\frac{\lambda^{p+2}e^{-2\lambda t}}{(p+2)[\frac{p+2}{p+1}]^{p+1}}\|\omega\|_{p+2}^{p+2},\\
\big|\lambda e^{-2\lambda t}(\tilde{a}(t,v),\omega)\big| &\le \Big(\frac{M_2+2M_3}{4}\Big)\|\Delta v\|^2+\frac{\lambda^2 M_3}{4}\|v\|^2\\
&\quad+e^{-2\lambda t}\Big(\frac{M_1^2\lambda^2}{2M_3}+\theta\lambda^2 M_2+M_3\Big)\|\omega\|^2.
\end{aligned}
$$

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Combining Estimations (1) and (2.13), we arrive at

$$\frac{d}{dt}E(t) \geq \Big(\sum_{i=1}^{3} M_i\Big)E(t) + \Big(\frac{\lambda p}{p+2} - \frac{1}{p+2}\sum_{i=1}^{3} M_i - 1\Big)e^{\lambda p t}\|v\|_{p+2}^{p+2}$$
$$+ \Big(2\lambda - \frac{M_3}{\lambda^2} - \theta M_2\Big)\|v_t\|^2 - D_1(t), \tag{2.14}$$

where

$$D_1(t) = e^{-2\lambda t}\Big[\frac{\|\Delta^2\omega\|^2}{2M_2} + \frac{\lambda^{p+2}}{(p+2)[\frac{p+2}{p+1}]^{p+1}}\|\omega\|_{p+2}^{p+2}$$
$$+ \Big(\frac{\lambda^2 M_1^2}{2M_3} + \theta\lambda^2 M_2 + M_3\Big)\|\omega\|^2\Big].$$

At this point, we choose

$$\lambda = \max\Big\{\mu, p^{-1}\Big(p + 2 + \sum_{i=1}^{3} M_i\Big)\Big\},$$

where $\mu$ is the maximum root of $2\lambda^3 - \theta M_2\lambda^2 - M_3 = 0$. Now (2.14) takes the form

$$\frac{d}{dt}E(t) \geq \Big(\sum_{i=1}^{3} M_i\Big)E(t) - D_1(t). \tag{2.15}$$

Therefore (2.15) with (2.11) yields the desired result. □

**Theorem 2.2.** *Assume that*

$$\|u_0\| > 0 \quad \textit{and} \quad E(0) \geq \frac{2B}{p+4} + D_1, \tag{2.16}$$

*where*

$$B = \frac{2}{p\lambda^2}\|\Delta^2\omega\|^2 + \frac{1}{(p+2)[\frac{p+2}{p+1}]^{p+1}}\|\omega\|_{p+2}^{p+2}$$
$$+ \frac{2M_3^2 + 2\lambda^2(M_1 + \theta M_2)^2}{p\lambda^2}\|\omega\|^2. \tag{2.17}$$

*Then there exists a finite time $t^*$ such that the solution of direct problem* (2.5)–(2.7) *blows up at $t^*$.*

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*Proof.* The choice of the functional is standard (see [4]),

$$\psi(t) = \|v(t)\|^2.$$

Then

$$\begin{aligned} \psi'(t) &= 2(v, v_t), \\ \psi''(t) &= 2(v, v_{tt}) + 2\|v_t\|^2. \end{aligned} \tag{2.18}$$

From (2.5) and (2.10) we deduce that

$$\begin{aligned} (v, v_{tt}) = \Big(\frac{p}{2} + 2\Big)E(t) &+ \frac{p}{4}\|\Delta v\|^2 + \frac{\lambda^2 p}{4}\|v\|^2 + \Big(1 + \frac{p}{4}\Big)\|v_t\|^2 - 2\lambda(v_t, v) \\ &+ \frac{p}{2p+4}e^{\lambda p t}\|v\|_{p+2}^{p+2} - e^{-\lambda t}(\tilde{a}(t, v), v) + e^{-\lambda t}(v, \Delta^2\omega) \\ &- e^{\lambda(p-1)t}(|v|^p v, \omega) + e^{-2\lambda t}(\tilde{a}(t, v), \omega). \end{aligned} \tag{2.19}$$

On exploiting condition (A2), inequalities (1.1), (1.2) and choosing suitable values for $a, b, q, q'$ and $\beta$ in (2.2), to estimate the last four terms in (2.19) we get the following estimations.

