Error Analysis of Airborne Fire Control System Radar via Monte Carlo Approach

A. Karsaz, H. Khaloozadeh

Abstract**--The designer of the airborne defense missile system is interested in defining the Fire Control System (FCS) error, and the seeker detection probability to make the defense system more powerful. This paper develops one radar error statistical mathematics model for the airborne defense and determined the distribution of errors for each error, which can affect the Probability of Hit (PH). Furthermore, the errors in Radar as the important subsystem of fire control system are determined, simulated and analysis. Then a new statistical mathematics approach based on Monte Carlo Method suggestion for error analysis of radar.**

Index Terms**--Error analysis, Fire control system, Monte Carlo simulation, Radar sensitive analysis, Hit Probability.**

I. Introduction

There exist two approaches for the error analysis, Analytical methods and stochastic simulations. The First method required a mathematical model for analysis. However, the stochastic simulations are a strong method for the complex systems analysis, based on the random input variables. In many applications stochastic simulations approach identify as Monte Carlo method. In Monte Carlo techniques an actual realization of process is simulated on computer and, after having observed the simulated process for some time, estimate the stochastic parameters [1], [2].

There are many papers for computing Circular Error Probability *(CEP)* for projectiles. [7], [8]. For example, in Ref.21 describes a concept for deriving information for gun system calibration from projectile track data. A common technique for improving the accuracy of gun fire against surface targets which has been in use since the earliest development of gun technology is the spotting procedure. In a typical application of this procedure, a number of shots are fired at a specific target. The centroid of the fall of shot is computed and incremental to the gun pointing angle are made to move the centroid on to the target. However, the concept of (CEP) and this types technique are for projectiles. In this paper there is a missile that beyond to the large categories of missiles that called Fire&Forget missiles. The design of the shipboard defense missile system is highly considered in the defense industry electronics, especially the missile fire control radar tracking accuracy design, and one system statistical error analysis and design approach is proposed [4].

Using minimum variance estimation techniques developed a model based, post-event missile trajectory and error analysis program for the National Air Intelligence Center. This paper bases its trajectory simulation on four different physical models-three for calculation the equation of motion and one for computing sensor data. Its propagation of uncertainties from launch to impact and the error ellipses plotted an dare expected to be, powerful aids for assisting military planners and sensors in and/or over a battlefield and what types of data to collect-are required for, meeting their specification for launch site and impact point accuracies [5].

The method of target-position location in a bistatic system is described, and the analysis about the position error is made. The Root Mean Square (RMS) is used to describe the position accuracy, and the constant-accuracy contours are drawn to show the location characteristics of the bistatic system [15]. All errors can belong to two large categories, random and systematic errors called kind of error and each of them can belong to three categories dynamic, disturbance and equipment errors called type error [6].

A typical fire control system can be divided to the three important subsystems in the airborne defense, as mentioned before Radar subsystem, environmental and carrier subsystem. In this paper, after airborne fire control system (AFCS) description in section 2, all errors in radar subsystem are determined in section 3. Error analysis technique which used in this paper are described in section 4, and finally the simulation results shows the radar elements effect on reduce of probability of hit in section 5.

II. Airborne Fire Control System (AFCS) Description

A modern AFCS are composed of number of systems, each tasked with specific functionality. Example includes aerial radar, for target parameters tracking such as speed, range, azimuth, bearing of target. Wind meter for measuring speed and direction of wind, temperature and pressure sensors for determine the environmental temperature and pressure. And helicopter subsystem sensors for measuring heading, roll, pitch by (AHRS) height by Altimeter sensor and velocity by Doppler sensor of helicopter. Central computer that receives each parameters of above subsystems and in the equations called Missile Fire Control Director (MFCD) evaluates two parameters for setting in the missiles, see Fig.1. In MFCD as

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show in Fig.2 after some recursive equations computation, and by using eleven inputs parameter, two-output parameter obtained. First parameter is ψ_m, that determined who much missile must edit its azimuth after lunch for vision the target in front of itself at C point. Second parameter is T_m that determine when the seeker of missile must be turn on in C point. After this step, the missile dynamic equations solved and the position of missile impact respect to the target determined.

