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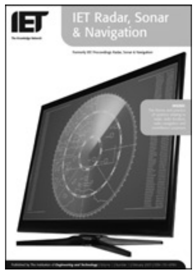

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Modified input estimation technique for tracking manoeuvring targets

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Abstract: A new input estimation (IE) model for problems in tracking manoeuvring targets is proposed. The proposed model is constructed by combining the two models of uncertainties, Bayesian and Fisher. The conventional model, which describes targets with manoeuvre, is based on the state vector of target position and velocity. The acceleration is treated as an additive input term in the corresponding state equation. The proposed method is a Kalman filter-based tracking scheme with the IE approach. The proposed model is a special augmentation in the state-space model which considers both the state vector and the unknown input vector as a new augmented state vector. In the proposed scheme, the original state and acceleration vectors are estimated simultaneously with a standard Kalman filter. The proposed tracking algorithm operates in both the non-manoeuvring and the manoeuvring modes and the manoeuvre detection procedure is eliminated. The theoretical development is verified by simulation results, which also contain some examples of tracking typical target manoeuvres. The results are compared with a traditional IE method. A comparison based on the Monte-Carlo simulation is also made to evaluate the performances of the proposed method in three scenarios: low, medium and high manoeuvring target.

1 Introduction

Target tracking is the determination of the present and often future states (position and velocity) of a moving target from noisy measurements of its present states. Tracking manoeuvring targets is an important problem complicated by the fact that radar cannot directly measure target accelerations [1].

A linear Kalman filter is widely used for the tracking problem, but its performance may be seriously degraded unless the estimation error because of the unknown target manoeuvre is compensated by an auxiliary estimation process.

Approaches based on the Kalman filter include the early work of Singer [2], who augmented the Kalman filter with the target acceleration equation represented by a first-order autoregressive process. The augmented filter tracks a manoeuvring target closely, but during constant-velocity, straight-line motions, its performance degrades when compared with a simple Kalman filter that assumes no manoeuvre. A common method in the application uses a non-manoeuvring target model to track a target moving at a constant velocity and then switches to a tracking filter for an appropriate manoeuvring model, when the target manoeuvre is detected.

Many different tracking filters to track a manoeuvring target have been considered in the literature. Two different approaches that have been widely studied to handle the case of unknown target manoeuvres are model-based adaptive filtering [3] and input estimation (IE).

1.1 Model-based adaptive filtering techniques

Model-based adaptive filtering techniques have evolved to the interacting multiple model (IMM) algorithm [4], in which the change of the plant is modelled as a Markovian parameter having a transition probability. Using a hypothesis merging technique for multiple model filtering, the IMM algorithm calculates the Bayesian sum of the filter outputs. IMM trackers hypothesise two or more manoeuvre modes and assume that the mode changes are modelled by a hidden Markov process. The measurements are filtered through each mode to produce a set of state estimates conditioned on the hypothesised manoeuvre mode. The outputs are then combined as a weighted sum where the weights are proportional to mode likelihoods [5].

The most basic IMM [6] has one low acceleration mode and one high acceleration mode and assumes that the Markov chain transition probabilities are stationary and known.

There exist IMMs [7–9] of varying structure. More complicated IMMs can have non-stationary transition probabilities, autocorrelated manoeuvres and adaptive mode sets [5]. They perform well when the modes accurately represent the true accelerations and are relatively robust to small modelling errors. Their computational complexity increases linearly with the number of manoeuvre modes.

The variable-dimension filtering approach has been proposed by Bar-Shalom and Birmiwal [10]. In this approach, the state model for the target is changed by introducing extra state components, the target accelerations, when a manoeuvre is detected. The manoeuvre, modelled as acceleration, is estimated recursively along with the other states associated with position and velocity while a target manoeuvres.

1.2 IE techniques

IE is a totally different approach which detects the existence of target manoeuvres and directly estimates the magnitude of the unknown manoeuvres [11–18].

The IE approach for tracking a manoeuvring target has been proposed by Chan et al., [11]. In this approach, the magnitude of the acceleration is identified by the least-squares estimation when a manoeuvre is detected. The estimated acceleration is then used in conjunction with a standard Kalman filter to compensate the state estimate of the target. The standard filter is alone used during periods when no manoeuvre takes place. Although this algorithm is attractive in several aspects, it suffers from a major deficiency that stems from a constant input assumption.

It is to be noted that the inputs of the IE are the filter residuals over a finite detection window, and the outputs are the estimated target manoeuvre inputs. Therefore the IE can be understood as an inverse process of the tracking kinematics. The basic idea of this approach is to explicitly estimate the unknown control input $u(n)$, and then estimate the state using the estimated input $\hat{u}(n)$. IE approach may be more accurate (but less tractable) to estimate the state and input jointly. The modified IE (MIE) algorithm proposed in [15] and enhanced IE (EIE) algorithm proposed in [12] are two modifications and enhancements of the original IE algorithm by relaxing certain assumptions.

Whereas the EIE and MIE algorithms were developed aimed at overcoming the deficiencies of the original IE algorithm because of ignoring the uncertainty in the manoeuvre onset time, the generalised IE algorithm, proposed in [16], is intended to relax the restrictive assumption concerning the evolution of the input, by modelling the unknown input as a linear combination of known basic time functions, defined over the detection window. In [16], by the use of the least-squares method the optimal coefficients were estimated by defining the unknown input as a sum of elementary time functions. Although the IE model is more general than the Constant-input model of the original IE algorithm, it does not cover the case with dynamics in input. In the work of Wang Varshney [19] the predicted and estimated states for the manoeuvring target are related to the corresponding states without manoeuvring, based on a constant acceleration assumption. The performance of the estimation is reduced when a target moves with non-constant acceleration. The conventional IE techniques based on a constant acceleration assumption have not been very successful, because the actual value of target acceleration during typical target manoeuvre is not constant.

We present a new IE model for problems in tracking manoeuvring targets. The acceleration is treated as an additive input term in the corresponding state equation. The proposed model is a special augmentation of the state-space model, which considers both the state and unknown input vectors as new augmented states. The proposed tracking algorithm operates in both the non-manoeuvre and manoeuvre modes.

This paper is organised as follows. The problem formulation is based on two major models of uncertainties as presented in Section 2. In this section, the basic mathematical formulations for optimal filtering of the Bayesian uncertainty models are also presented. In Section 3, the main results of the paper for manoeuvring target tracking are described. In Section 4, the tracking performance of the proposed filter is compared with that of the Wang's method for the scenarios of tracking a target having a coordinate-turn manoeuvre in a two-dimensional plane.

2 Problem formulation

2.1 Models of uncertainties

The two basic models of uncertainties to be considered in this paper are the Bayesian and Fisher models [20]. These models are specific cases of the state-space structure-white process. In the Bayesian model, uncertainty is either modelled by random variables and/or stochastic processes with completely specified probability distributions or completely specified first and second moments. In many applications, the input disturbance can be modelled as being completely unknown. A model where the input disturbance is completely unknown is a type of Fisher model.

