Performance Comparison of the Two-Stage Kalman filtering Techniques for Target Tracking

A. Karsaz and H. Khaloozadeh and M. Darbandi[1]

Abstract-- **The two-stage filtering methods, such as the well-known augmented state Kalman estimator (AUSKE) and the optimal two-stage Kalman estimator (OTSKE), suffer from some major drawbacks. These drawbacks stem from assuming constant acceleration and assuming the input term is observable from the measurement equation. In addition, these methodologies are usually computationally expensive. The innovative optimal partitioned state Kalman estimator (OPSKE) developed to overcome these drawbacks of traditional methodologies. In this paper, we compare performance of the OPSKE with the OTSKE and the AUSKE in the maneuvering target tracking (MTT) problem. We provide some analytic results to demonstrate the computational advantages of the OPSKE.**

Keywords- Optimal two-stage Kalman estimators; input estimation; augmented state Kalman estimators; maneuvering target tracking.

1. Introduction

The AUSKE or "full state" method, solves the general state estimation problems by including the input parameters as a part of an augmented state to be estimated [1], [2]. However "reduced state" methods do not augment the state, and usually yield a better performance [3]. The AUSKE suffers from complexity of computational effort and numerical problems when state dimensions are large. The input detection and estimation (IDE) algorithm was first developed by Chan et al., in [4] using a simplified batch least square data. The IDE approach suffers from a major deficiency, being that the little prior knowledge is available for dynamics estimation [5]. For example we can cite Wang et. al., [6] used the IDE approach in the maneuvering target tracking problem. In [6], the predicted states for the maneuvering target are related to the corresponding states without maneuvering assuming constant input or constant acceleration (CA). Therefore, the performance of the estimation is reduced when target moves with non-constant acceleration. In [7] the unknown input defined as a sum of elementary time functions. Although this input modeling is more general than the constant-input model of the original IDE algorithm, the performance is reduced if there is any input dynamics.

Friedland [8] introduced a method of separating estimation of the unknown input from the dynamic variables and Blair used this method in the MTT problem [9]. The basic idea was to decouple the augmented Kalman filter (AKF) into two-stage filters in order to reduce computation and memory requirements [10]-[13]. Recently, Hsieh and Chen [10], [11] derived an optimal two-stage Kalman estimator (OTSKE) for a general case to reduce the computational complexity of AUSKE. The two-stage filtering method, suggested for MTT problem in [9] suffers from two major drawbacks. These drawbacks stem from assuming constant acceleration and assuming the input term is observable from the measurement equation (also in [10] and [13]).

In this paper, the performance of the OPSKE is analyzed and compared with the AUSKE and the OTSKE to show the advantage of the proposed algorithm. The OPSKE may serve as an alternative solution of the OTSKE proposed in [10]. It is shown that the maneuver tracking algorithm proposed in [6] and [9] are special case of the OPSKE.

2. Statement of the Problem

The problem of interest is described by the discretized equation set

$$X_{k+1} = A_k X_k + B_k U_k + W_k^x \tag{1}$$

$$U_{k+1} = C_k U_k + W_k^u \tag{2}$$

$$Z_k = H_k X_k + V_k \tag{3}$$

Where $X_k \in R^n$ is the system state, $U_k \in R^m$ and $Z_k \in R^p$ are the input and the measurement vectors, respectively. Matrices A_k, B_k, C_k and H_k are assumed to be known functions of the time interval k and are of appropriate dimensions. Matrix C_k is assumed nonsingular. The process noises W_k^x, W_k^u and the measurement noise V_k are zero-mean white Gaussian sequences with the following covariances: $E[W_k^x (W_l^x)'] = Q_k^x \delta_{kl}$, $E[W_k^x (W_l^u)'] = Q_k^{xu} \delta_{kl}$, $E[W_k^u (W_l^u)'] = Q_k^u \delta_{kl}$ $E[V_k V_l'] = R_k \delta_{kl}$, $E[W_k^x V_l'] = 0$ and $E[W_k^u V_l'] = 0$, where $'$ denotes transpose and δ_{kl} denotes the Kronecker delta function. The initial states X_0 and U_0 are assumed to be uncorrelated with the sequences W_k^x, W_k^u and V_k. The initial conditions are assumed to be Gaussian random variables with $E[X_0] = \hat{X}_0$, $E[X_0 X_0'] = P_0^x$, $E[U_0] = \hat{U}_0$, $E[U_0 U_0'] = P_0^u$, $E[X_0 U_0'] = P_0^{xu}$.

