

# Tracking a High Maneuver Target Based on Intelligent Matrix Covariance Resetting

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## Abstract

*A high accurate tracking technique with the use of intelligent approach on matrix covariance resetting is proposed in this paper. In practice, the conventional Kalman filters have a fast convergence rate at the beginning. However, after some iteration the Kalman filter steps become very small. To overcome this defect and to make use of Kalman filter abilities, the matrix covariance resetting idea is used. The matrix covariance presetting usually is used to improve the tracking algorithm result especially for high maneuvering targets. To determine the optimal value of the unknown resetting parameter in each step, the intelligent fuzzy block is used. In this paper, an innovative technique is presented, which resets covariance matrix by using fuzzy logic. It is demonstrated by means of numerical acceleration examples that the tracking capability of the proposed method is essentially as good as that of the traditional methods, especially for high maneuver targets.*

## 1. Introduction

Track while scan (TWS) radars, which use phase array antenna are often used in air and sea surveillance. With the function of TWS, the search radars can track a target or targets while scanning [1]. The Kalman filter has been used widely in target tracking problems [2]-[4]. However, when the target maneuvers the quality of the position and velocity estimation could be degraded significantly. To solve this problem, some techniques have been introduced to modify the conventional Kalman filter. For instance, Korn, Gully and Willsky presented a generalized likelihood ratio (GLR) method for maneuver detection and target state estimation [5]. This algorithm proposed the use of two ssss. Null hypothesis for a target without maneuver, and alternative hypothesis for a target with maneuver. When the log likelihood ratio is over a threshold, a maneuver is detected. This system needs a bank of correlators to detect the maneuver onset time. In some situations, the Kalman filter solves the target tracking problem by including the parameters as part of an augmented state to be estimated [6, 7, 8]. In many papers such filters are called a “full state” estimator. Goodwin and Sin proposed an adaptive control of time varying systems

[9]. They used a finite data window for error covariance matrix in least square algorithm and reset that matrix periodically. In practice, the performance characteristic of this technique is very well. However, every time the error covariance matrix is reset, information from earlier updates is partially lost. Therefore, resetting error covariance should be logical.

On the other hand, fuzzy logic was applied to maneuvering target tracking with intelligent adaptation and capability to add human knowledge to the system [10, 11].

Continuing these efforts, in this paper we find some logical rules for resetting error covariance matrix and add them to the Goodwin and Sin method with the use of fuzzy logic.

## 2. Model of uncertainty [12]

The basic models to be considered in this paper are the Bayesian and Fisher models, which are used in [6]. Theses models are specific cases of the state space structure-white process. The Bayesian models are one of the most important and common used models of uncertainty. In Bayesian models, uncertainty is modeled by random variables and/or stochastic processes with either completely specified probability distributions or completely specified first and second moments.

The complete definition of the Bayesian, discrete time model for linear systems is summarized below.

$$\begin{aligned} X(n+1) &= F(n)X(n) + G(n)w(n) \\ z(n) &= H(n)X(n) + v(n) \\ X(n) &\quad \text{state} \\ z(n) &\quad \text{observation} \\ v(n) &\quad \text{white observation uncertainty} \\ w(n) &\quad \text{white system driving uncertainty} \\ X(0) &\quad \text{initial condition} \end{aligned} \tag{1}$$

$$E\{v(n_1)v^T(n_2)\} = \begin{cases} \mathcal{R}(n_1) & n_1 = n_2 \\ 0 & n_1 \neq n_2 \end{cases}$$

$$E\{w(n_1)w^T(n_2)\} = \begin{cases} Q(n_1) & n_1 = n_2 \\ 0 & n_1 \neq n_2 \end{cases}$$

$$E\{x(0)x^T(0)\} = \psi$$

$$E\{x(0)\} = 0, E\{w(0)\} = 0, E\{v(0)\} = 0$$

In many applications, the input disturbance,  $w(\cdot)$  can be modeled as being completely unknown. A model where  $w(\cdot)$  is completely unknown is a type of Fisher model. Of course, conceptually such Fisher models have to be handled in a fashion different from Bayesian models where  $w(\cdot)$  is viewed as a random vector with known covariance matrix  $Q(\cdot)$ .

### 3. Filtering of bayesian models

The desired form of the filtering solution is a difference equation (recursive relationship) expressing  $\hat{X}(N+1|N)$  in terms of  $\hat{X}(N|N)$  on  $z(N+1)$ .