The complete definition of the Bayesian, discrete-time uncertainty model for linear systems is summarised as

$$\begin{aligned} &\boldsymbol{X}(n+1)=\boldsymbol{F}(n)\boldsymbol{X}(n)+\boldsymbol{G}(n)w(n); && \text{Non-manoeuvring target state equation} \\ &z(n)=\boldsymbol{H}(n)\boldsymbol{X}(n)+v(n); && \text{Measurement equation} \end{aligned} \tag{1}$$

where (1) is the target state space dynamic equation. $\boldsymbol{F}(n)$ and $\boldsymbol{G}(n)$ are the transient and input uncertainty (plant noise) matrices, respectively; $w(n)$ and $v(n)$ are zero mean white plant and measurement noises, respectively. The noises $w(n)$ and $v(m)$ are assumed to be mutually uncorrelated. They are characterised as

$$\begin{aligned} &\boldsymbol{E}\{v(n_1)v^{\mathrm{T}}(n_2)\}=\begin{cases}\boldsymbol{R}(n_1) & n_1=n_2\\ 0 & n_1\neq n_2\end{cases}\\ &\boldsymbol{E}\{w(n_1)w^{\mathrm{T}}(n_2)\}=\begin{cases}\boldsymbol{Q}(n_1) & n_1=n_2\\ 0 & n_1\neq n_2\end{cases}\\ &\boldsymbol{E}\{\boldsymbol{X}(0)\boldsymbol{X}^{\mathrm{T}}(0)\}=\psi,\ \boldsymbol{E}\{\boldsymbol{X}(0)\}=0,\ \boldsymbol{E}\{w(n)\}\\ &\quad=0,\ \boldsymbol{E}\{v(n)\}=0,\ \text{for all } n\\ &\boldsymbol{E}\{w(n)v^{\mathrm{T}}(m)\}=0,\ \text{for all } n, m \end{aligned} \tag{2}$$

It is assumed that the target moves in a plane, which is a two-dimensional case, such as a ship. Thus, the state vector at time n is

$$\boldsymbol{X}(n)=[x(n)\ v_x(n)\ y(n)\ v_y(n)]^{\mathrm{T}} \tag{3}$$

where $x(n)$, $v_x(n)$ and $y(n)$, $v_y(n)$ are the target positions and speeds in x and y Cartesian coordinates, respectively.

The matrices $\boldsymbol{H}(n)$, $\boldsymbol{F}(n)$ and $\boldsymbol{G}(n)$ in (1) are assumed to be known functions of time n.

In (1) the state transition matrix $\boldsymbol{F}$ and the plant noise matrix $\boldsymbol{G}$, as functions of T (T is the time interval between two consecutive measurements) are

$$\boldsymbol{F}=\begin{bmatrix}1 & T & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & T\\ 0 & 0 & 0 & 1\end{bmatrix},\quad \boldsymbol{G}=\begin{bmatrix}T^2/2 & T & 0 & 0\\ 0 & 0 & T^2/2 & T\end{bmatrix}^{\mathrm{T}} \tag{4}$$

The measurement matrix $\boldsymbol{H}$ is given by

$$\boldsymbol{H}=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0\end{bmatrix} \tag{5}$$

2.2 Standard Kalman filter

The problem to be considered is how to use the observations up to time n_2, $z(1), \ldots, z(n_2)$, to estimate optimally – in the sense of the minimum error covariance matrix – the state $\boldsymbol{X}(n_1)$ at a certain time n_1. The solution of the problem filtering, after certain mathematical manipulations leads us to the Kalman filter with equations

$$\begin{aligned} &\hat{\boldsymbol{X}}(n+1|n+1)=\boldsymbol{F}(n)\hat{\boldsymbol{X}}(n|n)+\boldsymbol{K}(n+1)[z(n+1)\\ &\qquad -\boldsymbol{H}(n+1)\boldsymbol{F}(n)\hat{\boldsymbol{X}}(n|n)]\\ &\boldsymbol{K}(n+1)=\boldsymbol{P}(n+1|n+1)\boldsymbol{H}^{\mathrm{T}}(n+1)\boldsymbol{R}^{-1}(n+1)\\ &\boldsymbol{P}(n+1|n+1))=\boldsymbol{P}(n+1|n)-\boldsymbol{P}(n+1|n)\boldsymbol{H}^{\mathrm{T}}(n+1)\\ &\quad\times[\boldsymbol{R}(n+1)+\boldsymbol{H}(n+1)\boldsymbol{P}(n+1|n)\boldsymbol{H}(n+1)^{\mathrm{T}}]^{-1}\\ &\quad\times\boldsymbol{H}(n+1)\boldsymbol{P}(n+1|n)\\ &\boldsymbol{P}(n+1|n)=\boldsymbol{F}(n)\boldsymbol{P}(n|n)\boldsymbol{F}^{\mathrm{T}}(n)+\boldsymbol{G}(n)\boldsymbol{Q}(n)\boldsymbol{G}^{\mathrm{T}}(n)\\ &\boldsymbol{P}(0|0)=\psi,\ \hat{\boldsymbol{X}}(0|0)=0 \end{aligned} \tag{6}$$

where $\boldsymbol{P}(n|n)$ is the error covariance matrix and $\boldsymbol{P}(n+1|n)$ the error covariance matrix of the one-step prediction.

The manoeuvring model deals with the acceleration as an additive term

$$\boldsymbol{X}(n+1)=\boldsymbol{F}\boldsymbol{X}(n)+\boldsymbol{C}u(n)+\boldsymbol{G}w(n) \tag{7}$$

The acceleration input vector is

$$u(n)=[a_x(n)\quad a_y(n)]^{\mathrm{T}} \tag{8}$$

It is assumed that the acceleration u is a completely unknown input which models the target manoeuvres. When there is no manoeuvre, u is 0.

Remark 1: Although many types of targets motion are limited to the linear state-space model (7), there is another approach called the equivalent-noise approach [21–25]. The basic assumption of this method is that the manoeuvre effect can be modelled by (part of) a white or coloured noise process sufficiently well. In other words, it is assumed that the above equation that describes target motions can be simplified as (1) with adequate accuracy. In this case w is the equivalent noise that quantifies the error of this model. Of course, the statistics (e.g. the mean and covariance) of this noise w, non-stationary in general, are not known.

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3 Main results

In the manoeuvring target tracking problem, one of the difficulties is to decide when the target begins to manoeuvre. In many research studies, it has been attempted to detect the target manoeuvre as quickly as possible. In this paper, we present a new approach which does not need any manoeuvre detection procedure. The proposed tracking algorithm operates in both the non-manoeuvre and manoeuvre modes.

In this section, the tracking manoeuvring target scheme is proposed on the basis of an IE approach. The proposed scheme is based on the combination of both the Bayesian and Fisher uncertainty models [26, 27].

If we consider the additive manoeuvre term $u(n)$ as a deterministic signal in the manoeuvring (7), then we deal with two mixed uncertainties, $w(n)$ as a stochastic plant noise and $u(n)$ as a deterministic but unknown additive manoeuvre term where

$$\boldsymbol{C} = \begin{bmatrix} T^2/2 & T & 0 & 0 \\ 0 & 0 & T^2/2 & T \end{bmatrix}^{\mathrm{T}}$$

Now, we propose the additive manoeuvre term $u(n)$ as a new state and convert the manoeuvring model (7) to a non-manoeuvring model with an augmented state equation in the form of the standard Bayesian model with (7) as

$$\begin{bmatrix} \boldsymbol{X}(n+1) \\ u(n+1) \end{bmatrix} = \begin{bmatrix} \boldsymbol{F} & \boldsymbol{C} \\ 0 & \boldsymbol{I} \end{bmatrix} \begin{bmatrix} \boldsymbol{X}(n) \\ u(n) \end{bmatrix} + \begin{bmatrix} \boldsymbol{G} \\ 0 \end{bmatrix} w(n)$$
$$z(n) = \boldsymbol{H}(n)\boldsymbol{X}(n) + v(n) \qquad (9)$$