**Estimations (2):**

$$\begin{aligned} \big|e^{-\lambda t}(\tilde{a}(t, v), v)\big| &\le \frac{p}{8}\|\Delta v\|^2 + \frac{pM_3 + 2(M_1 + \theta M_2)^2}{p}\|v\|^2, \\ \big|e^{-\lambda t}(v, \Delta^2\omega)\big| &\le \frac{\lambda^2 p}{8}\|v\|^2 + \frac{2e^{-2\lambda t}}{\lambda^2 p}\|\Delta^2\omega\|^2, \\ \big|e^{\lambda(p-1)t}(|v|^p v, \omega)\big| &\le \frac{p}{2(p+2)}e^{\lambda p t}\|v\|_{p+2}^{p+2} \\ &\quad + \frac{1}{(p+2)[\frac{p}{2(p+1)}]^{p+1}}e^{-2\lambda t}\|\omega\|_{p+2}^{p+2}, \\ \big|e^{-2\lambda t}(\tilde{a}(t, v), \omega)\big| &\le \frac{p}{8}\|\Delta v\|^2 + \frac{2M_3^2 + 2\lambda^2(M_1 + \theta M_2)^2}{\lambda^2 p}\|\omega\|^2 \\ &\quad + \frac{\lambda^2 p}{8}\|v\|^2. \end{aligned}$$

Using Estimations (2) in (2.19), by conditions (2.16), (2.17) we obtain

$$(v_{tt}, v) \ge \Big(1 + \frac{p}{4}\Big)\|v_t\|^2 - \frac{pM_3 + 2(M_1 + \theta M_2)^2}{p}\|v\|^2 - 2\lambda(v_t, v). \tag{2.20}$$

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By using (2.20) in (2.18) we get

$$\psi''(t) \geq -2\Big(\frac{pM_3 + 2(M_1 + \theta M_2)^2}{p}\Big)\psi(t) + 4\Big(1 + \frac{p}{8}\Big)\|v_t\|^2 - 2\lambda\psi'(t),$$

then

$$\psi(t)\psi''(t) \geq -2\lambda\psi(t)\psi'(t) - \Big(\frac{2pM_3 + 4(M_1 + \theta M_2)^2}{p}\Big)[\psi(t)]^2 + \Big(1 + \frac{p}{8}\Big)[\psi'(t)]^2,$$

where

$$\|v_t\|^2 \geq \frac{1}{4}\psi^{-1}(t)[\psi'(t)]^2$$

has been used. Hence we see that the hypotheses of Lemma 1.1 are fulfilled with

$$\alpha = \frac{p}{8}, \quad C_1 = \lambda, \quad C_2 = \frac{2pM_3 + 4(M_1 + \theta M_2)^2}{p}$$

and the proof of Theorem 2.2 is complete. □

## 3 A stability result

In this section we study the asymptotic stability of solutions to the following inverse problem for a class of Petrovsky systems:

$$u_{tt} + \Delta^2 u + au_t + |u|^p u - bu = f(t)\omega(x), \quad x \in \Omega,\ t > 0, \tag{3.1}$$

$$u(x,t) = \partial_\nu(x,t) = 0, \quad x \in \partial\Omega,\ t > 0,$$

$$u(x,0) = u_0(x), \quad u_t(x,0) = u_1(x), \quad x \in \Omega,$$

$$\int_\Omega u(x,t)\omega(x)dx = \phi(t), \quad t > 0, \tag{3.2}$$

where $\Omega$ is a bounded domain of $R^n$ with smooth boundary $\partial\Omega$. The functions $\omega, u_0, u_1, \phi(t)$ and the positive constants $a$ and $b$ are given, while $\{u(x,t), f(t)\}$ is unknown.

We shall assume that the functions appearing in the data of the problem are measurable and satisfy the following conditions:

$$(u_0, u_1) \in \big(H_0^2(\Omega) \cap L^{p+2}(\Omega)\big) \times L^2(\Omega), \quad (u_0, \omega) = \phi(0). \tag{A5}$$

As was mentioned in the previous section, we multiply (3.1) by $\omega(x)$ and use relation (3.2) to express

$$f(t) = \phi''(t) + (\Delta u, \Delta\omega) + a\phi'(t) + (|u|^p u, \omega) - b\phi(t).$$

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**Theorem 3.1.** *Assume that* (A1) *and* (A5) *hold. Furthermore, let the damping coefficient* $b \in (\eta, \theta^{-2})$, $a > \eta > \frac{b\delta\theta^2}{4+\delta\theta^2}$ *for* $\delta$ *sufficiently small, and let* $\phi(\cdot) : R^+ \to R^+$ *be a continuous function of the class* $C^2$ *such that* $\phi''$ *is bounded and* $\phi(t), \phi'(t) \to 0$ *as* $t \to +\infty$. *Then*

$$\lim_{t\to+\infty}\left[\|u_t(t)\|^2 + \|\Delta u(t)\|^2 + \|u(t)\|_{p+2}^{p+2}\right] = 0. \tag{3.3}$$

