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Coordinates Transformation for Target Maneuver Detection

A. Karsaz, N. Pariz
Department of Electrical Engineering
Ferdowsi University & Kh. N. Toosi University
Mashhad, Iran
a_karsaz1@yahoo.com, n-pariz@ferdowsi.um.ac.ir,

H. Khaloozadeh, A. Khorami
Department of Electrical Engineering
Ferdowsi University
Mashhad, Iran
h_khaloozadeh@yahoo.com, Khorami_ali@yahoo.com

***Abstract*--Ship borne targets normally maneuver on circular paths which have lead to tracking filters on circular turns. In this paper, an innovation technique is presented to transform the tracking-maneuvering target problems from Polar coordinate to Cartesian coordinate, therefore a standard linear Kalman filter can be easily applied to them. Mathematical relation between measurement noise covariance in polar coordinate and the measurement noise covariance in Cartesian coordinate for Kalman implementation is obtained in this approach via a theorem.**

Keywords-Maneuvering targets, Polar to Cartesian transformation, target maneuver detection.

I. Introduction

THERE exist many approaches and methods for tracking maneuvering targets [1]-[3]. For example, switching between Kalman filters of different orders, estimation of acceleration as input during a maneuver and to correct the state using batch least squares methods or recursive estimation for on-line implementation. Lanka [4] and Korn, Gully and Willsky [5] have developed an extended Kalman filter using a circular model of motion. Singer [6] modeled target acceleration as a random process with known exponential autocorrelation. This model is capable of tracking a maneuvering target, but the performance of the estimation is reduced when target moved at a constant velocity. A generalized likelihood ratio (GLR) method for maneuver detection and estimation was presented by Korn, Gully and Willsky [5]. This algorithm proposes the use of two hypotheses, null hypothesis for a target without maneuver, and alternative hypothesis for a target with maneuver. When the log likelihood ratio is over a threshold, a maneuver is detected. This system needs a bank of correlators to detect the maneuver onset time.

In this Paper, we presented a new approach to transform measurement noise covariance matrix from Polar coordinate to Cartesian coordinate.

II. MODELS OF UNCERTAINTY [7]

The basic models to be considered in this paper are the Bayesian and Fisher models which use in [8]. Theses models are specific cases of the state space structure-white process.

The Bayesian models are one of the most important and common used models of uncertainty. In Bayesian models, uncertainty are modeled by random variables and/or stochastic processes with completely specified either probability distributions or completely specified first and second moments.

The complete definition of the Bayesian, discrete time model for linear systems is summarized now.

$$X(n+1) = F(n)X(n) + G(n)w(n)$$
$$z(n) = H(n)X(n) + v(n)$$

$X(n)$	*state*	
$z(n)$	*observation*	(1)
$v(n)$	*white observation* uncertainty	
$w(n)$	*white* system driving uncertainty	
$X(0)$	initial condition	

$$E\{v(n_1)v^T(n_2)\} = \begin{cases} \Re(n_1) & n_1 = n_2 \\ 0 & n_1 \neq n_2 \end{cases}$$
$$E\{w(n_1)w^T(n_2)\} = \begin{cases} Q(n_1) & n_1 = n_2 \\ 0 & n_1 \neq n_2 \end{cases}$$
$$E\{x(0)x^T(0)\} = \psi$$
$$E\{\mathrm{x}(0)\} = 0 \quad , \quad E\{\mathrm{w}(0)\} = 0 \quad , \quad E\{\mathrm{v}(0)\} = 0$$

In many applications, the input disturbance, w(.) can be modeled as being completely unknown. A model where w(.) is completely unknown is a type of Fisher model. Of course, conceptually such Fisher models have to be handled in a different fashion from Bayesian models where w(.) is viewed as a random vector with known covariance matrix Q(.). For some applications the Fisher modeling of w(.), can be viewed as the limiting Bayesian case, where.

III. FILTERING OF THE BAYESIAN MODELS

The desired form of the filtering solution is a difference equation (recursive relationship) expressing $\hat{X}(N+1|N)$ in terms of $\hat{X}(N|N)$ on *z(N+1).*

The logic, which yields the desired equation, can be summarized in the following steps:

1. Assume that $\hat{X}(N+1|N+1)$ is to be calculate just $\hat{X}(N|N)$ and *z(N+1).*
2. Use the one-step prediction logic to change the problem to calculate $\hat{X}(N+1|N+1)$ from $\hat{X}(N+1|N)$ and *z(N+1).*
3. Solve a Fisher estimation problem where $\hat{X}(N+1|N+1)$ and *z(N+1)* are considered on an unknown vector *X(N+1).*

In the Bayesian model, stochastic probabilistic models are used for all the uncertainties. Thus *x(0), v(n)* and *w(n)* are modeled as zero mean uncertainty random variables.