By defining an augmented state and an innovative posteriori measurement $z(n+1)$ via some manipulations, we can write

$$\boldsymbol{X}_{\mathrm{Aug}}(n) = \begin{bmatrix} \boldsymbol{X}(n) \\ u(n) \end{bmatrix}, \quad \boldsymbol{F}_{\mathrm{Aug}} = \begin{bmatrix} \boldsymbol{F} & \boldsymbol{C} \\ 0 & \boldsymbol{I} \end{bmatrix}$$
$$\boldsymbol{G}_{\mathrm{Aug}} = \begin{bmatrix} \boldsymbol{G} \\ 0 \end{bmatrix}, \quad \boldsymbol{W}_{\mathrm{Aug}} = w \qquad (10)$$

The innovative posteriori measurement $z(n+1)$ is also written based on the new augmented state as

$$z(n+1) = \boldsymbol{H}\boldsymbol{X}(n+1) + v(n+1) = \boldsymbol{H}\{\boldsymbol{F}\boldsymbol{X}(n) + \boldsymbol{C}u(n) + \boldsymbol{G}w(n)\} + v(n+1)$$
$$z(n+1) = [\boldsymbol{HF}\ \boldsymbol{HC}] \begin{bmatrix} \boldsymbol{X}(n) \\ u(n) \end{bmatrix} + \boldsymbol{HG}w(n) + v(n+1)$$

Thus, the new augmented measurement equation $\boldsymbol{Z}_{\mathrm{Aug}}(n)$ is obtained

$$\boldsymbol{Z}_{\mathrm{Aug}}(n) = \boldsymbol{H}_{\mathrm{Aug}}\boldsymbol{X}_{\mathrm{Aug}}(n) + \boldsymbol{V}_{\mathrm{Aug}}(n) \qquad (11)$$

where

$$\boldsymbol{H}_{\mathrm{Aug}} = [\boldsymbol{HF}\ \boldsymbol{HC}] \quad \text{and} \quad \boldsymbol{V}_{\mathrm{Aug}}(n) = \boldsymbol{HG}w(n) + v(n+1) \qquad (12)$$

Now, we have a standard non-manoeuvring augmented state model as

$$\boldsymbol{X}_{\mathrm{Aug}}(n+1) = \boldsymbol{F}_{\mathrm{Aug}}\boldsymbol{X}(n) + \boldsymbol{G}_{\mathrm{Aug}}\boldsymbol{W}_{\mathrm{Aug}}(n)$$
$$\boldsymbol{Z}_{\mathrm{Aug}}(n) = z(n+1) = \boldsymbol{H}_{\mathrm{Aug}}(n)\boldsymbol{X}_{\mathrm{Aug}}(n) + \boldsymbol{V}_{\mathrm{Aug}}(n) \qquad (13)$$

where, $\boldsymbol{X}_{\mathrm{Aug}}$, $\boldsymbol{F}_{\mathrm{Aug}}$, $\boldsymbol{G}_{\mathrm{Aug}}$, $\boldsymbol{W}_{\mathrm{Aug}}$ and $\boldsymbol{Z}_{\mathrm{Aug}}$, $\boldsymbol{H}_{\mathrm{Aug}}$, $\boldsymbol{V}_{\mathrm{Aug}}$ are obtained as (10), (12), and (13).

It is seen that the augmented measurement noise $\boldsymbol{V}_{\mathrm{Aug}}(n) = \boldsymbol{HG}w(n) + v(n+1)$ is time correlated with the process noise w. Thus, there is a cross term between the process and the measurement noises in the augmented measurement (11). It should be noted that the augmented measurement noise $\boldsymbol{V}_{\mathrm{Aug}}$ is still a white process.

There is a standard solution to overcome this problem. This is done by a modification on the Kalman gain. By defining the cross-covariance between augmented process noise $\boldsymbol{W}_{\mathrm{Aug}}(n)$ and the augmented measurement noise $\boldsymbol{V}_{\mathrm{Aug}}(n)$ as the matrix $\boldsymbol{T}_{\mathrm{Aug}}(n)$, the new Kalman gain is modified on the basis of $\boldsymbol{T}_{\mathrm{Aug}}(n)$ as

$$\boldsymbol{K}_{\mathrm{Aug}}(n+1) = [\boldsymbol{P}_{\mathrm{Aug}}(n+1|n)\boldsymbol{H}^{\mathrm{T}}_{\mathrm{Aug}}(n+1) + \boldsymbol{G}_{\mathrm{Aug}}(n)\boldsymbol{T}_{\mathrm{Aug}}(n)]\boldsymbol{R}^{-1}_{\mathrm{Aug}}(n+1)$$
$$\boldsymbol{P}_{\mathrm{Aug}}(n+1|n+1) = \boldsymbol{P}_{\mathrm{Aug}}(n+1|n) - \boldsymbol{P}_{\mathrm{Aug}}(n+1|n) \times \boldsymbol{H}^{\mathrm{T}}_{\mathrm{Aug}}(n+1) \times [\boldsymbol{R}_{\mathrm{Aug}}(n+1) + \boldsymbol{H}_{\mathrm{Aug}}(n+1) \times \boldsymbol{P}_{\mathrm{Aug}}(n+1|n)\boldsymbol{H}^{\mathrm{T}}_{\mathrm{Aug}}(n+1)]^{-1}\boldsymbol{H}_{\mathrm{Aug}}(n+1)\boldsymbol{P}_{\mathrm{Aug}}(n+1|n)$$
$$\boldsymbol{P}_{\mathrm{Aug}}(n+1|n) = \boldsymbol{F}_{\mathrm{Aug}}(n)\boldsymbol{P}_{\mathrm{Aug}}(n|n)\boldsymbol{F}^{\mathrm{T}}_{\mathrm{Aug}}(n) + \boldsymbol{G}_{\mathrm{Aug}}(n)\boldsymbol{Q}_{\mathrm{Aug}}(n)\boldsymbol{G}^{\mathrm{T}}_{\mathrm{Aug}}(n) \qquad (14)$$

The optimal target manoeuvre estimator for the augmented system (13) is

$$\hat{\boldsymbol{X}}_{\mathrm{Aug}}(n+1|n+1) = \boldsymbol{F}_{\mathrm{Aug}}(n)\hat{\boldsymbol{X}}_{\mathrm{Aug}}(n|n) + \boldsymbol{K}_{\mathrm{Aug}}(n+1) \times [\boldsymbol{Z}_{\mathrm{Aug}}(n+1) - \boldsymbol{H}_{\mathrm{Aug}}(n+1)\boldsymbol{F}_{\mathrm{Aug}}(n) \times \hat{\boldsymbol{X}}_{\mathrm{Aug}}(n|n)] \qquad (15)$$

It remains to compute $\boldsymbol{Q}_{\mathrm{Aug}}(n)$, $\boldsymbol{R}_{\mathrm{Aug}}(n)$ and $\boldsymbol{T}_{\mathrm{Aug}}(n)$. Since $v(n)$ and $w(n)$ are uncorrelated white processes, we can obtain the new covariance matrix of the augmented process noise $\boldsymbol{W}_{\mathrm{Aug}}(n)$ and measurement noise $\boldsymbol{V}_{\mathrm{Aug}}(n)$ as

$$\boldsymbol{E}\begin{bmatrix}\boldsymbol{W}_{\mathrm{Aug}}(n_1)\\ \boldsymbol{V}_{\mathrm{Aug}}(n_1)\end{bmatrix}\begin{bmatrix}\boldsymbol{W}^{\mathrm{T}}_{\mathrm{Aug}}(n_2) & \boldsymbol{V}^{\mathrm{T}}_{\mathrm{Aug}}(n_2)\end{bmatrix} = \begin{cases}\begin{bmatrix}\boldsymbol{Q}_{\mathrm{Aug}}(n_1) & \boldsymbol{T}_{\mathrm{Aug}}(n_1)\\ \boldsymbol{T}^{\mathrm{T}}_{\mathrm{Aug}}(n_1) & \boldsymbol{R}_{\mathrm{Aug}}(n_1)\end{bmatrix}, & n_1 = n_2\\ 0, & n_1 \neq n_2\end{cases} \tag{16}$$