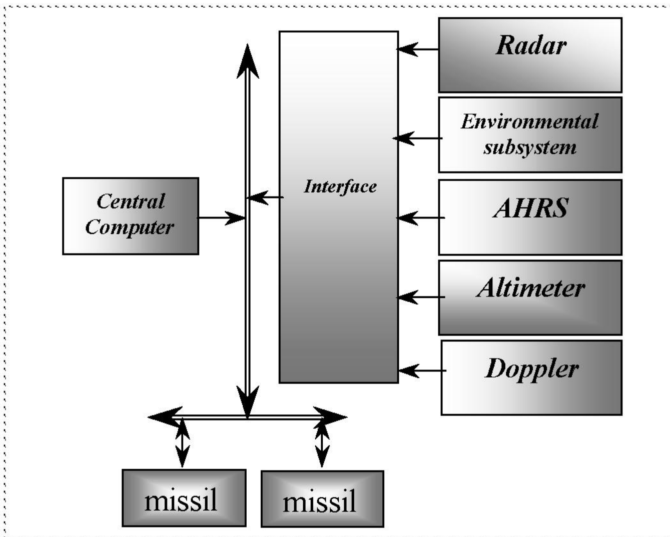

Fig. .1. Airborne Fire Control System

III. Errors Determination

All errors of radar can categories as below:

1. Target azimuth measurement
2. Target range measurement
3. Target speed measurement
4. Target bearing measurement
5. Radar antenna zero setting
6. Error in helicopter vibration
7. Error in installation of radar antenna
8. Error irises from delay of transition radar information
9. Radar encoder error
10. Dynamic DC motor error
11. Resolution error
12. Radar antenna production error
13. Enternal circuit error
14. Wind fluctuation on antenna error
15. Aliment of the installation area
16. Shock error

IV. Error Analysis techenique

A typical airborne fire control system consist of deferent sensors such as radar for target movement and position detection, wind meter for measurement of wind speed and directional attitude heading reference system (AHRS) for detection of fly angles and speeds, altimeter for height measuring, Doppler as a measurement for helicopter speed, that divided to three subsystem, as mentioned before. The radar subsystem, consist of radar sensor is an important subsystem on FCS. In section 3, all of errors in this subsystem determined. In addition, in the section 4, the fundamental basis for error analysis is illustrated. Now the tools for error analysis are available. As mentioned in the introduction for Monte Carlo simulation the input variables are required. All errors in section3, for radar can analysis in fourth important element that are the radar errors. See Fig.2)

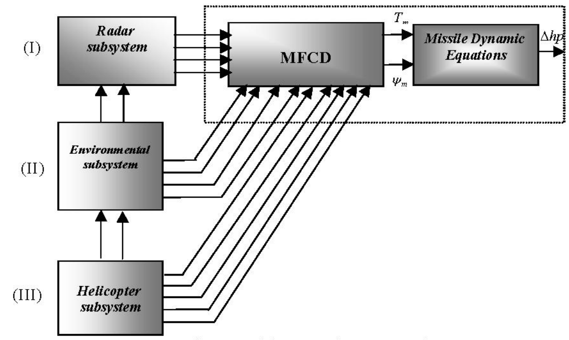

Fig. 2. Airborne Fire Control System

Therefore, these MFCD's input elements (specially four elements of radar subsystem) can effect on the two elements that are MFCD's outputs and these two elements can effect on PH directly. In this paper radar subsystem (that is the most important subsystem) analysis are proposed, one can determines, and analysis other errors subsystems. For this reason Error analysis consist of two important steps, first step analysis effects of radar errors on MFCD's outputs, and analysis of effects of MFCD's outputs on the PH as second step. Than these two steps combined together. It is clear that each of sixth errors of radar subsystem effect to each of four elements as MFCD's inputs. For ranking and generation of input variables to MFCD, used of mse criteria as bellow;

$$mse = E\left\{(X_i - X)^2\right\} \quad (1)$$

Where:

X_i : input variable at ith itreation

X : true value when error is zero

It is clear that for many of X_i samples;

$$E(X_i - \hat{\mu}) = 0$$

Therefore:

$$E\left\{(X_i - \hat{\mu} + \hat{\mu} - X)^2\right\} = E\left\{(X_i - \hat{\mu})^2\right\} + E\left\{(X - \hat{\mu})^2\right\} + 2E\left\{(X_i - \hat{\mu})(\hat{\mu} - X)\right\}$$

$$mse = E\left\{(X_i - X)^2\right\} = E\left\{(X_i - \hat{\mu})^2\right\} + E\left\{(X - \hat{\mu})^2\right\} = Var(X_i) + bias^2(X_i) \quad (2)$$

Therefore, for each of input variables mse equals to summation of variance and bias of that input, because both variance and bias are important on the deviation of correct value.