*Proof.* Multiplying both sides of (3.1) by $u_t + \eta u$ scalarly in $L^2(\Omega)$ gives the relation

$$\tilde{E}'(t) + (a-\eta)\|u_t\|^2 + \eta\|\Delta u\|^2 + \eta\|u\|_{p+2}^{p+2} - b\eta\|u\|^2 = (\Delta u, \Delta\omega)(\phi' + \eta\phi)$$
$$+(|u|^p u, \omega)(\phi' + \eta\phi) + \phi''(\phi' + \eta\phi) + (a\eta - b)\phi\phi' - \eta b\phi^2 + a[\phi']^2, \tag{3.4}$$

where

$$\tilde{E}(t) = \frac{1}{2}\|u_t\|^2 + \frac{1}{2}\|\Delta u\|^2 + \frac{a\eta - b}{2}\|u\|^2 + \frac{1}{p+2}\|u\|_{p+2}^{p+2} + \eta(u_t, u).$$

Now, we estimate the first two terms on the right-hand side of (3.4). The subsequent estimations follow from Schwartz' and Young's inequalities:

**Estimations (3):**

$$\|\Delta u\|\|\Delta\omega\|\big(|\phi'(t)| + \eta|\phi(t)|\big)$$
$$\le \varepsilon\|\Delta u\|^2 + \frac{1}{4\varepsilon}\big(|\phi'(t)| + \eta|\phi(t)|\big)^2\|\Delta\omega\|^2,$$
$$\big|(|u|^p u, \omega)\big|\big(|\phi'(t)| + \eta|\phi(t)|\big)$$
$$\le \varepsilon\|u\|_{p+2}^{p+2} + \rho(\varepsilon, p)\big(|\phi'(t)| + \eta|\phi(t)|\big)^{p+2}\|\omega\|_{p+2}^{p+2},$$

where

$$\rho(\varepsilon, p) = \frac{1}{(p+2)[\varepsilon\frac{p+2}{p+1}]^{p+1}}.$$

Inserting Estimations (3) and the definition of $\delta\tilde{E}(t)$ ($\delta > 0$) into (3.4), we deduce that

$$\tilde{E}'(t) + \delta\tilde{E}(t) + (a-\eta)\|u_t\|^2 + (\eta - \varepsilon)\|\Delta u\|^2$$
$$+ (\eta - \varepsilon)\|u\|_{p+2}^{p+2} - b\eta\|u\|^2 - \delta\tilde{E}(t) \le H(t), \tag{3.5}$$

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where

$$H(t) = |\phi''\phi'| + \frac{1}{4\varepsilon}\big(|\phi'| + \eta|\phi|\big)^2 \|\Delta\omega\|^2 + (b + a\eta)|\phi'\phi| + \eta|\phi''\phi| + a|\phi'|^2 + \rho(\varepsilon, p)\big(|\phi'(t)| + \eta|\phi(t)|\big)^{p+2} \|\omega\|_{p+2}^{p+2}.$$

We can rewrite (3.5) as follows:

$$\tilde{E}'(t) + \delta\tilde{E}(t) \le -\Big(a - \eta - \frac{\delta}{2}\Big)\|u_t\|^2 - \Big(\eta - \varepsilon - \frac{\delta}{2}\Big)\|\Delta u\|^2 - \Big(\eta - \varepsilon - \frac{\delta}{p+2}\Big)\|u\|_{p+2}^{p+2} + \Big(b\eta + \frac{\delta}{2}(a\eta - b)\Big)\|u\|^2 + \delta\eta|(u_t, u)_\Omega| + H(t). \tag{3.6}$$

By virtue of the Poincaré–Steklov inequality (1.1) and

$$|(u_t, u)| \le \frac{1}{2}\|u_t\|^2 + \frac{\theta^2}{2}\|\Delta u\|^2,$$

inequality (3.6) implies

$$\tilde{E}'(t) + \delta\tilde{E}(t) \le -\Big(a - \eta - \frac{\delta}{2} - \frac{\delta\eta}{2}\Big)\|u_t\|^2 - \bigg(\eta - \varepsilon - \frac{\delta}{2} - \eta\theta^2\Big(b + \frac{a\delta}{2}\Big) + \frac{\delta(b-\eta)\theta^2}{2}\bigg)\|\Delta u\|^2 - \Big(\eta - \varepsilon - \frac{\delta}{p+2}\Big)\|u\|_{p+2}^{p+2} + H(t).$$

At this point, taking $a > \eta > \varepsilon$, $\varepsilon = \frac{\delta}{4}(b-\eta)\theta^2$ and assuming $\delta$ to be a sufficiently small number and $\eta < b < \theta^{-2}$, we derive from the last inequality the inequality

$$\tilde{E}'(t) + \delta\tilde{E}(t) \le H(t)$$

thanks to the assumptions on $\phi(t), \phi'(t)$ and $\phi''$. Indeed, $\phi(t), \phi'(t)$ tend to zero as $t$ tends to infinity and $\phi''$ is a bounded function, so we get

$$H(t) \to 0 \quad \text{as} \quad t \to \infty.$$

Therefore for some positive constant $\gamma$, result (3.3) follows from

$$\|u_t(t)\|^2 + \|\Delta u(t)\|^2 + \|u(t)\|_{p+2}^{p+2} \le \gamma\tilde{E}(t),$$

and the proof of Theorem 3.1 is complete. □

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