where

$$\boldsymbol{Q}_{\mathrm{Aug}}(n) = \boldsymbol{E}\left\{\boldsymbol{W}_{\mathrm{Aug}}(n)\boldsymbol{W}^{\mathrm{T}}_{\mathrm{Aug}}(n)\right\} = \boldsymbol{E}\left\{w(n)w^{\mathrm{T}}(n)\right\} = \boldsymbol{Q} \tag{17}$$

$$\begin{aligned}\boldsymbol{R}_{\mathrm{Aug}}(n) &= \boldsymbol{E}\{\boldsymbol{V}_{\mathrm{Aug}}(n)\boldsymbol{V}^{\mathrm{T}}_{\mathrm{Aug}}(n)\} = \\ &\boldsymbol{E}\{(\boldsymbol{H}(n)\boldsymbol{G}(n)w(n) + v(n+1))\\ &\quad \times (\boldsymbol{H}(n)\boldsymbol{G}(n)w(n) + v(n+1))^{\mathrm{T}}\}\\ &= \boldsymbol{H}(n)\boldsymbol{G}(n)\boldsymbol{E}\{w(n)w(n)^{\mathrm{T}}\}\boldsymbol{G}(n)^{\mathrm{T}}\boldsymbol{H}(n)^{\mathrm{T}}\\ &\quad + \boldsymbol{E}\{v(n+1)v(n+1)^{\mathrm{T}}\}\\ \Rightarrow \boldsymbol{R}_{\mathrm{Aug}}(n) &= \boldsymbol{E}\{\boldsymbol{V}_{\mathrm{Aug}}(n)\boldsymbol{V}^{\mathrm{T}}_{\mathrm{Aug}}(n)\}\\ &= \boldsymbol{H}(n)\boldsymbol{G}(n)\boldsymbol{Q}(n)\boldsymbol{G}(n)^{\mathrm{T}}\boldsymbol{H}(n)^{\mathrm{T}} + \boldsymbol{R}(n)\end{aligned} \tag{18}$$

and finally $\boldsymbol{T}_{\mathrm{Aug}}(n)$, is derived as

$$\begin{aligned}\boldsymbol{T}_{\mathrm{Aug}}(n) &= \boldsymbol{E}\left\{\boldsymbol{W}_{\mathrm{Aug}}(n)\boldsymbol{V}^{\mathrm{T}}_{\mathrm{Aug}}\right\}\\ &= \boldsymbol{E}\left\{w(n)[\boldsymbol{H}((n)\boldsymbol{G}(n)w(n) + v(n+1)]^{\mathrm{T}}\right\}\\ &= \boldsymbol{E}\left\{w(n)w^{\mathrm{T}}(n)G^{\mathrm{T}}(n)H^{\mathrm{T}}(n)\right\} + \boldsymbol{E}\left\{w(n)v(n+1)^{\mathrm{T}}\right\}\\ &= \boldsymbol{Q}\boldsymbol{G}^{\mathrm{T}}(n)\boldsymbol{H}^{\mathrm{T}}(n)\end{aligned} \tag{19}$$

Remark 2: Although $\boldsymbol{F}_{\mathrm{Aug}}$ has a zero vector and it seems that for the vector u we do not have any transition $u(n+1) = u(n)$, the estimation of $\hat{u}(n+1)$ is modified through the new Kalman gain $\boldsymbol{K}_{\mathrm{Aug}}$ corresponding to the augmented state system (13).

Remark 3: It should be noted that by introducing a posteriori measurement $z(n+1)$, the standard Bayesian model can be derived based on the new augmented state. Since $\boldsymbol{Z}_{\mathrm{Aug}}(n) = z(n+1)$, when the proposed optimal target manoeuvre estimator (15) is applied, the estimated state $\hat{\boldsymbol{X}}_{\mathrm{Aug}}(n+1|n+1)$ depends on $z(n+2)$. This fact causes a delay of one time interval.

Remark 4: Since target tracking is a real-time problem, timing is a crucial factor, and many researchers want to detect the target manoeuvre as quickly as possible. However,

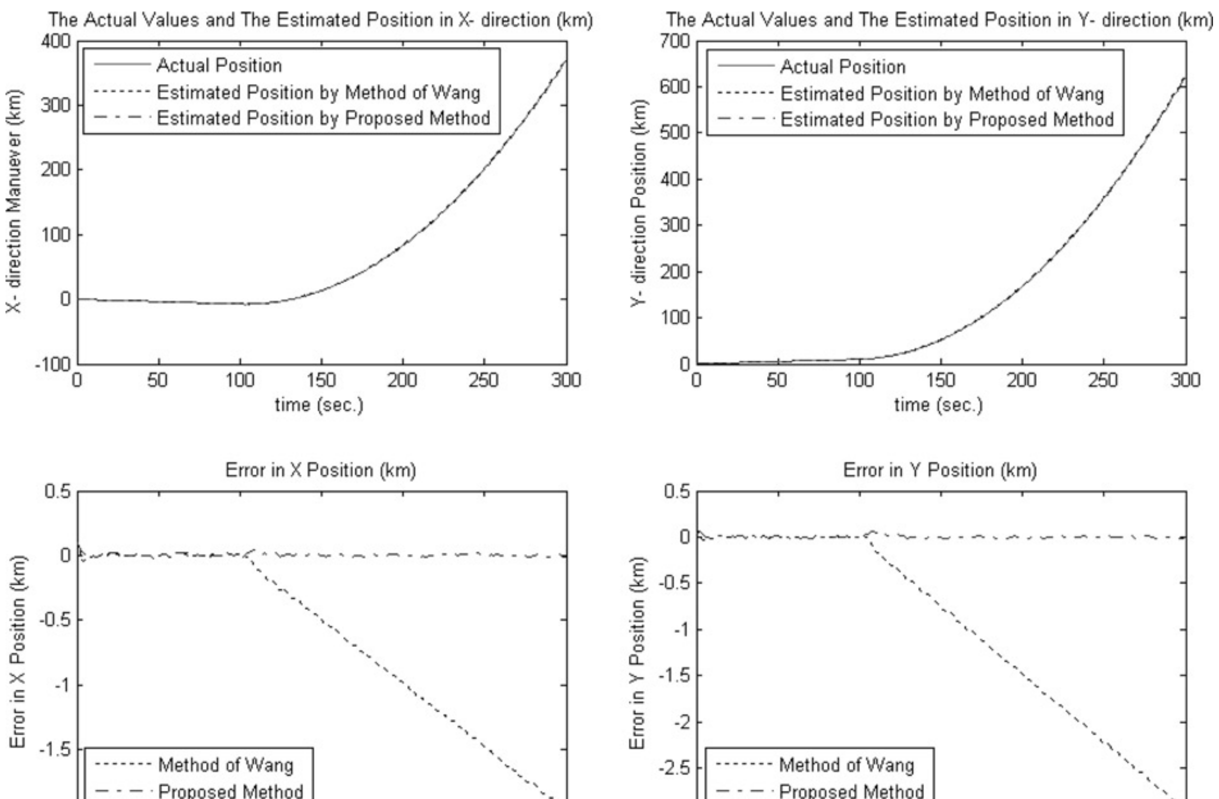

Figure 1 *The actual values, the estimations of x(t) and y(t), and their corresponding errors*