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Treating X_k and U_k as the augmented system state [10], the AUSKE is described by

$$X^{Aug}_{k+1|k+1} = X^{Aug}_{k+1|k} + K^{Aug}_{k+1}(Z_{k+1} - H^{Aug}_{k+1} X^{Aug}_{k+1|k}) \quad (4)$$

$$X^{Aug}_{k+1|k} = A^{Aug}_k X^{Aug}_{k|k} \quad (5)$$

$$K^{Aug}_{k+1} = P_{k+1|k}(H^{Aug}_{k+1})'[H^{Aug}_{k+1} P_{k+1|k}(H^{Aug}_{k+1})' + R_k]^{-1} \quad (6)$$

$$P_{k+1|k} = A^{Aug}_k P_{k|k}(A^{Aug}_k)' + Q_k \quad (7)$$

$$P_{k+1|k+1} = (I - K^{Aug}_{k+1} H^{Aug}_{k+1})P_{k+1|k} \quad (8)$$

Where

$$X^{Aug}_k = \begin{bmatrix} X_k \\ U_k \end{bmatrix}, \quad K^{Aug}_k = \begin{bmatrix} K^x_k \\ K^u_k \end{bmatrix}, \quad P_k = \begin{bmatrix} P^x_k & P^{xu}_k \\ (P^{xu}_k)' & P^u_k \end{bmatrix},$$

$$A^{Aug}_k = \begin{bmatrix} A_k & B_k \\ 0_{m\times n} & C_k \end{bmatrix}, H^{Aug}_k = \begin{bmatrix} H_k \\ 0_{p\times m} \end{bmatrix}', Q_k = \begin{bmatrix} Q^x_k & Q^{xu}_k \\ (Q^{xu}_k)' & Q^u_k \end{bmatrix}$$

Where the superscript 'Aug' denotes the augmented system state, I denotes the identity matrix of any dimension and $0_{m\times n}$ is a $m \times n$ zero matrix. It is clear from (4)-(8) that the computational cost of the AUSKE increases with the augmented state dimension [14]. The proposed approach in [14] intends to relax restrictive assumptions concerning the input dynamics modeling and using a new optimal partitioned Kalman estimator. The OPSKE formulation is based on the following equations (for details see [16]):

$$\hat{\hat{X}}_{k+1|k+1} = \hat{\hat{X}}_{k+1|k} + K_{k+1}(Z_{k+1} - H_{k+1}\hat{\hat{X}}_{k+1|k}) \quad (9)$$

$$\hat{\hat{X}}_{k+1|k} = A_k \hat{\hat{X}}_{k|k} \quad (10)$$

$$K_{k+1} = P^x_{k+1|k} H'_{k+1}[H_{k+1} P^x_{k+1|k}(H_{k+1})' + R_k]^{-1} \quad (11)$$

$$P^x_{k+1|k} = A_k P^x_{k|k}(A_k)' + Q^x_k \quad (12)$$

$$P^x_{k+1|k+1} = (I - K_{k+1}H_{k+1})P^x_{k+1|k} \quad (13)$$

$$N_{k+1} = [I - K_{k+1}H_{k+1}]M_{k+1} \quad (14)$$

$$\hat{U}_{k+1|k+1} = \hat{U}_{k+1|k} + K^u_{k+1}[\tilde{\tilde{Z}}_{k+1} - H_{k+1}M_{k+1}\hat{U}_{k+1|k}] \quad (15)$$

$$\hat{U}_{k+1/k} = C_k \hat{U}_{k/k} \quad (16)$$

$$K^u_{k+1} = 2P^u_{k+1|k}M'_{k+1}H'_{k+1} \times [3H_{k+1}M_{k+1}P^u_{k+1|k}M'_{k+1}H'_{k+1} + P^z_{k+1|k}]^{-1} \quad (17)$$

$$\begin{aligned} P^u_{k+1|k+1} &= P^u_{k+1|k} + 3K^u_{k+1}H_{k+1}M_{k+1}P^u_{k+1|k}M'_{k+1}H'_{k+1}(K^u_{k+1})' \\ &+ K^u_{k+1}P^z_{k+1|k}(K^u_{k+1})' - 2P^u_{k+1|k}M'_{k+1}H'_{k+1}(K^u_{k+1})' - 2K^u_{k+1}H_{k+1}M_{k+1}P^u_{k+1|k} \end{aligned} \quad (18)$$

