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Optimization of Multi-Echelon Closed-Loop Supply Chain Networks Using a Deterministic Approach

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***Abstract*— This study develops a deterministic multi-period mixed-integer programming model for the strategic design and operational planning of multi-echelon Closed-Loop Supply Chain (CLSC) networks. The formulation integrates forward and reverse logistics flows within a unified optimization framework, encompassing manufacturing facilities, potential distribution centers, customer demand zones, prospective collection centers, and designated disposal facilities. The model captures critical network design decisions, including capacity-level selection for potential facilities, production and remanufacturing quantities, inter-echelon shipment volumes, inventory dynamics, and end-of-life product disposition, while enforcing constraints on capacities, flow conservation, and recovery/scrap ratios. The objective function minimizes the aggregate cost of production, transportation, processing, remanufacturing, disposal, and facility establishment. A synthetic dataset is employed to illustrate the framework, and the results demonstrate that the proposed model yields efficient network configurations and balanced flow allocations. Overall, the formulation offers a rigorous and adaptable foundation for cost-effective closed-loop supply chain design.**

Keywords*— Closed-loop supply chain, Reverse logistics, Capacitated network design, Deterministic optimization, Multi-period optimization, Network design

I. Introduction

Closed-Loop Supply Chains (CLSCs) have emerged as a pivotal paradigm in sustainable operations management, integrating forward logistics activities with reverse flows to enable the recovery, remanufacturing, recycling, or responsible disposal of end-of-life products. This integration not only enhances resource efficiency and environmental stewardship but also generates economic value by reclaiming materials and extending product lifecycles. Designing such networks requires the simultaneous consideration of strategic facility location, capacity planning, and operational flow decisions across multiple echelons, each with distinct roles and constraints.

A typical CLSC structure is illustrated in Fig. 1, which depicts the major echelons—manufacturing plants, distribution centers, customer segments, collection centers, and disposal facilities—and the flows that connect them. The forward channel involves the flow of newly produced items from manufacturers to customers through distribution centers, while the reverse channel involves the collection of returned products, separation of repairable and scrapped items, and subsequent remanufacturing or disposal. This schematic highlights the dual nature of CLSC networks, where both economic performance and environmental impact are strongly shaped by the efficiency of reverse logistics processes.

Despite its conceptual clarity, designing and operating CLSC networks is a highly complex task. Decision-makers must simultaneously determine which facilities to establish, at what capacity levels, and how to allocate flows of new, returned, repairable, and scrapped products. These challenges are amplified in multi-period settings, where inventory management and temporal interactions between production and remanufacturing decisions play a crucial role. Deterministic mathematical programming provides one of the most rigorous approaches for analyzing such problems, offering exact solutions that capture the trade-offs among production, processing, transportation, and facility costs.

A substantial body of research has investigated CLSC network design through mixed-integer programming frameworks. In [1], a multi-period CLSC design model was proposed to coordinate strategic and operational decisions across time, showing how temporal planning influences network efficiency. The study in [2] examined facility location problems in logistics networks with reverse flows, emphasizing the complexity of integrating product returns into traditional forward networks. A deterministic multi-period MILP for CLSC optimization was developed in [3], demonstrating the effectiveness of location–allocation formulations in capturing cost trade-offs. In [4], a deterministic, multi-echelon CLSC model incorporating multiple products and hybrid facilities was presented, highlighting scalability and solution tractability. A capacity-aware CLSC design solved via exact optimization was introduced in [5], illustrating how mathematical programming can address facility sizing and routing simultaneously. More recently, [6] formulated a risk-averse two-stage stochastic programming model under uncertain demand, showing that incorporating Conditional Value-at-Risk (CVaR) yields more