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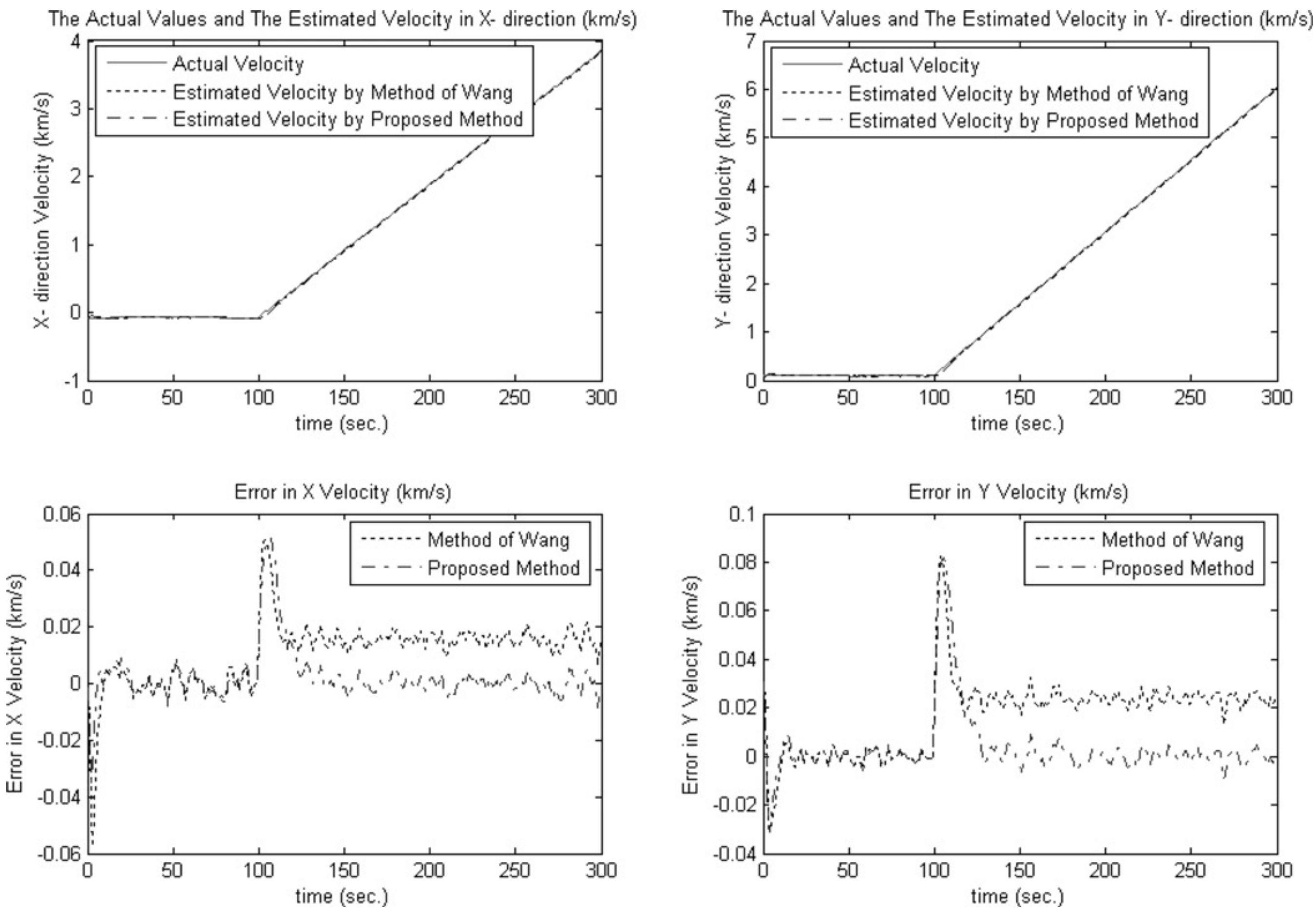

Figure 2 *The actual values and the estimations of* $v_x(t)$ and $v_y(t)$ *and their corresponding errors*

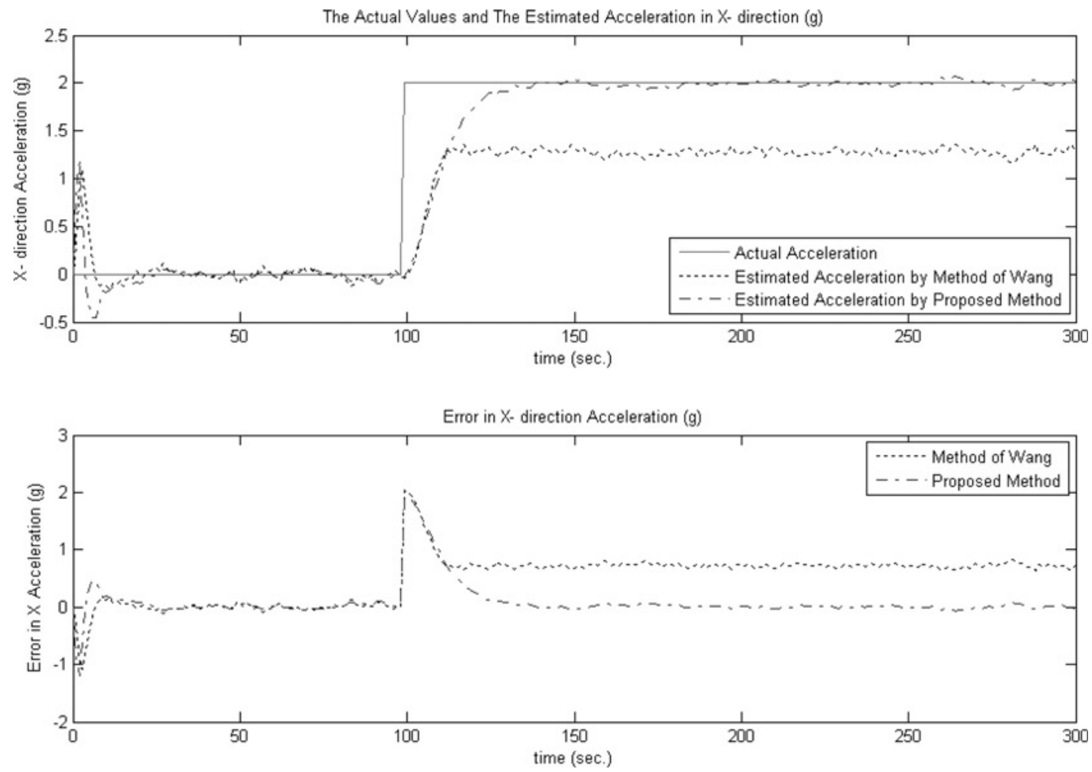

Figure 3 *The actual values, the X-acceleration estimations and their corresponding errors*