$$P^u_{k+1|k} = C_k P^u_{k|k} C'_k + Q^u_k \quad (19)$$

$$P^z_{k+1|k} = H_{k+1}P^x_{k+1|k}H'_{k+1} + R_{k+1} \quad (20)$$

$$P^{zu}_{k+1|k} = H_{k+1}M_{k+1}P^u_{k+1|k} \quad (21)$$

$$\begin{aligned} \hat{X}_{k+1|k} &= \hat{\hat{X}}_{k+1|k} + M_{k+1}U_{k+1} \\ \hat{X}_{k+1|k+1} &= \hat{\hat{X}}_{k+1|k+1} + N_{k+1}U_{k+1} \end{aligned} \quad (22)$$

$$\begin{aligned} M_{k+1} &= [A_k M_k + B_k]C^{-1}_k, \qquad k = 2,3,.... \\ M_1 &= B_0 C^{-1}_0 \end{aligned} \quad (23)$$

$$N_{k+1} = [I - K_{k+1}H_{k+1}]M_{k+1} \quad (24)$$

3. PERFORMANCE EVALUATIONS

To demonstrate the computational advantage of the OPSKE over the AUSKE, the number of arithmetic operations are considered, i.e., multiplications and summations, as suggested in [10]. The arithmetic operations of a standard Kalman estimator with state dimension n and measurement dimension p, are listed in Table 1. It is clear from the equations (4)-(8) and Table 1, that the arithmetic operations required for the AUSKE which has state dimension $n+m$ and measurement dimension p, are $M(n+m, p)$ for multiplications and $S(n+m, p)$ for summations. Table 2 shows the arithmetic operations of the input estimation and the auxiliary matrices needed by the OPSKE which has state dimension n, measurement dimension p and input vector dimension m. Note that the number of the arithmetic operations of the AUSKE increases with the augmented state dimension, which makes the algorithm computationally inefficient. In contrast, the OPSKE based on the two-stage decoupling technique required fewer computations. The efficiency of the OPSKE is due to order reduction, i.e., implementing two less order n and m partitioned filters. This enables the proposed algorithm to have much better computational efficiency than the AUSKE. So, the arithmetic operations required (AOR) for the AUSKE are

$$\begin{aligned} &AOR(AUSKE) = M(n+m, p) + S(n+m, p) \\ &= [3(n+m)^3 + 2(n+m)^2 p + 2(n+m)p^2 + p^3 + (n+m)^2 + 2(n+m)p] \\ &+ [3(n+m)^3 + 2(n+m)^2 p + 2(n+m)p^2 + p^3 - (n+m)^2 - (n+m)] \end{aligned} \quad (25)$$

The arithmetic operations required for the input estimation and auxiliary matrices, by the OPSKE as shown in Table 2 and using equations (15)-(24) are

$$\begin{aligned} &AOR(OPSKE) \\ &= M(n, p) + S(n, p) + M^{OP}(n,m,p) + S^{OP}(n,m,p) \\ &= [3n^3 + 2n^2 p + 2np^2 + p^3 + n^2 + 2np] \\ &+ [3n^3 + 2n^2 p + 2np^2 + p^3 - n^2 - n] \\ &+ [3mp + 2m^2 + 2m^2 p + 2mp^2 + p^3 + p^2 \\ &+ 4m^3 + 2n^2 p + 2nm + n^2 m + nm^2 + nmp] \\ &+ [-mp - m^2 - m + 2m^2 p + 2mp^2 + p^3 + 4m^3 \\ &+ 2n^2 p - 2np + p^2 - n + 2n^2 m + nm^2 + nmp] \end{aligned} \quad (26)$$

Using (25) and (26), the operational savings, denoted by OS^{OPSKE}_{AUSKE}, of the OPSKE as compared to the AUSKE are

$$\begin{aligned} &OS^{OPSKE}_{AUSKE} = AOR(AUSKE) - AOR(OPSKE) = \\ &M(n+m, p) + S(n+m, p) - M(n, p) \\ &- S(n, p) - M^{OP}(n,m,p) - S^{OP}(n,m,p) \\ &= -2m^3 + 15n^2 m + 17nm^2 - 4n^2 p + 6nmp \\ &- 2p^3 + 2np + n - m^2 - 2p^2 - 2nm \end{aligned} \quad (27)$$

and the operational savings of the OTSKE over the AUSKE as reported in [10] are

$$\begin{aligned} &OS^{OTSKE}_{AUSKE} = AOR(AUSKE) - AOR(OTSKE) = -4m^3 + \\ &12n^2 m + 12nm^2 + 4nmp + m - 2m^2 - p^3 - 2nm \end{aligned} \quad (28)$$