The matrix *H(n), F(n)* and *G(n)* in Eq. (1) assumed to be known function of time n. The problem to be considered in how to use the observation up to time n_2, *z(1),...,z(*n_2*),* to estimate the state $X(n_1)$ at some time n_1.

The solution of the problem filtering, after some manipulation leads us to the Kalman filter with equations:

$$\begin{aligned}
&\hat{X}(N+1|N) = F(N)\hat{X}(N|N) + K(N+1)[z(N+1) \\
&-H(N+1)F(N)\hat{X}(N|N)] \\
&K(N+1) = \Sigma(N+1|N)H^T(N+1)\Re^{-1}(N+1) \\
&\Sigma(N+1|N+1)) = \Sigma(N+1|N) - \\
&\Sigma(N+1|N)H^T(N+1)[\Re(N+1) + H(N+1) \\
&\Sigma(N+1|N)H(N+1)^T]^{-1}H(N+1)\Sigma(N+1) \\
&\Sigma(N+1|N) = F(N)\Sigma(N|N)F^T(N) \\
&+G(N)Q(N)G^T(N) \\
&\Sigma(0|0) = 0, \hat{X}(0|0) = 0
\end{aligned} \tag{2}$$

$K(N)$ is the Kalman gain and notation $\hat{X}(N+1|N)$ denotes the prediction at the $(K+1)^{th}$ sample point given the measurement up to and including the K^{th} whilst $\hat{X}(N|N)$ denotes the estimation at the N^{th} sample point given the measurement up to and including the N^{th}. $\Sigma(N|N)$ is the error covariance matrix and $\Sigma(N+1|N)$ is the error covariance matrix of the one-step prediction. The measurement covariance matrix $\Re$ is usually introduced in polar coordinate, but we need it in Cartesian coordinate for Kalman filter implementation at Eq. (2).

$$\Re = \begin{bmatrix} \delta_x^2 & 0 \\ 0 & \delta_y^2 \end{bmatrix} \tag{3}$$

δ_x *: Standard diversion of target trajectory measurement at X axis*

δ_y *: Standard diversion of target trajectory measurement at Y axis*

Maneuvering Targets are difficult to track with Kalman filter since the target model of tracking filter might not fit the real target trajectory [9].

IV. Transformation between Polar Coordinate and Cartesian Coordinate Measurement Covariance Matrix

The measurement noise in the maneuvering model is related to the measurement noise in Cartesian coordinates by Roecker and McGillem [10]. Tracking with a polar model of motion is not easily implemented by using a Kalman filter in Cartesian coordinate because the process model is nonlinear, and the covariance matrix of each radar obtained in polar coordinate. Therefore for Kalman filter implementation in Cartesian coordinate, transformation between polar coordinate and Cartesian coordinate measurement noise covariance matrix is necessary.

The transformation between Cartesian coordinates and polar coordinates is given by

$$R = \sqrt{x^2 + y^2} \tag{4}$$

$$\theta = \tan^{-1}(\frac{y}{x}) \tag{5}$$

$$X = R\cos\theta \tag{6}$$

$$Y = R\sin\theta \tag{7}$$

In polar coordinates the state is radius and angle of target $([R\ \theta])$.

A. Theorem

If the states R and θ in polar coordinates are two independent measurements with white noise zero mean variables, then the states in Cartesian coordinates are two independent measurements with white noise zero mean variables. In addition the measurement noise vector in Cartesian coordinates is related to the measurement noise vector in polar coordinates by the following equation,

$$\begin{bmatrix} \delta_x^2 \\ \delta_y^2 \end{bmatrix} = \begin{bmatrix} \cos^2\theta_0 & R_0^2\sin^2\theta_0 \\ \sin^2\theta_0 & R_0^2\cos^2\theta_0 \end{bmatrix} \begin{bmatrix} \delta_R^2 \\ \delta_\theta^2 \end{bmatrix} \tag{8}$$

where δ_R and δ_θ are too small.