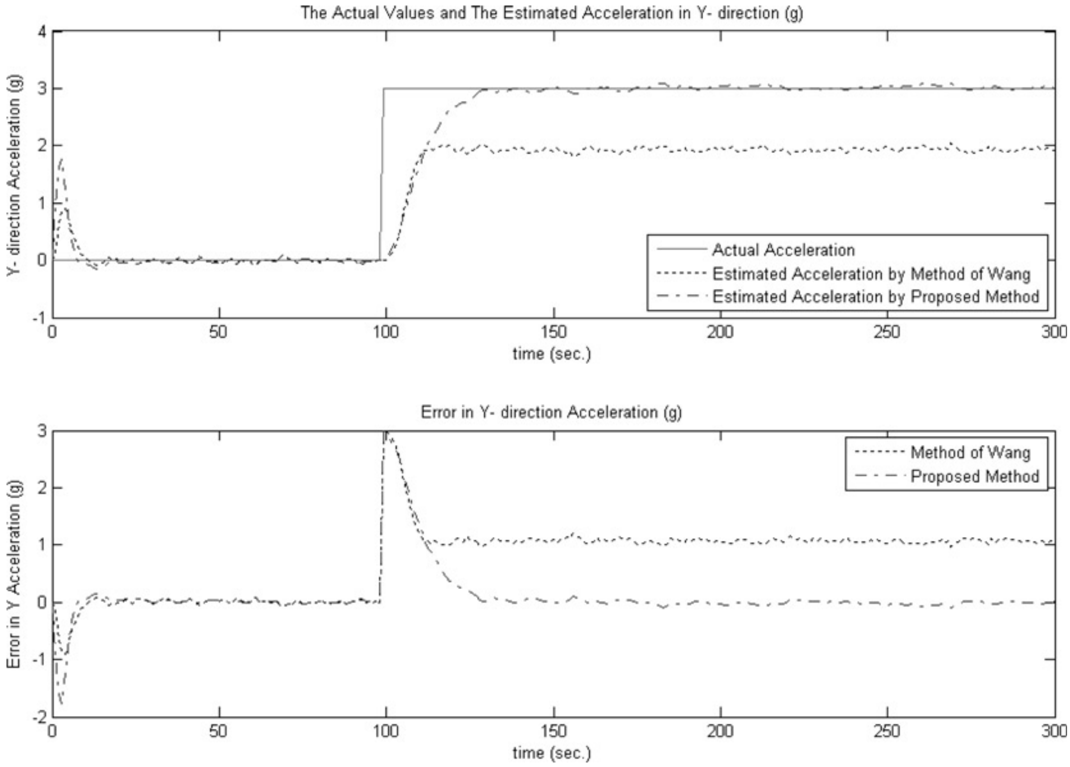

Figure 4 *The actual values, the Y-acceleration estimations and their corresponding errors*

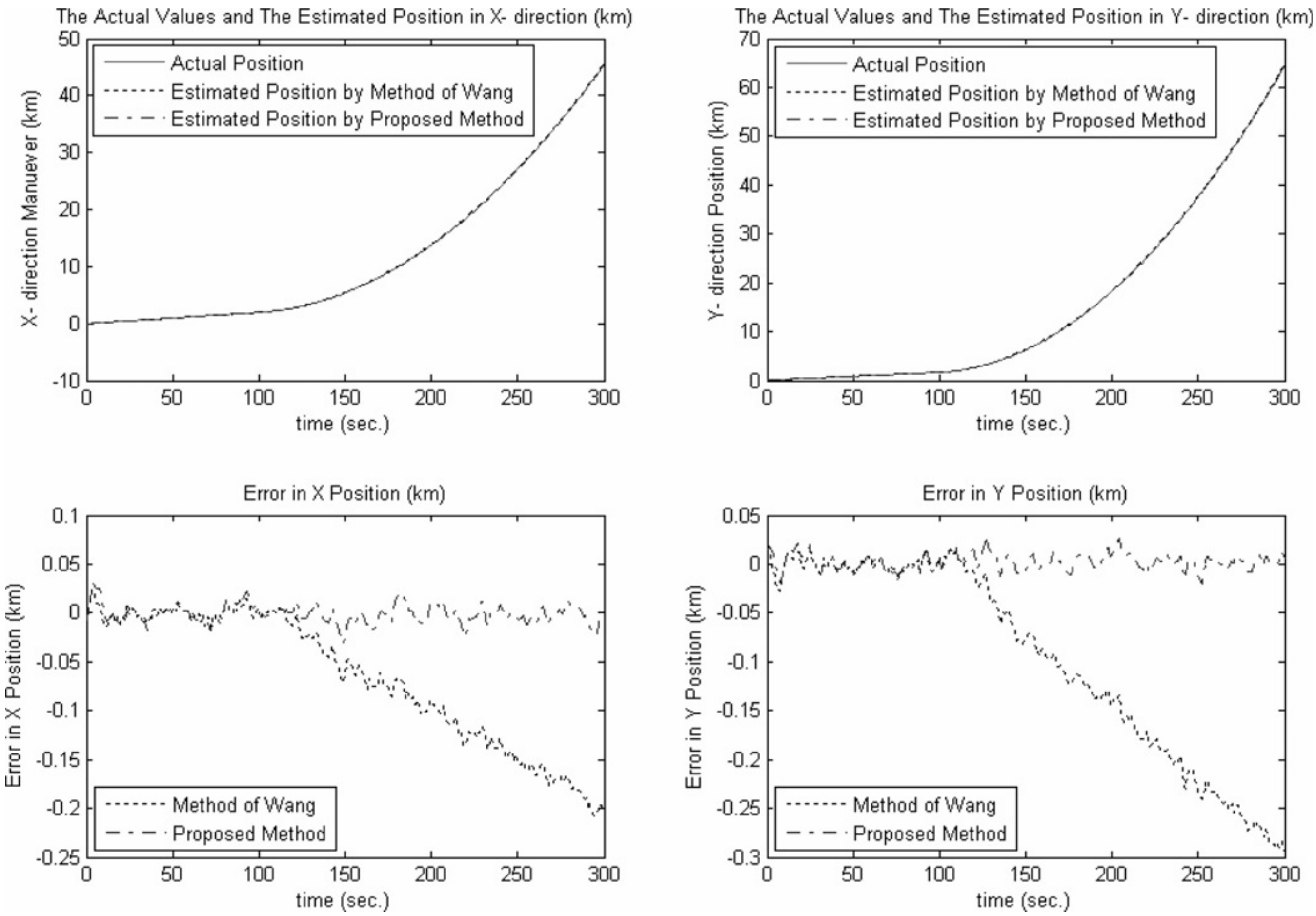

Figure 5 *The actual values and the estimations of x(t) and y(t) and their corresponding errors*