Therefore, using (27) and (28) the operational savings of the OPSKE over the OTSKE are

$$\begin{aligned} &OS^{OPSKE}_{OTSKE} = AOR(OTSKE) - AOR(OPSKE) = 2m^3 + 3n^2 m \\ &+ 5nm^2 - 4n^2 p + 2nmp - p^3 + 2np + n - m + m^2 - 2p^2 \end{aligned} \quad (29)$$

It is clear from (27) and (29) that for m and $p \le n$, the proposed scheme has computational advantage over the

AUSKE and it is comparable to the OTSKE. The operational savings discussed here will be tested as an example in the simulation results section. To measure the relative operational savings of the OPSKE with respect to the arithmetic operation required by the AUSKE ($AOR(AUSKE)$), the percentage of the operational savings defined as below:

$$POS_{AUSKE}^{OPSKE} = \frac{OS_{AUSKE}^{OPSKE}}{AOR(AUSKE)} \times 100 \quad (30)$$

Using (27), (29) and (30), the operational savings and the percentage of the operational savings, of the OPSKE comparing to the OTSKE and the AUSKE for different values of n , m and p are shown in Table 3. It can be inferred from Table 3 that the OPSKE has better overall performance than the AUSKE (averaged 32%) and the OTSKE (averaged 7.3%).

TABLE 1: ***STANDARD KALMAN ESTIMATOR ARITHMETIC OPERATION REQUIREMENTS***

	Variable	*Number of Multiplications,* $M(n,p)$	*Number of summations,* $S(n,p)$
1	$X_{k+1\|k+1}$	$2np$	$2np$
2	$X_{K+1\|k}$	n^2	$n^2 - n$
3	K_{k+1}^{x}	$n^2 p + 2np^2 + p^3$	$n^2 p + 2np^2 + p^3 - 2np$
4	$P_{K+1\|k}^{x}$	$2n^3$	$2n^3 - n^2$
5	$P_{K+1\|k+1}^{x}$	$n^3 + n^2 p$	$n^3 + n^2 p - n^2$
	Totals	$3n^3 + 2n^2 p + 2np^2 + p^3 + n^2 + 2np$	$3n^3 + 2n^2 p + 2np^2 + p^3 - n^2 - n$

TABLE 2: ***INPUT ESTIMATION AND AUXILIARY MATRICES ARITHMETIC OPERATION REQUIREMENTS FOR THE OPSKE***

	Variable	*Number of Multiplications* $M^{OP}(n,m,p)$	*Number of summations* $S^{OP}(n,m,p)$
1	$U_{k+1\|k+1}$	$2mp$	$2mp$
2	$U_{K+1\|k}$	m^2	$m^2 - m$
3	K_{k+1}^{u}	$m^2 p + 2mp^2 + p^3 + p^2 + mp$	$m^2 p + 2mp^2 + p^3 - 2mp$
4	$P_{K+1\|k}^{u}$	$2m^3$	$2m^3 - m^2$
5	$P_{K+1\|k+1}^{u}$	$m^3 + m^2 p + m^2$	$m^3 + m^2 p - m^2$
6	$P_{k+1\|k}^{z}$	$2n^2 p$	$2n^2 p - 2np + p^2$
7	$\hat{X}_{k+1\|k}$	mn	mn
8	$\hat{X}_{k+1\|k+1}$	mn	$mn - n$
9	M_{k+1}	$n^2 m + m^3 + nm^2$	$n^2 m + m^3 + nm^2 - nm$
10	N_{k+1}	$n^2 m$	$n^2 m - nm$
11	$H_{k+1} M_{k+1}$	nmp	$nmp - mp$
	Totals	$3mp + 2m^2 + 2m^2 p + 2mp^2 + p^3 + p^2 + 4m^3 + 2n^2 p + 2nm + n^2 m + nm^2 + nmp$	$-mp - m^2 - m + 2m^2 p + 2mp^2 + p^3 + 4m^3 + 2n^2 p - 2np + p^2 - n + 2n^2 m + nm^2 + nmp$