B. Proof

Define,

$$\begin{aligned} R &= R_o + \Delta R \\ \theta &= \theta_o + \Delta\theta \end{aligned} \tag{9}$$

where, ΔR and $\Delta\theta$ are two zero mean independent random white noise variables as bellow:

$\Delta R : N(0, \delta_R)$

$\Delta\theta : N(0, \delta_\theta)$

δ_R *: Standard diversion of target radius measurement*

δ_θ *: Standard diversion of target angle measurement*

R_o *: Target radius without noise*

θ_o *: Target angle without noise*

Suppose that x and y in Cartesian coordinates have been perturbed by a noise as bellow

$$\begin{aligned} X &= X_o + \Delta X \\ Y &= Y_0 + \Delta Y \end{aligned} \tag{10}$$

where the states X_o and Y_o define the target trajectory in Cartesian coordinates without noise. The relation equation between variables ΔX, ΔY and ΔR, $\Delta\theta$ is desired.

By Euler equation;

$$X_o + jY_o = R_o e^{j\theta_o} = R_o \cos\theta_o + jR_o \sin\theta_o \qquad (11)$$

$$\begin{aligned} X + jY &= Re^{j\theta} \\ &= R\cos\theta + jR\sin\theta \\ &= (R_o + \Delta R)\cos(\theta_o + \Delta\theta) + j(R_o + \Delta R)\sin(\theta_o + \Delta\theta) \quad (12) \\ &= (R_o + \Delta R)(\cos\theta_o \cos\Delta\theta - \sin\theta_o \sin\Delta\theta) \\ &\quad + j(R_o + \Delta R)(\sin\theta_o \cos\Delta\theta + \cos\theta \sin\Delta\theta) \end{aligned}$$

By using Eq. (8) we have,

$$\begin{aligned} X_o + \Delta X + j(Y_o + \Delta Y) &= R_o \cos\theta_o \cos\Delta\theta - R_o \sin\theta_o \sin\Delta\theta \\ &\quad + \Delta R\cos\theta_o \cos\Delta\theta - \Delta R\sin\theta_o \sin\Delta\theta \\ &\quad + \mathrm{j}(R_o \sin\theta_o \cos\Delta\theta + R_o \cos\theta_o \sin\Delta\theta \qquad (13) \\ &\quad + \Delta R\sin\theta_o \cos\Delta\theta + \Delta R\cos\theta_o \sin\Delta\theta) \end{aligned}$$

Suppose we know that ΔR and $\Delta\theta$ both have small values. so we have

$$\cos\Delta\theta \approx 1$$

$$\sin\Delta\theta \approx \Delta\theta$$

$$\Delta\theta\Delta R \approx 0$$

By substitution (11) in (13) and using above relation we have,

$$\begin{aligned} X_o + jY_o + \Delta X + j\Delta Y &= R_o \cos\theta_o - R_o\Delta\theta\sin\theta_o \\ &\quad + \Delta R\cos\theta_o - \Delta R\Delta\theta\sin\theta_o \\ &\quad + \mathrm{j}(R_o \sin\theta_o + R_o\Delta\theta\cos\theta_o \\ &\quad + \Delta R\sin\theta_o + \Delta R\Delta\theta\cos\theta_o) \qquad (14) \\ &= R_o\cos\theta_o + \mathrm{j}R_o\sin\theta_o \\ &\quad - R_o\Delta\theta\sin\theta_o + \Delta R\cos\theta_o \\ &\quad + \mathrm{j}(R_o\Delta\theta\cos\theta_o + \Delta R\sin\theta_o) \end{aligned}$$

Therefore,

$$\Delta X = \Delta R\cos\theta_o - R_o\Delta\theta\sin\theta_o \qquad (15)$$

$$\Delta Y = R_o\Delta\theta\cos\theta_o + \Delta R\sin\theta_o \qquad (16)$$

As mentioned before ΔR and $\Delta\theta$ are two zero mean independent random white noise variables, then X, Y are zero mean white noise with following variances :

$$\mathrm{var}(\Delta X) = \cos^2\theta_0 \,\mathrm{var}(\Delta R) + R_0^2 \sin^2\theta_0 \,\mathrm{var}(\Delta\theta)$$

$$\mathrm{var}(\Delta Y) = \sin^2\theta_0 \,\mathrm{var}(\Delta R) + R_0^2 \cos^2\theta_0 \,\mathrm{var}(\Delta\theta)$$