Definition:

$$IF_{ij} = \frac{mse_j}{\delta_i} \qquad (3)$$

IF_{ij} : Important Factor of element i th radar output that effect on j th MFCD output.
i: each of radar outputs
j: each of MFCD's output

This definition determined the mse of jth output of MFCD from i th random input variable with standard deviation from correct value, δ_i when other elements of radar are zeros. For this, execute the error analysis program by deferent values of standard deviations ($\delta_R, \delta_a, \delta_s, \delta_b$) of radar elements and obtain mean square errors of each MFCD's output (mse_{T_m}, mse_{ψ_m}).This is clear that if there is;

$IF_{xj} > IF_{yj} \Rightarrow xismoreimpor tan tthaty$

Because, with equal mse on the j th MFCD's arise by x,y, x hase smaller δ_x than y.

$$\theta : R^4 \to R^2$$

$$\begin{bmatrix} mse_{T_m} \\ mse_{\psi_m} \end{bmatrix} = \theta_{2*4}[\delta_R\ \delta_A\ \delta_S\ \delta_B]^T \qquad (4)$$

By using the least square error criteria can obtain the efficient coefficient for *IF*s.

$$\theta^* = (\varphi^T . \varphi)^{-1} . \varphi . y$$

θ^* *: Efficentparameters*

φ *: Inputmatrixeachrowsare* $[\delta_R\ \delta_A\ \delta_S\ \delta_B]$

y *: crospondingoutputstoinputvectors*

For second step on the Missile Dynamic Equations (MDE) box :

$$IF_{hj} = \frac{\Delta PH}{mse_j} \qquad (5)$$

$$\Phi : R^2 \to R^1$$

$$\Delta PH = \Phi_{1\times2} \begin{bmatrix} mse_{T_m} \\ mse_{\psi_m} \end{bmatrix} \qquad (6)$$

ΔPH : Reduced Probability of Hit that arises by j th MFCD's output

$\Phi_{1*2} : \begin{bmatrix} IF_{hT_m} & IF_{h\psi_q} \end{bmatrix}$

The ΔPH for some iterations can calculated with using circular error probability (CEP), which used in many papers for projectiles as mentioned insection2. The radius of circle is dependent on target dimensions. ΔPH directly calculated by dividing the outputs contact numbers of this circle to all iterations numbers. (see Fig. 6 for details)

$$\Delta PH = \frac{N_i}{N} \qquad (7)$$

N_i *: Insightpoint s*

N *: Iterationsnumber*

By combination of these two subsystems (16), (18) the formulation of probability of hit obtained as below and the percent of importance of each system can obtain by equation (21);

$$\Delta PH = \Phi_{1\times2} \begin{bmatrix} mse_{T_m} \\ mse_{\psi_m} \end{bmatrix} = \Phi_{1\times2} * \theta_{2*4}[\delta_R\ \delta_A\ \delta_S\ \delta_B]^T$$
$$= \begin{bmatrix} IF_{hT_m} & IF_{h\psi_q} \end{bmatrix} \begin{bmatrix} IF_{RT} & IF_{AT} & IF_{ST} & IF_{BT} \\ IF_{R\psi} & IF_{R\psi} & IF_{R\psi} & IF_{R\psi} \end{bmatrix} [\delta_R\ \delta_A\ \delta_S\ \delta_B]^T \qquad (8)$$

$$HP = 100 - \Delta HP \qquad (9)$$

Definition;

$$PIF_i = \frac{IF_{hT} * IF_{iT} + IF_{h\psi} * IF_{i\psi}}{\sum_{i=1}^{4} PIF_i} * 100 \qquad (10)$$

This definition determined the percentage of importunacy for each of elements that are effect on the PH.

V. Simulation Results

There are several advantages of error analysis, as first results, is determined the percentage of importunacy for each of four radar elements that effects on the PH. For using the (10), first by 200 iterations, mse_{T_m} and mse_{ψ_m} are determined for one random input variable of radar. For example, the target azimuth with specific random error (example 0.5 deg (std)) is used. Fig. 3 shows the T_m values irises for this standard deviation error of target azimuth.