TABLE 3: *THE OPERATIONAL SAVINGS AND THE PERCENTAGE OF THE OPERATIONAL SAVINGS OF THE OPSKE COMPARED TO THE AUSKE AND THE OTSKE*

The state vector dimensions	OS_{AUSKE}^{OPSKE}	POS_{AUSKE}^{OPSKE} *(%)*	OS_{OTSKE}^{OPSKE}	POS_{OTSKE}^{OPSKE} *(%)*
$n = 4, m = 4, p = 2$	*1340*	*35.7*	*592*	*15.7*
$n = 4, m = 2, p = 2$	*578*	*33.7*	*102*	*5.9*
$n = 4, m = 2, p = 1$	*553*	*37.5*	*155*	*10.5*
$n = 4, m = 1, p = 1$	*242*	*27.5*	*23*	*2.6*
$n = 4, m = 3, p = 3$	*978*	*32.7*	*247*	*8.2*
$n = 10, m = 2, p = 2$	*2954*	*25.1*	*132*	*1.12*
Average	*≅ 1107*	*32.0*	*≅ 208*	*7.3*

4. SIMULATION RESULTS

To evaluate the proposed algorithm, an example of maneuvering target tracking problem which turns, in two-dimensional space is simulated such as a ship or an aircraft with constant elevation. In this simulation example, the performance of the OPSKE for the maneuvering target tracking has been compared with the work suggested in [2] as an example of the AUSKE method. As mentioned before in the augmented state method the state vector includes the input vector i.e., acceleration and jerk parameter in maneuvering target tracking problem. The sampling interval is T=0.01 (sec) and target maneuver is applied at 9th second (900th sample). The initial conditions are selected similar for the AUSKE as well as the OPSKE. The state vectors are

$X_k = [x_k \quad v_k^x \quad y_k \quad v_k^y]'$, $U_k = [u_k^x \quad j_k^x \quad u_k^y \quad j_k^y]'$,

$X_k^{Aug} = [x_k \quad v_k^x \quad y_k \quad v_k^y \quad u_k^x \quad j_k^x \quad u_k^y \quad j_k^y]'$

where x_k, v_k^x, u_k^x and j_k^x denote the position, velocity, acceleration and jerk of the target along the x axis, respectively. We consider the target initial conditions for the state and the acceleration vectors as below:

$X_0 = [2165\,m \quad -80\,m/s \quad 1250\,m \quad 25\,m/s]'$,

$U_0 = [0\,g \quad 0\,g/sec \quad 0\,g \quad 0\,g/sec]'$

$X_0^{Aug} = [2165\,m \quad -80\,m/s \quad 1250\,m \quad 25\,m/s \quad 0\,g \quad 0\,g/\sec \quad 0\,g \quad 0\,g/\sec]'$

The target begins to maneuver as $U_{900} = [0g \quad -0.7\,g/\sec \quad 0g \quad 0.4\,g/\sec]'$ for $9\ (\sec) \le t \le 90\,(\sec)$. The system matrices are given by

$$A_k = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad B_k = \begin{bmatrix} T^2/2 & T^3/6 & 0 & 0 \\ T & T^2/2 & 0 & 0 \\ 0 & 0 & T^2/2 & T^3/6 \\ 0 & 0 & T & T^2/2 \end{bmatrix},$$

$$C_k = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad H_k = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}'$$

$$Q_k^u = 2\alpha\sigma_j \begin{bmatrix} T^3/3 & T^2/2 & 0 & 0 \\ T^2/2 & T & 0 & 0 \\ 0 & 0 & T^3/3 & T^2/2 \\ 0 & 0 & T^2/2 & T \end{bmatrix},$$

$$Q_k^x = 2\alpha\sigma_j \begin{bmatrix} T^7/252 & T^6/72 & 0 & 0 \\ T^6/72 & T^5/20 & 0 & 0 \\ 0 & 0 & T^7/252 & T^6/72 \\ 0 & 0 & T^6/72 & T^5/20 \end{bmatrix}$$

$$Q_k^{xu} = 2\alpha\sigma_j \begin{bmatrix} T^5/30 & T^4/24 & 0 & 0 \\ T^4/8 & T^3/6 & 0 & 0 \\ 0 & 0 & T^5/30 & T^4/24 \\ 0 & 0 & T^4/8 & T^3/6 \end{bmatrix}, \quad P_0^x = 10 I_{4\times4},$$