$$\begin{bmatrix} \delta_x^2 \\ \delta_y^2 \end{bmatrix} = \begin{bmatrix} \cos^2\theta_0 & R_0^2\sin^2\theta_0 \\ \sin^2\theta_0 & R_0^2\cos^2\theta_0 \end{bmatrix} \begin{bmatrix} \delta_R^2 \\ \delta_\theta^2 \end{bmatrix}$$

$$\Re = \begin{bmatrix} \delta_x^2 & 0 \\ 0 & \delta_y^2 \end{bmatrix}$$

Because, R_o, $\cos\theta_0$ and $\sin\theta_0$ are constant at each time. The values of δ_R, δ_θ can obtained from radar catalogs and have known values.

V. SIMULATION RESULTS

As an evaluation of this transformation technique, a Monte Carlo simulation was performed and various paths with circular and straight trajectories were implemented.

A target maneuvering tracking algorithm which turns in two-dimensional space is simulated. Several values of measurement noise covariance elements, range measurement and target azimuth measurement was used.

It is assumed that the target moves in a plane, which is the two-dimensional case, such as a ship.

At first example the target is traveling at zero acceleration $u(t)_{2*1}$ in X and Y direction equal $a_x(t) = 0\,\mathrm{m/s^2}$, $a_y(t) = 0\,\mathrm{m/s^2}$. In this simulation, the sampling time is T=0.015 second and the matrices of Q is 0.1. A Monte Carlo simulation of 10000 runs was performed on these paths ($R_0 = 5000\,(\mathrm{m})$ and $\theta_0 = 30\,(\mathrm{deg})$ with constant velocity). As Fig.1 and Fig.2 shows, measurement noise covariance elements generated for R and θ axis are both Gaussian random variable. Fig. 3 shows actual and noisy Trajectory (measurement trajectory) in polar coordinate with constant velocity. Fig. 3 is only apparent on finer scale (600 time index point from 10000).

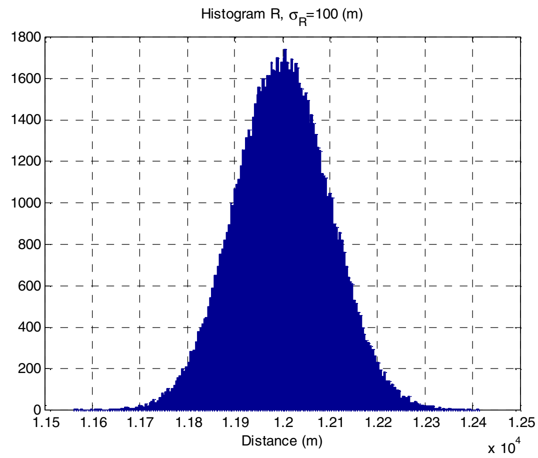

Fig. 1. Histogram of state R in polar coordinate with a Gaussian measurement noise ($\delta_R = 100\,m$)

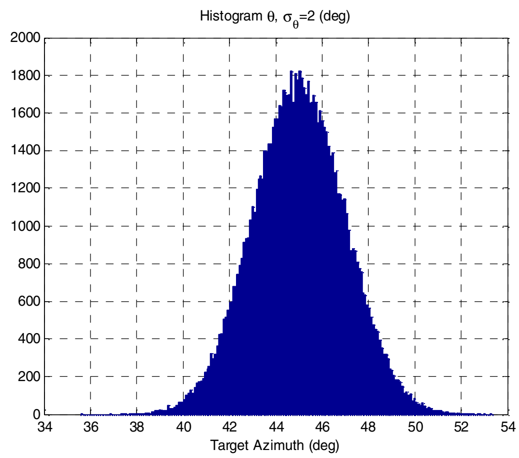

Fig. 2. Histogram of state θ in polar coordinate with a Gaussian measurement noise ($\delta_\theta = 2^o$)

As proof by Theorem A, and using Eq. (6), (7) for evaluate the X and Y directly from noisy R and θ (show in Fig.4, Fig.5), Gaussian distribution can see for both X and Y too.

Fig. 6 shows the simulation result when target has a constant velocity and maneuver detection is done by using linear Kalman filter (Eq. 2).