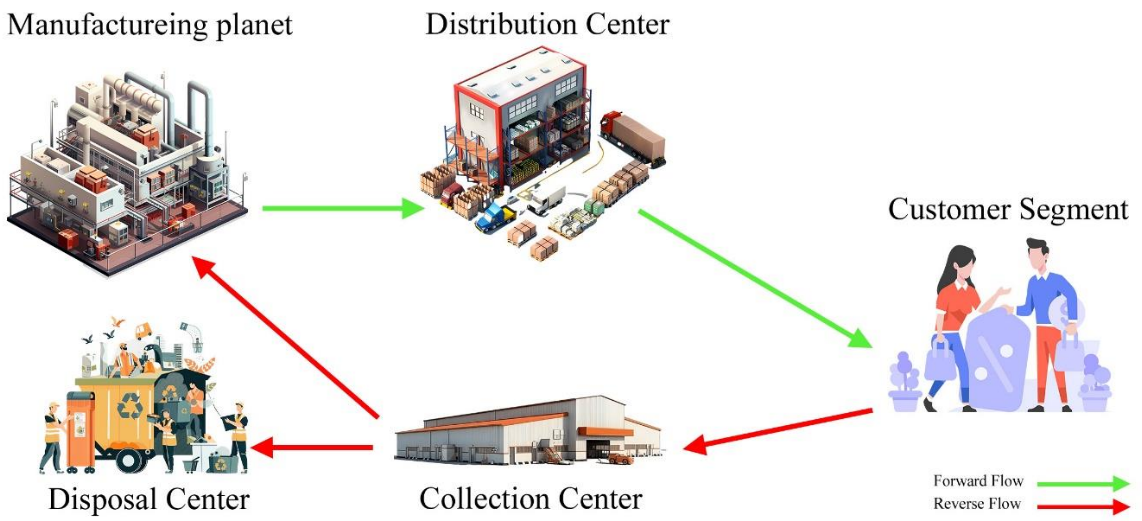

Fig. 1. Multi-echelon closed-loop supply chain including manufacturers, distribution centers, customers, collection centers, and disposal facilities.

resilient and stable CLSC configurations compared to risk-neutral approaches.

In this study, a deterministic multi-period mixed-integer programming framework is developed to capture the interplay between strategic and operational decisions in CLSC networks. Rather than focusing on a specific product type, the formulation is designed to be adaptable to diverse industrial contexts. The model simultaneously considers facility siting with discrete capacity options, flow routing in both forward and reverse channels, inventory transitions over time, and disposition pathways for recovered products. This integrated structure ensures that strategic network design remains aligned with operational feasibility, thereby supporting cost-efficient and environmentally responsible supply chain configurations.

The remainder of the paper is organized as follows. Section II presents the methodology, including the network structure, notation, and the mathematical model. Section III reports the results: Part A describes the dataset, and Part B provides the computational results together with the optimal CLSC configuration. Section IV concludes the study and outlines directions for future research.

II. Methodology

A. Network Structure

The proposed CLSC network consists of five echelons: manufacturing facilities, Distribution Centers (DCs), customer demand zones, Collection Centers (CCs), and disposal facilities. Forward flows originate at manufacturing facilities, pass through DCs, and are delivered to customer zones. Reverse flows start at customer zones, where used products are collected at CCs for inspection and sorting. Recoverable products are sent to manufacturing facilities for remanufacturing, while non-recoverable products are directed to disposal facilities. The network is modeled over a finite multi-period planning horizon, enabling temporal coupling between production, remanufacturing, and inventory decisions.

B. Notation

Sets and Indices

M	Set of fixed locations of manufacturing plants, $m \in M$
J	Set of candidate locations for distribution centers, $j \in J$
K	Set of fixed locations for customer segments, $k \in K$
L	Set of candidate locations for collection centers, $l \in L$
N	Set of fixed locations for disposal centers, $n \in N$
Q	Set of capacity levels available for distribution centers and collection centers, $q \in Q$
T	Set of time periods, $t \in T$

Parameters

ξ_k^t	Demand of customer segment k in period t
τa	Product recovery rate
τb	Product scrap rate
h	Holding cost per unit of product
ω	Price per unit of product
ca_m	Capacity of manufacturing plant m
cb_{jq}	Capacity of distribution center j with capacity level q
cc_{lq}	Capacity of collection center l with capacity level q
cd_n	Capacity of disposal center n
da_{mj}	Transportation cost per unit of product from manufacturing plant m to distribution center j
db_{jk}	Transportation cost per unit of product from distribution center j to customer segment k
dc_{kl}	Transportation cost per unit of product from customer segment k to collection center l
dd_{lm}	Transportation cost per unit of product from collection center l to manufacturing plant m
de_{ln}	Transportation cost per unit of product from collection center l to disposal center n
fa_{jq}	Fixed opening cost of distribution center j with capacity level q

fb_{lq}	Fixed opening cost of collection center l with capacity level q
va_m	Production cost per unit of product in manufacturing plant m
vb_j	Processing cost per unit of product in distribution center j
vc_l	Processing cost per unit of product in collection center l, including inspection cost and sorting cost
vd_m	Remanufacturing cost per unit of product in manufacturing plant m
ve_n	Disposal cost per unit of product at disposal center n