$$P_o^u = 0.1 I_{4\times4}, \quad P_0^{xu} = I_{4\times4}, \quad H_k^{Aug} = \begin{bmatrix} H_k \\ 0_{2\times4} \end{bmatrix}'$$

$$A_k^{Aug} = \begin{bmatrix} A_k & B_k \\ 0_{4\times4} & C_k \end{bmatrix}, Q_k = \begin{bmatrix} Q_k^x & Q_k^{xu} \\ (Q_k^{xu})' & Q_k^u \end{bmatrix}, P_k = \begin{bmatrix} P_k^x & P_k^{xu} \\ (P_k^{xu})' & P_k^u \end{bmatrix}.$$

where $\sigma_j = 0.09 (ms^{-3})$ is the variance of the target jerk and $\alpha = 0.0123\,(s^{-1})$ is the reciprocal of the jerk time constant $\tau = 1/\alpha$. The measurement standard deviations of x and y target positions are: $\sigma_x = 10\sqrt{10}\,(m)$, $\sigma_y = 20\,(m)$. Thus, the measurement covariance matrix is $R_k = \begin{bmatrix} 1000 & 0 \\ 0 & 400 \end{bmatrix}$ for both methods. The Root Mean Square Error (RMSE) index is used for the results evaluation.

Fig. 1 shows the actual value and the estimation of x and y and RMS errors of x and y positions estimations by the proposed OPSKE and the AUSKE.

Fig. 2 shows the actual value and the estimations of v^x, v^y and the RMS errors of the x and y velocities estimations by the proposed method compared with the augmented method. The actual value and the accelerations estimations in the x and y directions and their corresponding averaged RMS errors can be seen in Fig. 3.Fig. 4 displays the actual value and the estimated jerk parameters are evaluated by the OPSKE and the AUSKE methodologies.

It is clear that the performance of the proposed OPSKE is as well as the results obtained by the AUSKE in the maneuvering target tracking problem. Note that in this example $n = 4$, $m = 4$ and $p = 2$, and the operation savings for the OPSKE over the AUSKE and the OTSKE as shown in Table 3 are 1340 (or 35.7%) and 592 (or 15.7%), respectively.

5. CONCLUSIONS

The OPSKE proposed in [16] was based on a new partitioned dynamic modeling and intends to overcome the computational expensiveness drawbacks of the other works which are based on the augmented methods. The proposed OPSKE provides the optimal state estimate, which is equivalent to that of the AUSKE. Comparison with two estimators shows that the proposed scheme has computational advantage over the AUSKE and the OTSKE.

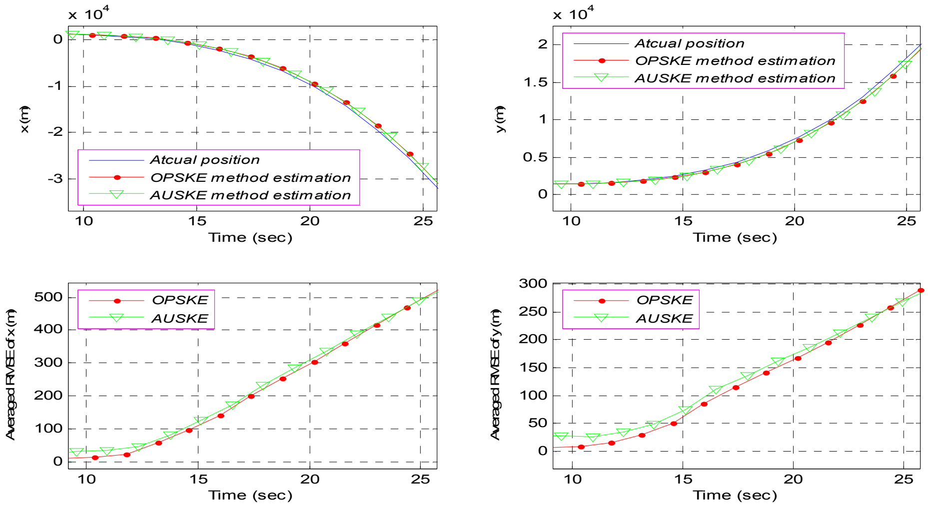

Fig. 1. The actual value and the estimation of the x, y positions and RMS errors estimations by the OPSKE and the AUSKE methods.