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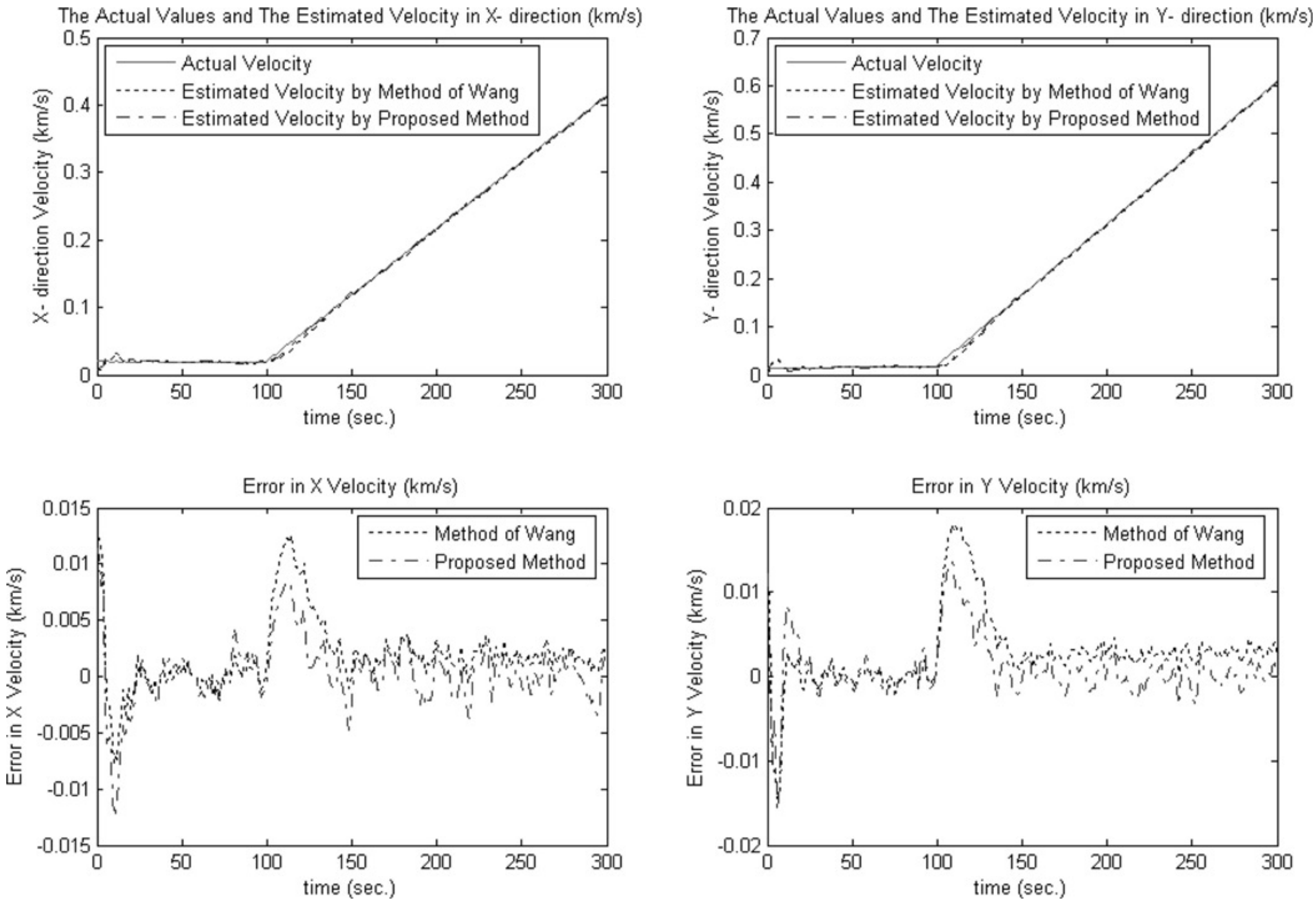

Figure 6 *The actual values and the estimations of* $v_x(t)$, and $v_y(t)$ *and their corresponding errors*

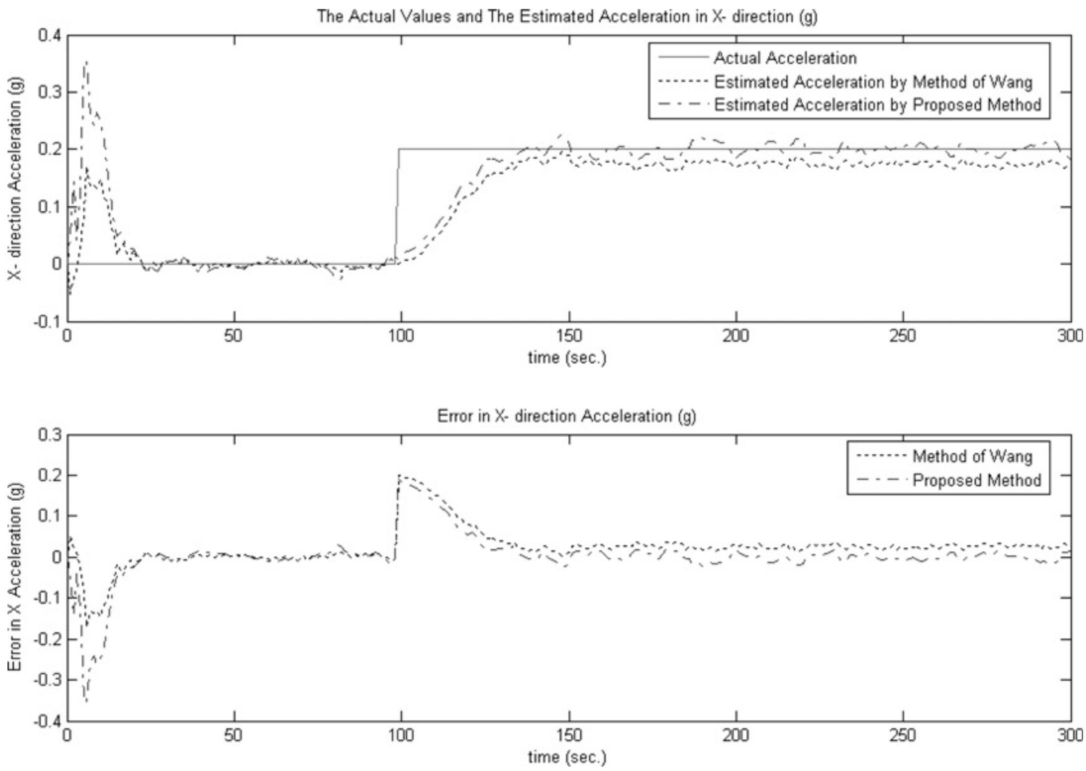

Figure 7 *The actual values, the X-acceleration estimations and their corresponding errors*

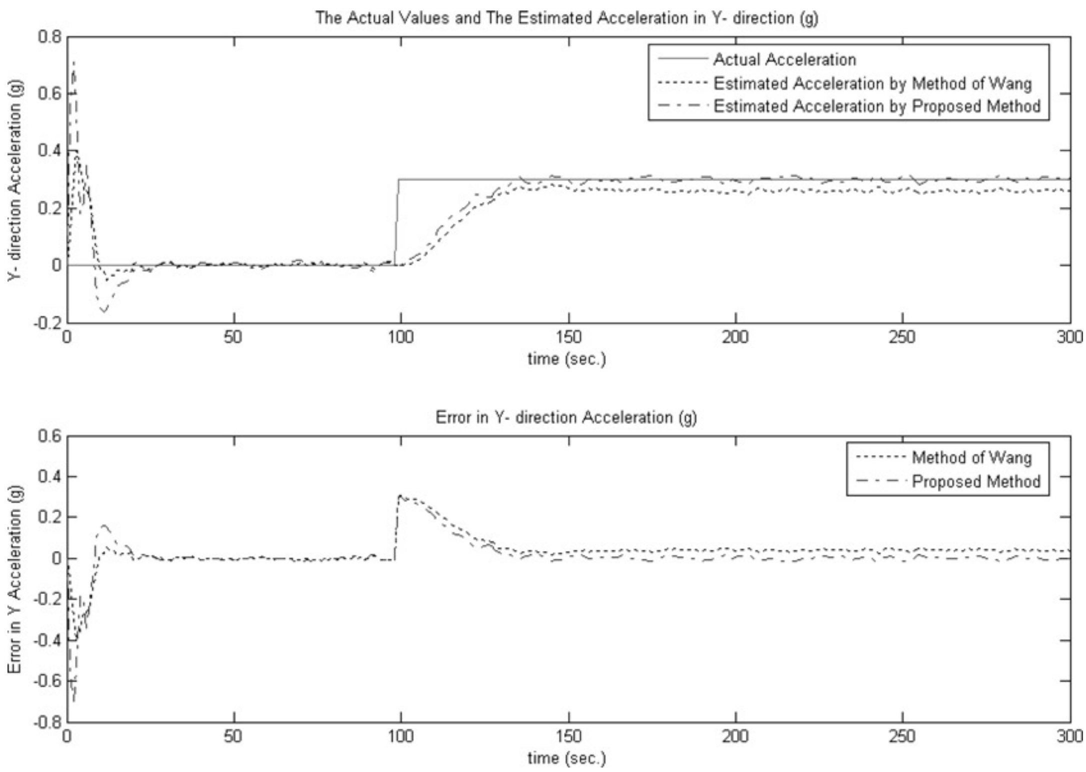

Figure 8 *The actual values, the Y-acceleration estimations and their corresponding errors*

in the proposed method, the manoeuvre detection algorithm does not exist and therefore the consumption time of the manoeuvre detection is zero.