Decision Variables

x_{jq}	If the distribution center with capacity level q opens at location j, it is equal to 1; otherwise, it is equal to 0
y_{lq}	If the collection center with capacity level q opens at location l, it is equal to 1; otherwise, it is equal to 0
δ_m^t	Quantity of products produced in manufacturing plant m in period t
I_m^t	Quantity of products holds in manufacturing plant m in period t
θ_{mj}^t	Quantity of products shipped from manufacturing plant m to distribution center j in period t
β_{jk}^t	Quantity of products shipped from distribution center j to customer segment k in period t
γ_{kl}^t	Quantity of products shipped from customer segment k to collection center l in period t
ψ_{lm}^t	Quantity of repairable products shipped from collection center l to manufacturing plant m in period t
φ_{ln}^t	Quantity of scrapped products shipped from collection center l to disposal center n in period t

C. Mathematical model

The model minimizes the total system cost, including production, processing, remanufacturing, transportation, disposal, inventory holding, and fixed facility-opening costs. This integrated objective ensures simultaneous optimization of strategic and operational decisions.

$$\text{Min } z = \sum_j\sum_q x_{jq}\, fa_{jq} + \sum_l\sum_q y_{lq}\, fb_{lq} + \sum_m\sum_t \delta_m^t \times va_m + \sum_l\sum_m\sum_t \psi_{lm}^t \times vd_m + \sum_m\sum_j\sum_t \theta_{mj}^t \times da_{mj} + \sum_m\sum_j\sum_t \theta_{mj}^t \times vb_j + \sum_j\sum_k\sum_t \beta_{jk}^t \times db_{jk} + \sum_k\sum_l\sum_t \gamma_{kl}^t \times dc_{kl} + \sum_k\sum_l\sum_t \gamma_{kl}^t \times vc_l + \sum_l\sum_m\sum_t \psi_{lm}^t \times dd_{lm} + \sum_l\sum_n\sum_t \varphi_{ln}^t \times de_{ln} + \sum_l\sum_n\sum_t \varphi_{ln}^t \times ve_n + \sum_m\sum_t I_m^t \times h - \sum_j\sum_k\sum_t \beta_{jk}^t \times \omega \quad (1)$$

s. t.

$$\delta_m^t + I_m^{t-1} + \sum_l \psi_{lm}^t = \sum_j \theta_{mj}^t + I_m^t \quad \forall\, m,t \quad (2)$$

$$\sum_m \theta_{mj}^t = \sum_k \beta_{jk}^t \quad \forall\, j,t \quad (3)$$

$$\sum_j \beta_{jk}^t = \xi_k^t \quad \forall\, k,t \quad (4)$$

$$\sum_l \gamma_{kl}^t = \tau a \times \xi_k^{t-1} \quad \forall\, k,t \quad (5)$$

$$\sum_m \psi_{lm}^t = (1-\tau b)\sum_k \gamma_{kl}^t \quad \forall\, l,t \quad (6)$$

$$\sum_n \varphi_{ln}^t = \tau b \sum_k \gamma_{kl}^t \quad \forall\, l,t \quad (7)$$

$$\delta_m^t + \sum_l \psi_{lm}^t \le ca_m \quad \forall\, m,t \quad (8)$$

$$\sum_k \beta_{jk}^t \le \sum_q cb_{jq} \times x_{jq} \quad \forall\, j,t \quad (9)$$

$$\sum_k \gamma_{kl}^t \le \sum_q cc_{lq} \times y_{lq} \quad \forall\, l,t \quad (10)$$

$$\sum_l \varphi_{ln}^t \le cd_n \quad \forall\, n,t \quad (11)$$

$$\sum_q x_{jq} \le 1 \quad \forall\, j \quad (12)$$

$$\sum_q y_{lq} \le 1 \quad \forall\, l \quad (13)$$

$$\sum_j\sum_q x_{jq} \ge 1 \quad (14)$$

$$\sum_l\sum_q y_{lq} \ge 1 \quad (15)$$

$$x_{jq}, y_{lq} \in \{0,1\} \quad \forall j,l,q \quad (16)$$

$$\theta_{mj}^t,\ \beta_{jk}^t, \gamma_{kl}^t, \psi_{lm}^t, \varphi_{ln}^t, \delta_m^t, I_m^t \ge 0 \quad \forall m,j,k,l,n,t \quad (17)$$

Constraint “(2)” enforces the production and reverse flow balance at manufacturers. Constraints “(3)~(7)” are logistics balance equations ensuring that the total inflow to each facility equals its total outflow for every period and echelon. Constraints “(8)~(11)” impose capacity restrictions, ensuring that the quantity of products handled at each facility does not exceed its assigned capacity. Constraints “(12)~(13)” define that each facility can operate at only one discrete capacity level. Constraints “(14)~(15)” ensure that the CLSC configuration includes at least one active facility in each echelon, thereby guaranteeing a fully connected and operational closed-loop network.