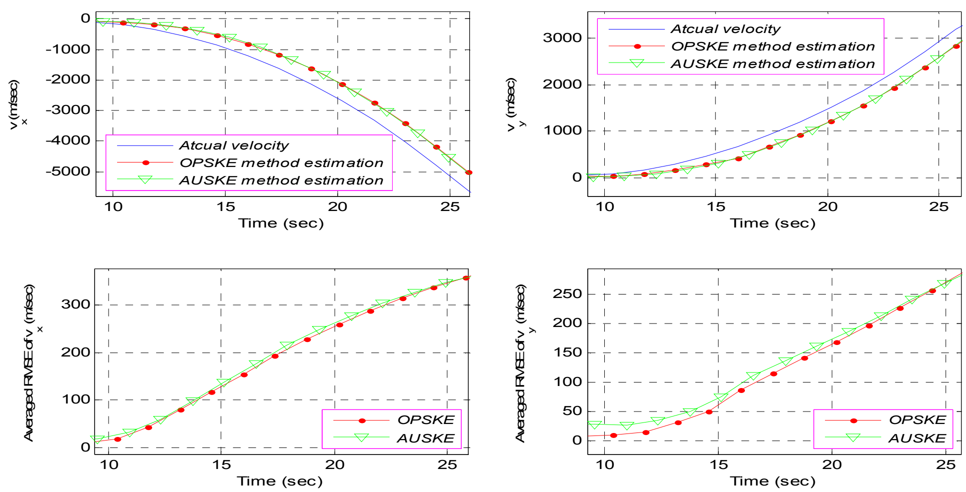

Fig. 2. The actual value and the estimation of v^x, v^y *and RMS errors of x and y velocities estimations by the OPSKE and the AUSKE methods.*

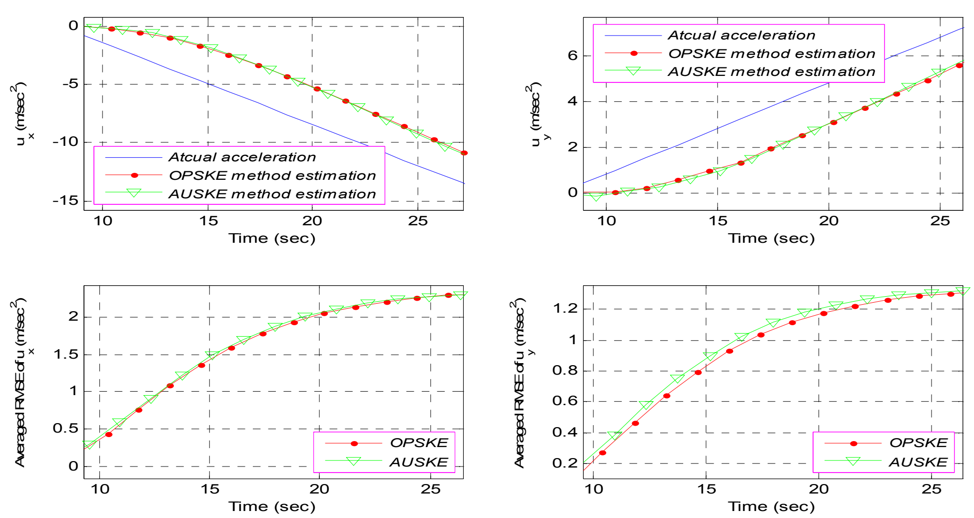

Fig. 3. The actual value and the estimation of acceleration in x and y directions and corresponding RMS errors by the proposed method compared with the augmented methods.