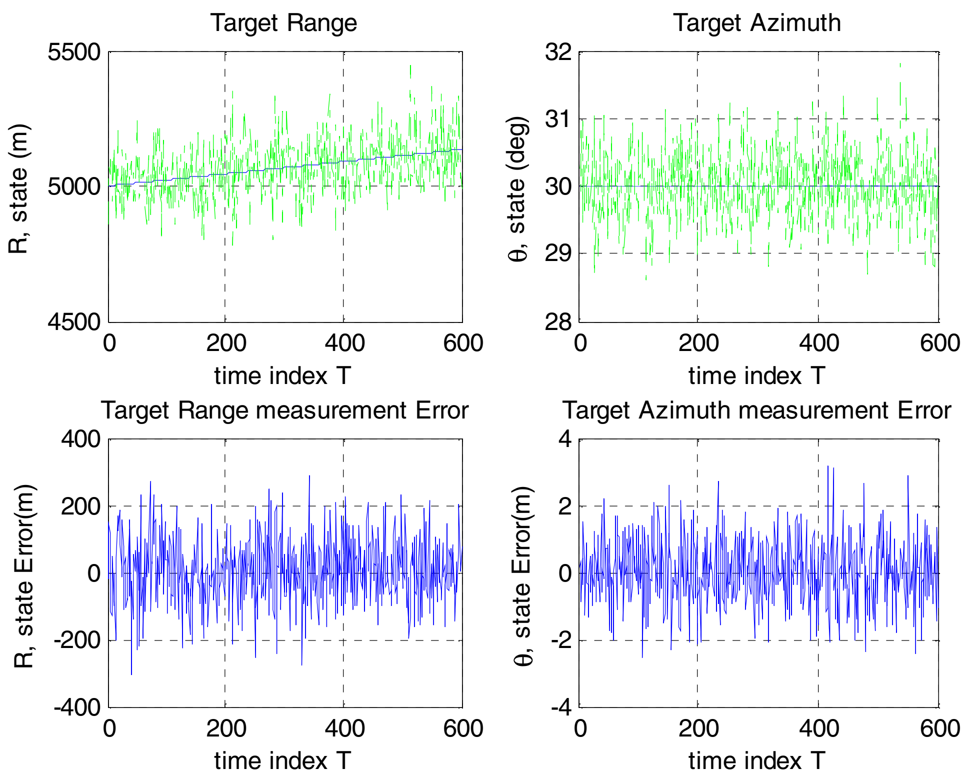

Figure 3: Actual and noisy Trajectory in polar coordinate with constant velocity and their Gaussian measurement errors

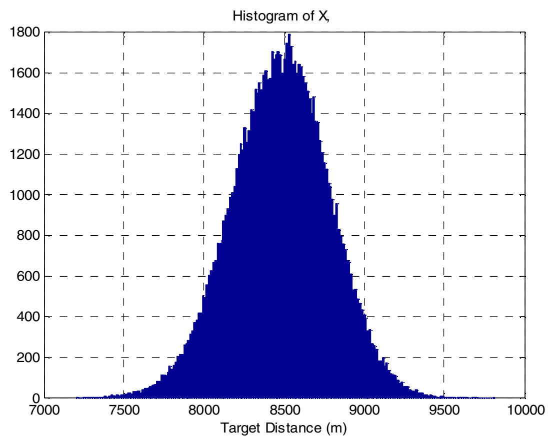

Fig.4. Histogram of X in Cartesian coordinate evaluated by Eq. (6)

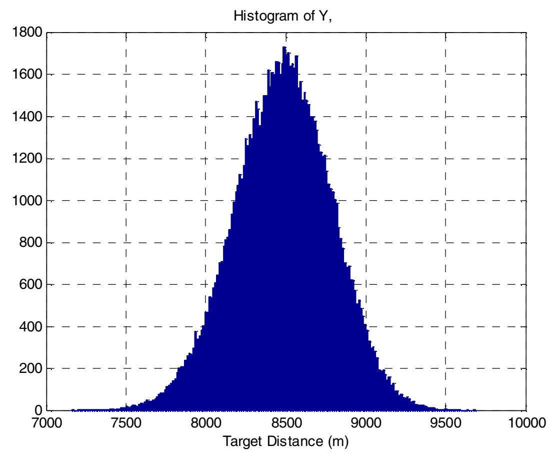

Fig.5. Histogram of Y in Cartesian coordinate evaluated by Eq. (7)