4 Simulation results

In this section the theoretical development is verified by simulation results. The effectiveness of the proposed algorithm is compared with the work of Wang and Varshney [19] as a traditional IE method. In [19], the predicted and estimated states for the manoeuvring targets are related to the corresponding, non-manoeuvring states according to the appropriate filters. The methodology in [19] includes a quick detection scheme based on the innovation sequence which has been developed for a prompt detection of target manoeuvres. The optimal length of a sliding window that minimises the manoeuvre detection delay for a given false alarm rate is determined. After manoeuvre detection, the system model is modified by adding a manoeuvre term. A recursive algorithm is proposed in [19] to estimate the manoeuvre magnitude using a modified Kalman.

It is assumed that the target moves, in a two-dimensional space and its dynamics is given by (7).

Example 1: As the first example, we consider a target initial condition with state $X(0) = [100\ \text{m}\ -80\ \text{m/s}\ 400\ \text{m}\ 100\ \text{m/s}]^{\text{T}}$ and the acceleration $u(t) = \begin{bmatrix} 0\,g & 0\,g \end{bmatrix}^{\text{T}}$ where $g = 9.8\ \text{m/s}^2$ for $0 \le t \le 100$ s. The target begins to manoeuvre as $u(t) = \begin{bmatrix} 2\,g & 3\,g \end{bmatrix}^{\text{T}}$ for $100\ \text{s} < t \le 300$ s. In this simulation, the sampling time is $T = 1$ s. The elements of the covariance matrices of the system and the measurement noises are selected as $Q_{ii} = 1$ and $R_{ii} = (100)^2\ \text{m}^2$. The initial augmented state $X_{\text{Aug}}(0, -1)$ is randomly selected. The elements of the augmented state error matrix $P_{\text{Aug}}(0, -1)$ for the proposed Kalman filter are chosen ten.

Figs. 1–4 are the actual values and the estimations of positions $x(t)$ and $y(t)$, the actual values and the estimations of $v_x(t)$, and $v_y(t)$, the actual values and the estimations of $a_x(t)$ and $a_y(t)$, and also their corresponding errors by the proposed method and the method of Wang, respectively.

Example 2: In this example, we consider a target initial condition with state $X(0) = [-10\ \text{m}\ \ 20\ \text{m/s}\ \ 100\ \text{m}\ \ 15\ \text{m/s}]^{\text{T}}$ and acceleration $u(t) = \begin{bmatrix} 0\,g & 0\,g \end{bmatrix}^{\text{T}}$ for $0 \le t \le 100$ s and the target begins to manoeuvre as $u(t) = \begin{bmatrix} 0.2\,g & 0.3\,g \end{bmatrix}^{\text{T}}$ for $100\ \text{s} < t \le 300$ s. In this simulation, the sampling time is $T = 1$ s. The elements of the covariance matrices of the system and measurement noises are chosen as $Q_{ii} = 0.1$ and $R_{ii} = (100)^2\ \text{m}^2$, respectively. The initial augmented state $X_{\text{Aug}}(0, -1)$ is randomly selected. The elements of the augmented state error matrix $P_{\text{Aug}}(0, -1)$ for the proposed Kalman filter are chosen ten.

Fig. 5 shows the actual values and the estimations of $x(t)$ and $y(t)$ of the proposed method and the method of Wang

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Table 1 Monte-Carlo simulation results for 200 runs at $t = 100 * T$ (ignoring initial transitions)

	Statistics →	ME		MAE		RMSE	
Manoeuvre level ↓	Error ↓	Method of Wang	Proposed method	Method of Wang	Proposed method	Method of Wang	Proposed method
Low manoeuvre $T = 0.1$ s $a_x = 0.02\,g$, $a_y = 0.03\,g$	X-position (m)	8.3516	0.4193	16.273	6.6862	8.8831	2.9970
	Y-position (m)	13.513	0.3178	120.09	6.1593	13.802	2.6911
	X-velocity (m/s)	1.5985	0.1620	5.0920	33.408	2.0848	1.4290
	Y-velocity (m/s)	2.7970	0.0893	5.5894	3.2705	3.0672	1.2492
	X-acceleration (m/s^2)	0.0375	0.0030	0.6409	0.0638	0.2576	0.0278
	Y-acceleration (m/s^2)	0.0134	0.0008	0.5642	0.0630	0.2425	0.0238
Medium manoeuvre $T = 1$ s $a_x = 0.2\,g$, $a_y = 0.3\,g$	X-position (m)	10.177	0.2448	24.522	12.417	11.990	4.2956
	Y-position (m)	15.786	0.7011	37.479	12.513	717.11	4.6901
	X-velocity (m/s)	0.1254	0.1051	1.8740	1.7823	0.7794	0.7425
	Y-velocity (m/s)	0.1908	0.0607	2.7749	2.8095	0.8757	0.8464
	X-acceleration (m/s^2)	0.0253	0.0039	0.1461	0.1289	0.0564	0.0481
	Y-acceleration (m/s^2)	0.0372	0.0010	0.1515	0.1596	0.0643	0.0526
High manoeuvre $T = 1$ s $a_x = 2\,g$, $a_y = 3\,g$	X-position (m)	98.614	0.2557	118.85	14.228	98.854	5.0543
	Y-position (m)	149.36	0.2888	168.23	18.974	149.56	5.7017
	X-velocity (m/s)	1.5512	0.1144	7.9328	5.4271	2.9741	1.8075
	Y-velocity (m/s)	2.1112	0.2408	10.503	6.5241	3.5622	2.1203
	X-acceleration (m/s^2)	0.7474	0.0164	1.5193	0.6181	0.8214	0.2274
	Y-acceleration (m/s^2)	1.0171	0.0699	1.8330	0.5495	1.0744	0.2570

in the presence of target manoeuvring. Fig. 6 illustrates the actual values and the estimations of the velocities in the X and Y directions and their corresponding errors. Also, the actual values and the estimations of the acceleration in the X and Y directions and the corresponding errors can be seen in Figs. 7 and 8, respectively.

Example 3: Since the performance of the proposed algorithm is presented as an estimation of the tracking of positions, velocities and accelerations, a comparison based on the Monte-Carlo simulation [28, 29] is also made to evaluate the performances between the proposed method and the method of Wang.

Performance evaluation of the methods is measured by the mean value of error (ME), the maximum of absolute error (MAE) and the root mean square error (RMSE) indices over 200 runs of simulation. These statistics are computed ignoring initial transitions.

In Table 1, the results of the state estimations for the above mentioned methods are summarised. The error indices ME, MAE and RMSE are computed with 200 Monte-Carlo runs in three scenarios: low, medium and high manoeuvring target. Table 1 shows that the proposed scheme can significantly improve the results of the state estimation and it is superior to the traditional IE method.

5 Conclusions

This paper deals with a new modelling of an IE approach for tracking manoeuvring targets and proposes a modification scheme for a Kalman filter-based target tracker. This method is based on a mixed Bayesian–Fisher uncertainty model. The main idea of this paper is converting a manoeuvring target problem to a non-manoeuvring problem via a special state augmentation to derive a standard Bayesian model. Then, a conventional Kalman filter can be applied to this model to estimate the original state and the acceleration vector simultaneously. The proposed manoeuvring target algorithm does not need the manoeuvre detection stage and also it does not consume any time for manoeuvre detection. Simulation results are provided to confirm the theoretical development. The results are compared with the work of Wang. Simulation results also show a high performance of the proposed IE model and the effectiveness of this scheme in the estimation of the parameters of manoeuvring targets.

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