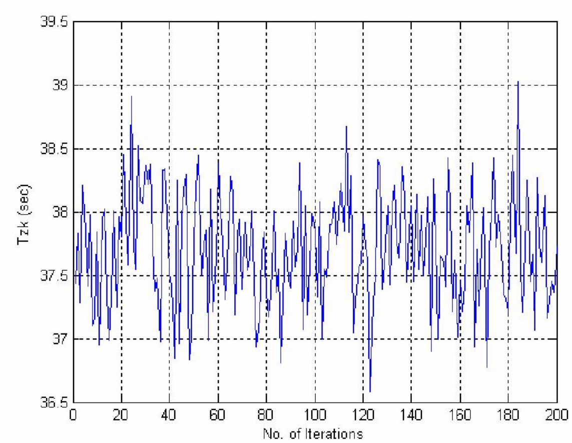

Fig. 3. T_m values for 200 iterations

The histogram of T_m perturbations is illustrated in Fig. 4, and the mse_{T_m} can easily calculated using (2). Same calculation is for ψ_m.

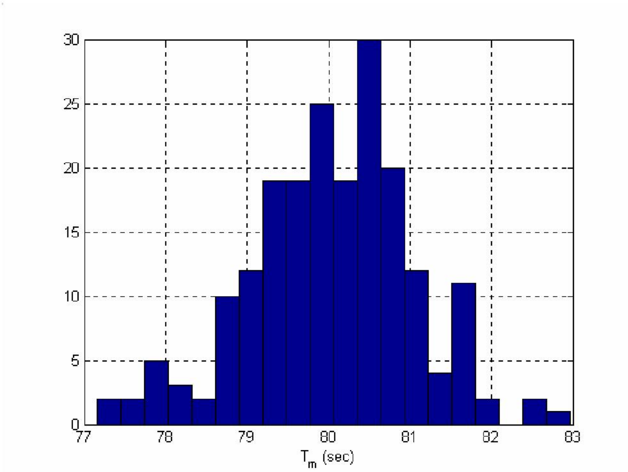

Fig. 4. Histogram of T_m

Fig. 5, 6 shows the ψ_m values irises for this standard deviation error of target azimuth and its histogram.

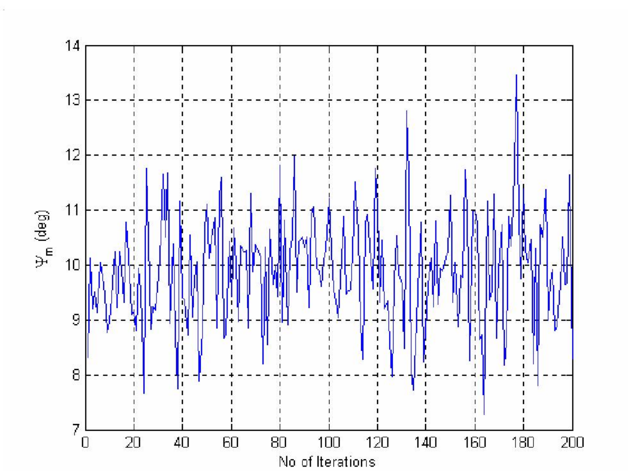

Fig. 5. ψ_m values for 200 iterations

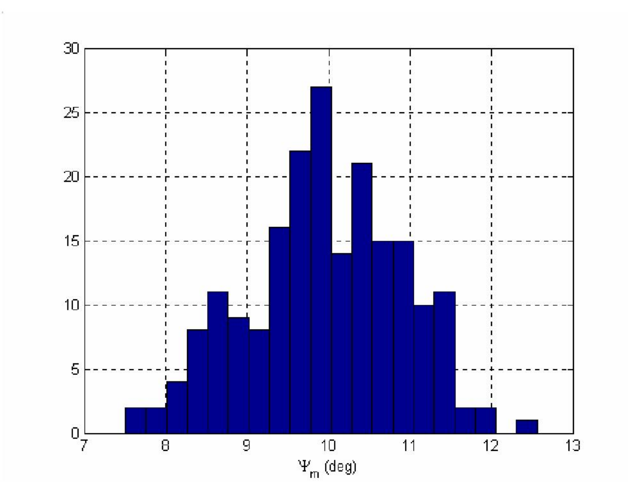

Fig. 6. Histogram of ψ_m

After determined mse_{T_m}, mse_{ψ_m} by using the (4) all-internal parameters in θ is obtained. Therefore, for using (10), only $IF_{hT}, IF_{h\psi}$ are unknown and must be calculated independently. By using (5), and deferent mse for both T_m, ψ_m, as variable inputs the corresponding output (i.e. ΔPH) obtained from (7), for example 200 iterations (see Fig. 7). Fig. 7, shows the missile and target trajectories for 200 iterations with specific mean square error Of T_m or ψ_m.