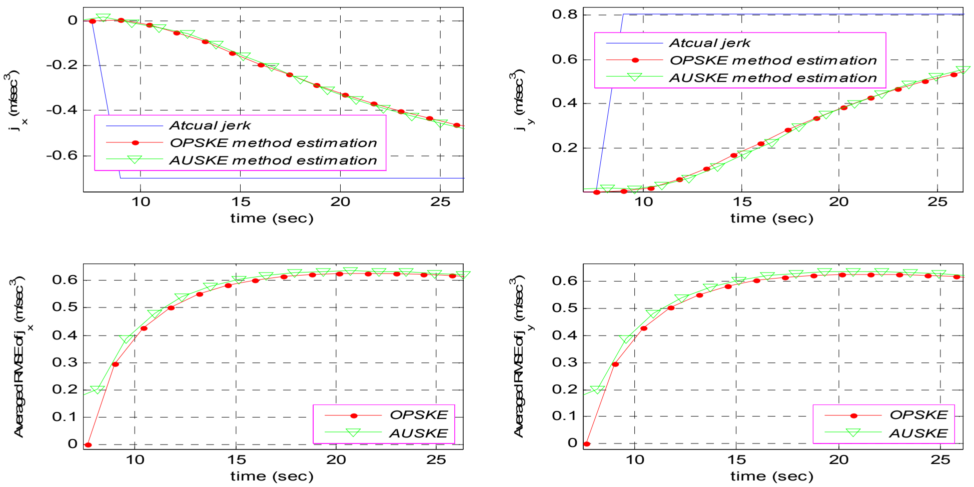

Fig. 4. The actual value and the estimation of jerk parameters and RMS errors by the OPSKE method compared with the AUSKE method.

REFERENCES

[1] H. Khaloozadeh, and A. Karsaz, 'A new state augmentation for maneuvering targets detection'. *Proc. 3th IEEE Int. Conf. on Signal Processing & Communication (SPCOM), Dec. 2005, pp. 65-69*

[2] K. Mehrotra and P. R. Mahapatra, 'A jerk model for tracking highly maneuvering targets', *IEEE Trans. Aerosp. Electron. Syst., 1997, 33, (4), pp. 1094-1105*

[3] P. Mookerjee and F. Reifler, 'Reduced state estimators for consistent tracking of maneuvering targets', *IEEE Trans. Aerosp. Electron. Syst., 1999, 41, (2), pp. 608-619*

[4] Y. T. Chan, A. G. C. Hu and J. B. Plant, 'A Kalman filter based tracking scheme with input estimation', *IEEE Trans. Aerosp. Electron. Syst., 1979, (15), pp. 237–244*

[5] X. R. Li and V. P. Jilkov, 'A survey of maneuvering target tracking–Part IV: Decision-based methods'. *Proc. Conf. on Signal and Data Processing of Small Targets, vol. 4728, Orlando, FL, Apr. 2002, pp. 511-534*

[6] T. C. Wang and P. K. Varshney, 'A tracking algorithm for maneuvering targets', *IEEE Trans. Aerosp. Electron. Syst., 1993, 29, (3), pp. 910-924*

[7] H. Lee and M. J. Tahk, 'Generalized input-estimation technique for tracking maneuvering targets', *IEEE Trans. Aerosp. Electron. Syst., 1999, 35, (4), pp. 1388-1402*

[8] B. Friedland, 'Treatment of bias in recursive filtering', *IEEE Trans. Automat. Contr., 1969, 14, pp. 359–367*

[9] W. D. Blair, 'Fixed-gain two-stage estimator for tracking maneuvering targets', *IEEE Trans. Aerosp. Electron. Syst., 1993, 29, (3), pp. 1004-1014*

[10] C. S. Hsieh and F. C. Chen, 'Optimal solution of the two-stage Kalman estimator', *IEEE Trans. Automat. Contr., 1999, 44, (1), pp.195-199*

[11] C. S. Hsieh and F. C. Chen, 'General two-stage Kalman filters', *IEEE Trans. Automat. Contr., 2000, 45, (4) pp. 819–824*

[12] T. Kawase, H. Tsurunosono, N. Ehara and I. Sasase, 'Two-stage Kalman estimator using an advanced circular prediction for tracking highly maneuvering targets', *Proc. Int. Conf. on Acoustics, Speech and signal processing, 1998, 4, pp. 2453-2456*

[13] H. Z. Qiu, H. Y. Zhang and X. F. Sun, 'Solution of two-stage Kalman filter', *IEE Proc. Control Theory Appl., 2005, 152, (2), pp.152-156*

[14] H. Khaloozadeh, and A. Karsaz, 'Modified input estimation technique for tracking manoeuvring targets', *IET Radar, sonar and Navig., 2009, 3, (1), pp.30-41*

[15] F. C. Schweppe, *'Uncertain Dynamic Systems'* (Prentice-Hall Press, 1973)

[16] A. Karsaz and H. Khaloozadeh, 'An optimal two-stage algorithm for highly maneuvering targets tracking', *Elsevier Signal Processing, 2009, 89, (4), pp. 532–547*

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