III. Results

A. Data Description

To evaluate the proposed deterministic CLSC model, a synthetic dataset was generated. The data are not case-specific but are instead randomly produced to provide a representative testbed for the model. The network consists of multiple echelons, including manufacturers, candidate distribution centers, customer segments, collection centers, and a disposal facility, all linked through both forward and reverse flows over a four-period planning horizon.

The input data were constructed to reflect a balance between realism and generality. Customer demand and cost parameters were sampled from uniform distributions to introduce variability across facilities and time periods. Fixed facility capacities and opening costs were specified at two discrete levels to represent scalable investment decisions. Recovery and scrap rates were set at constant values, ensuring that returned products are systematically divided between remanufacturing and disposal.

The following tables summarize the network configuration, parameter distributions, facility capacities, and fixed costs used in the experiments.

TABLE I. NETWORK CONFIGURATION AND HORIZON

Sets	Count
M	3
J	3
K	5
L	2
N	1
Q	2
T	4

TABLE II. INPUT PARAMETERS

Parameter	Value
ξ_k^t	Uniform (80,250)
τa	0.1
τb	0.1
h	Uniform (40,80)
ω	Uniform (200,600)
ca_m	Uniform (280,380)
cd_n	Uniform (15,20)
$da_{mj}, db_{jk}, dc_{kl}$	Uniform (30,240)
dd_{lm}, de_{ln}	Uniform (40,160)
va_m	Uniform (80,150)
vb_j, vc_l, ve_n	Uniform (10,40)
vd_m	Uniform (40,60)

TABLE III. DISTRIBUTION CENTERS CAPACITIES AND FIXED OPENING COST BY LEVEL

Distribution (j/q)	q1		q2	
	Capacity (cb_{jq})	*Fixed opening cost* (fa_{jq})	*Capacity* (cb_{jq})	*Fixed opening cost* (fa_{jq})
j1	300	20000	400	30000
j2	100	12000	380	28000
j3	340	24000	360	26000

TABLE IV. COLLECTION CENTERS CAPACITIES AND FIXED OPENING COST BY LEVEL

Collection (l/q)	q1		q2	
	Capacity (cc_{lq})	*Fixed opening cost* (fb_{lq})	*Capacity* (cc_{lq})	*Fixed opening cost* (fb_{lq})
l1	60	12000	40	13000
l2	80	12200	70	12800

B. Computational Performance

The proposed deterministic CLSC model was implemented in the GAMS environment and solved using the IBM ILOG CPLEX solver. All computational experiments were performed on a workstation equipped with an Intel® Core™ i5 processor and 8 GB of RAM.

The solver obtained an optimal solution within 0.191 seconds, highlighting the tractability of the formulation under the given dataset. The optimal objective value was $-50{,}587.947$, which due to the minimization formulation corresponds to a maximum achievable profit of 50,587.947 units over the four-period planning horizon. This result demonstrates that the model can effectively integrate strategic facility decisions with operational flows to identify profit-maximizing configurations within very short computation times.

The optimal network configuration establishes three distribution centers x_{11}, x_{22}, and x_{31} along with two collection centers y_{11} and y_{21}. This structure balances fixed investment costs with transportation and processing efficiencies, ensuring both forward and reverse flows are effectively supported. The facility opening pattern indicates that distributed coverage of customers and adequate collection capacity are critical for achieving system wide profitability.

IV. CONCLUSION

This paper developed a deterministic multi-period mixed-integer programming model for the design and planning of closed-loop supply chain networks. The formulation integrates forward and reverse logistics decisions across manufacturers, distribution centers, customer segments, collection centers, and disposal facilities, while explicitly considering discrete facility capacities, flow balances, and recovery/disposal processes.

Computational experiments on a synthetic dataset demonstrated the model's tractability, with the GAMS–CPLEX implementation achieving optimal solutions in fractions of a second. The optimal configuration highlighted the importance of balancing fixed facility investments with transportation and processing costs, resulting in the opening of three distribution centers and two collection centers to ensure profitability and feasibility of both forward and reverse flows.

The deterministic structure of the model provides a rigorous foundation for strategic and operational decision-making in CLSC design. Future research may extend this work by incorporating stochastic or robust optimization approaches to account for uncertainty in demand, returns, and processing yields, as well as by applying the model to industry-specific case studies with real data.

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