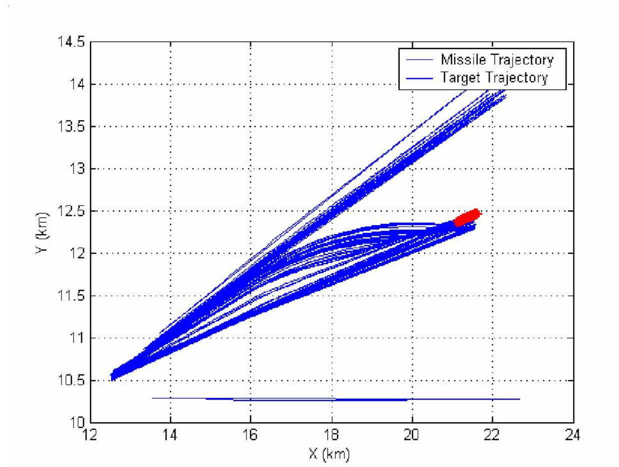

Fig. 7. Missile and target trajectories for 200 iterations

The end points of missile respect to target are easily obtained from Fig. 7, as illustrated at Fig. 8. In this figure the circle, which defined the target are shows with specific radius.

Table I illustrated the lunch missile scenario, and Table II illustrated the error analysis results, probability of hit obtained by equation (9), the mse of MFCD's outputs and the percentage of importance by using equation (10). These results are ranked by the PIF's of each parameter. For obtain one raw of this table the error analysis program executed 200 epochs as illustrated in the fig. 9. The end points of missile respect to target are easily obtained from Fig. 6, as illustrated at Fig. 8. In this figure the circle, which defined the target are shows with specific radius.

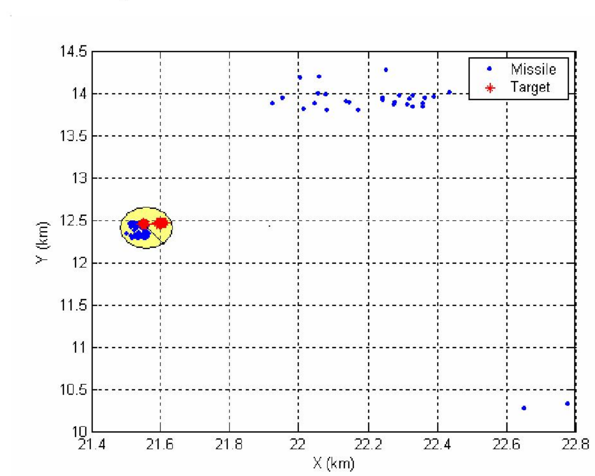

Fig. 8. End points of missile and target and its CEP

VI. CONCLUSIONS

In this paper, a analysis technique for a typical radar of airborne FCS was illustrated. This method based on Monte Carlo approach that deals with input random variables and its statistical momentums. All errors of radar by executed of this technique are ranked at the Table II. From this table it is clear that which parameter in radar catalog is most important and which is not. This table can help the designer of FCS to choice the efficient radar with proper price too.

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VIII. BIOGRAPHIES

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TABLE I: MISSILE LAUNCH SCENARIO

Parameter	Parameter values	***Parameter***	Parameter values	***Parameter***	Parameter values
Fly height (m)	900	Target range (km)	38.44	Wind speed (m/sec)	8.5
Helicopter azimuth fly (deg)	10	Target Azimuth (deg)	13.0023	Wind direction (deg)	100
Helicopter roll fly (deg)	3	Target speed (m/sec)	13.0023	Air pressure (mbar)	1009
Helicopter pitch fly (deg)	2-	Target bearing (deg)	35.979	Air temperature (^{o}c)	32.437