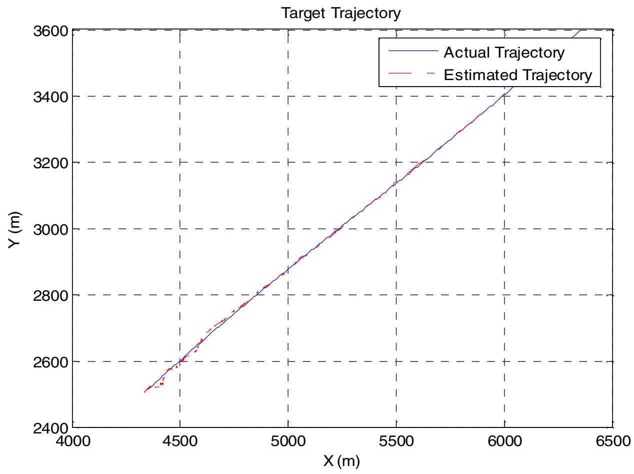

Fig. 6. Actual and Estimation Trajectory in Cartesian coordinate with constant velocity using linear Kalman filter (Eq. 2).

By using Eq. (4), (5) trajectory of target in polar coordinate are obtained (Fig .7).

Fig. 8, shows the actual and estimation of target parameter in R and θ direction in the present of target maneuvering.

VI. CONCLUSIONS

In this paper, a new transformation is presented to transform Polar coordinate to Cartesian coordinate tracking target problems. Linear Kalman filter in Cartesian coordinate can easily implemented by using this technique. Simulation results show a high performance of the proposed innovation technique and effectiveness of this scheme in tracking maneuvering targets.

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BIOGRAPHIES

Ali Karsaz received the B.S. degree in Electrical Engineering from the Amirkabir University of Technology, Tehran, Iran, in 1999. He Obtained his M.Sc. degree in Electrical Engineering in 2004 at Ferdowsi University of Mashhad. He is currently as a Ph.D. student in Control Engineering at the Ferdowsi University of Mashhad, Mashhad, Iran. His interest area is Artificial Neural Networks, Stochastic and nonlinear Modeling and Estimation, System Identification, Time Series Analysis and prediction, Inertial Navigation Systems.

Nasser Pariz with honors from the Ferdowsi University, Mashhad, Iran, in 1988, 1991, and 1999, respectively. He is the Director of Electrical Engineering Department, Ferdowsi University. His research interests are power and control systems. received the B.Sc., M.Sc., and Ph.D. degrees in electrical engineering

Hamid Khaloozadeh, received the B.S. degree in control engineering from Sharif University of technology (Tehran, Iran), in 1990, the M.S. degree in control engineering from K.N.Toosi university of technology Tehran, Iran), in 1993, and the Ph.D. degree in control engineering from Tarbiat Moddaress university (Tehran, Iran), in 1998. Since 1998 to 2004 he was a faculty member at Ferdowsi University of Mashhad. He is currently an assistant Professor teaches in the Department of Electrical Engineering in K.N.Toosi University of Technology (Tehran, Iran). His interest area is Digital Control, Nonlinear Modeling, System Identification, Optimal Control, Adaptive Control, Stochastic Estimation and Control, Neural Networks and Time Series Analysis.

Ali Khorami was born in Tehran in Iran, on July 9, 1975. He studied at the Ferdowsi University of Mashhad/Iran. He received B.S. degree in Electrical Engineering from this University. Obtained his M.Sc. in Electrical Engineering at Ferdowsi university of Mashhad.

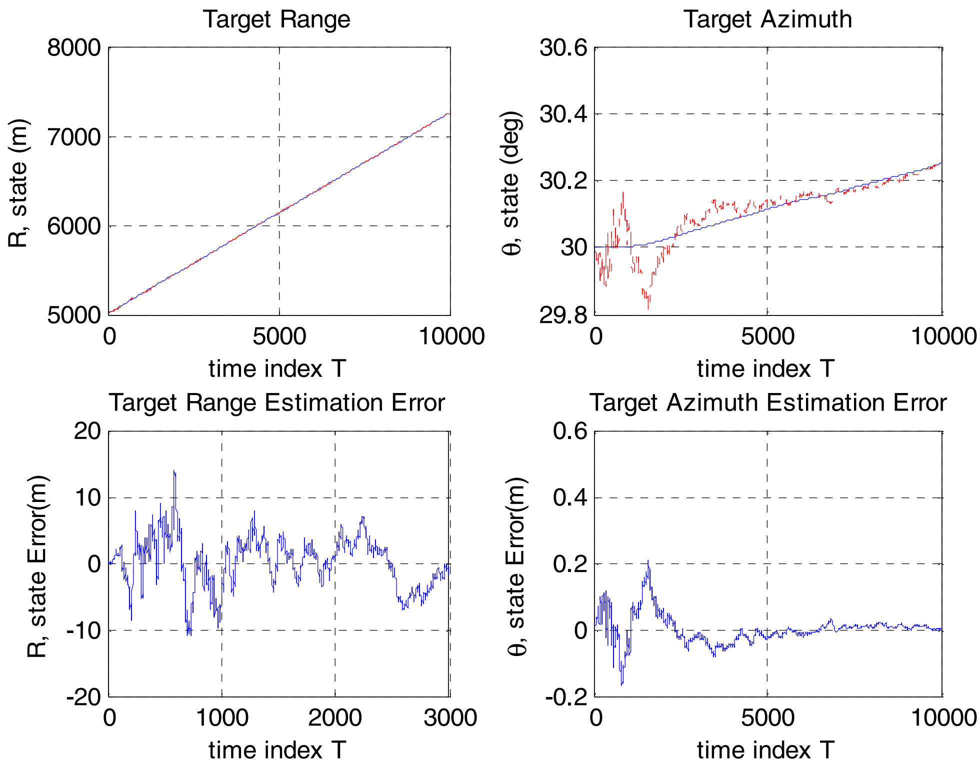

Figure 7: Actual and Estimation Trajectory in polar coordinate with constant velocity and their errors

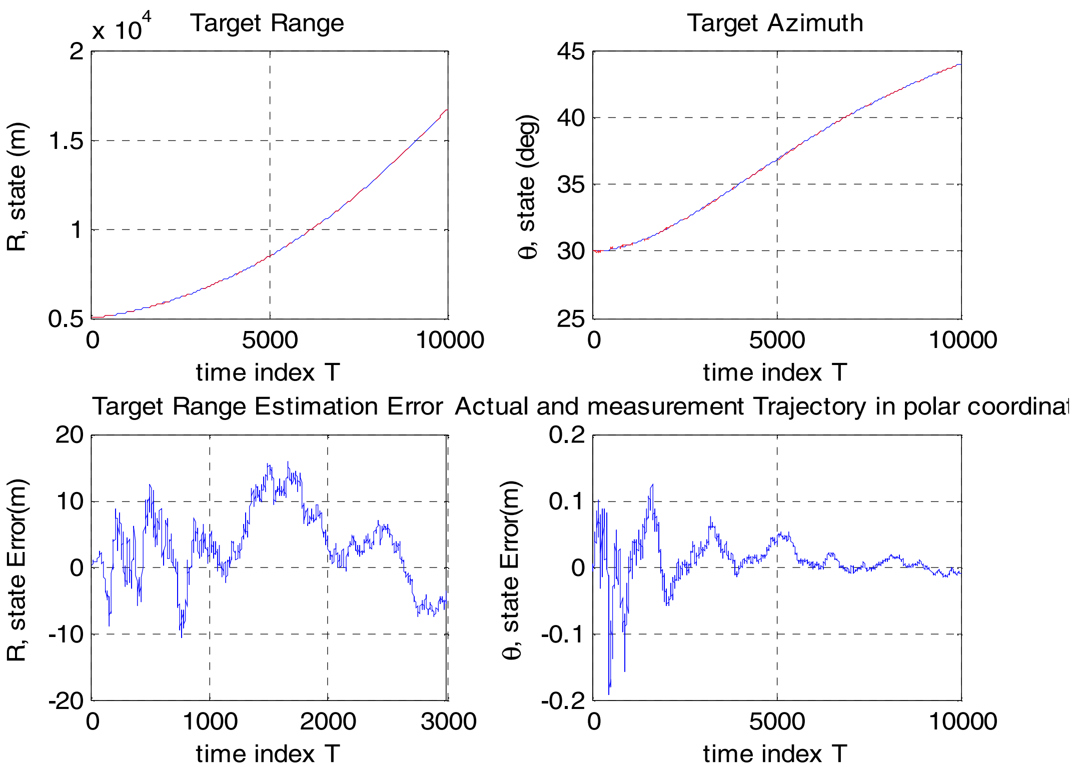

Figure 8: Actual and Estimation Trajectory in polar coordinate with maneuvering target and their errors $a(t) = \sqrt{a_x^2(t) + a_y^2(t)} = 0.1\, m/s^2$