TABLE II. OUTPUT RESULTS OF RADAR ERROR ANALYSIS									
elements	errors	Missile angel correction ψ_m			Time of missile seeker turn on T_m			PIF	Probability of Hit
	rms	mse_{ψ}	δ_{ψ}	$bias_{\psi}$	mse_{T_m}	δ_{T_m}	$bias_{T_m}$	%	PH
Target azimuth measurement	0.6 deg	6.404	2.42	0.74	2.6984	1.22	1.10	42.3	95.488
Target range measurement	350 m	2.92	0.19	1.7	6.76	.038	2.6	20.6	97.653
Target speed measurement	3 m/s	1.2725	0.25	1.1	4.50	0.3	2.1	9.5	98.901
Target bearing measurement	5 deg	0.7489	.33	.8	6.012	0.23	0.85	6.9	99.181
Radar antenna zero setting	0.1 deg	0.8302	0.9	0.1423	0.0389	0.17	0.1	5.2	99.431
Helicopter vibration	2g 9ms	0.6574	0.8	0.132	0.0377	0.16	0.11	4..3	99.549
Installation of radar antenna	0.01 deg	0.5248	0.72	0.08	0.0181	0.1	0.09	3.4	99.640
Delay of transaction radar informations	200 ms	0.4289	0.65	0.08	0.0410	0.17	0.11	2.6	99.70
Radar encoder error	0.05 deg	0.4158	0.64	0.079	0.0125	0.05	0.1	2.5	99.715
DC motor error	.03 deg	0.121	0.33	0.11	0.13	0.3	0.2	.8	99.910
Resolution error	1.5 deg, 100m	0.0554	0.23	0.05	0.9344	0.4	0.88	0.8	99.914
Radar antenna production error	0.005 deg	0.0125	0.11	0.02	0.0265	0.11	0.12	0.1	99.990
Enternal circuit error	$5-10\ \mu$sec	0.0029	0.05	0.02	0.0544	0.12	0.2	0.05	99.995
Wind fluctuation error	0.033 deg	0.0017	0.04	0.01	0.0829	0.1	0.27	0.05	99.995
Aliment of the installation area	0.02 deg	0.001	0.03	0.01	0.0743	0.08	0.26	0.05	99.995
Shock error	20g	0.0000	0.02	0.02	0.0481	0.09	0.2	0.05	99.996

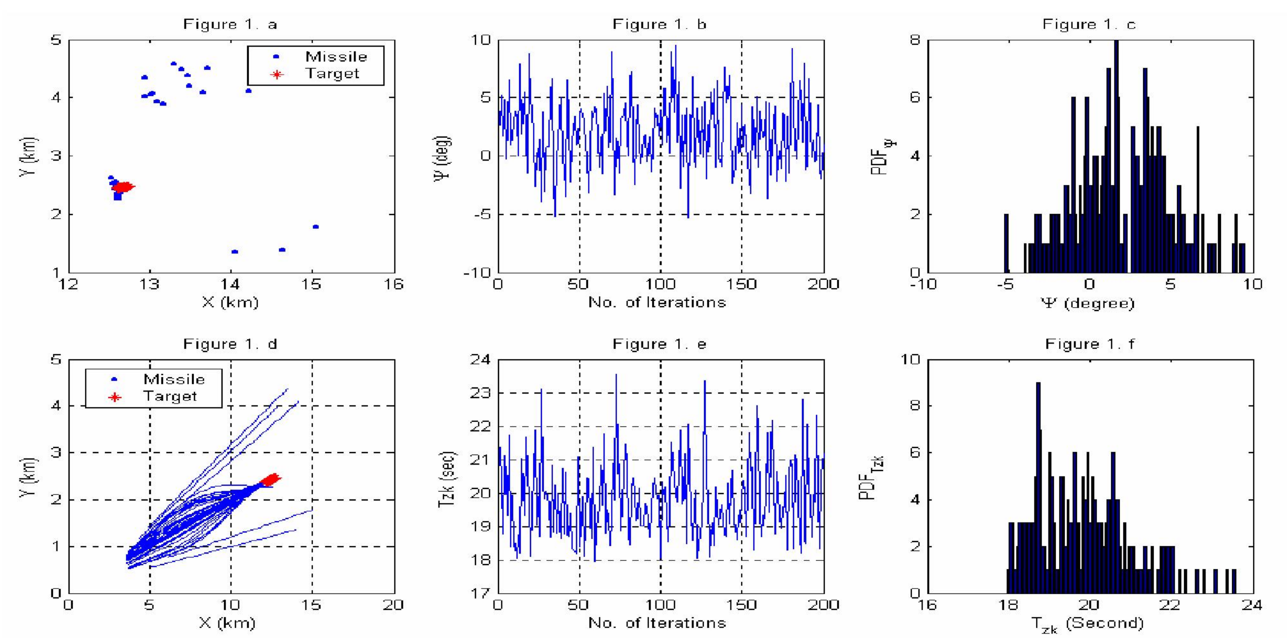

Fig. 9. (a) The end points of missile and target (b.e)-the valus of T_m, Ψ_m for 200 iterations (c.f) T_m, Ψ_m histograms (d) Missile and target trajectories.