The solution of the filtering problem is the Kalman filter with equations:

$$\begin{aligned} \hat{X}(N+1|N) &= F(N)\hat{X}(N|N) + K(N+1)[z(N+1) \\ &\quad - H(N+1)F(N)\hat{X}(N|N)] \\ K(N+1) &= \Sigma(N+1|N)H^T(N+1)\mathcal{R}^{-1}(N+1) \\ \Sigma(N+1|N+1) &= \Sigma(N+1|N) - \\ &\quad \Sigma(N+1|N)H^T(N+1)[\mathcal{R}(N+1) + H(N+1) \\ &\quad \Sigma(N+1|N)H(N+1)^T]H(N+1)\Sigma(N+1) \\ \Sigma(N+1|N) &= F(N)\Sigma(N|N)F^T(N) \\ &\quad + G(N)Q(N)G^T(N) \\ \Sigma(0|0) &= 0, \hat{X}(0|0) = 0 \end{aligned} \quad (2)$$

$K(N)$  is the Kalman gain and notation  $\hat{X}(N+1|N)$  denotes the prediction at the  $(N+1)^{th}$  sample point given the measurement up to and including the  $N^{th}$  whilst  $\hat{X}(N|N)$  denotes the estimation at the  $N^{th}$  sample point given the measurement up to and including the  $N^{th}$ .  $\Sigma(N|N)$  is the error covariance matrix and  $\Sigma(N+1|N)$  is the error covariance matrix of the one-step prediction.

Maneuvering targets are difficult to track with Kalman filter since the target model of tracking filter might not fit the real target trajectory [13].

### 4. Intelligent error covariance matrix resetting

#### 4.1. Error covariance matrix resetting

Kalman filter is known to provide extremely rapid initial convergence rate and optimal tracking. However, the algorithm was developed with some assumptions. The most important assumption is the constant speed of

target movements. To be more precise, when the target maneuvers, the quality of the position and velocity estimation could be decreased significantly. Therefore, using the kalman filter is suitable until the target starts to maneuver.

As we know, in this algorithm error covariance matrix ( $\Sigma(n|n)$ ) gets small after a few iterations [9]. So, when target begins to maneuver with high acceleration, tracker which uses Kalman filter would not be functionally accurate. This motivates a related scheme in which  $\Sigma(n|n)$  is reset at various times. In other words, old data is discarded to keep the algorithm alive. The main idea of resetting  $\Sigma(n|n)$  is to retain the fast initial convergence of Kalman filter and track a target immediately when it maneuvers. However, resetting the error covariance matrix descends tracking performance when the target moves at a constant velocity because, as we know, Kalman filter provides optimal tracking for targets with constant velocity. Therefore, resetting error covariance matrix should be logical. To be more accurate, when the target moves with constant velocity,  $\Sigma(n|n)$  should not be reset and when the target starts to maneuver, the system should reset  $\Sigma(n|n)$ . According to this fact, the key of this dilemma is detection of target maneuver.

#### 4.2. Fuzzy maneuver detector

In this research, fuzzy logic is employed in order to detect target maneuver immediately. It is the fuzzy maneuver detector system, which we proposed previously in [14] and modified it in [15]. When maneuver has been detected, covariance matrix will be reset automatically.

The radar output signal has no exact mathematical relation with target maneuver. However, with no doubt, there exists a complex nonlinear mapping between them. To map the input vector to target acceleration vector it is important to find the effective input elements. Two features are used as inputs of the fuzzy acceleration estimator system.

1. *Absolute value of difference between last target course ( $\psi$ ) and observation target course ( $\xi$ ):* It is shown as  $\Delta\theta$  in Fig.1.  $\Delta\theta$  is one of the most useful elements to detect the target maneuver [14].

When  $|\Delta\theta|$  is low, then with a high probability the target is moving around its last direction and when  $|\Delta\theta|$  is high, then with a high probability the target is moving toward sensor's observation. This fact was used as a fuzzy rule in fuzzy acceleration estimator.

$|\Delta\theta|$ ,  $\psi$  and  $\xi$  were calculated with the use of following equations.

$$\Delta\theta = \psi - \xi \quad (3)$$

Where:

$\psi$  = Last Target Course

$\xi$  = Observation Target Course

Last Target Course = angle( $H\hat{X}(N-1|N) - H\hat{X}(N-2|N)$ )

Observation Target Course = angle( $\hat{Z}(N|N) - H\hat{X}(N-1|N)$ ) (4)

2. *Absolute value of measurement residual (R):* The objective in this section is to develop a maneuver detection algorithm, which detects the acceleration and jerk of a maneuvering target. A similar idea of quickest detection and change detection algorithm, only for constant acceleration has been investigated in [16]. The standard KF “(2)” is an efficient and unbiased filter, so the following sequence is the residue of observation.

$$\tilde{Z}(n+1) = Z(n+1) - \hat{Z}(n+1|n) = Z(n+1) - H(n+1)\hat{X}(n+1|n) \quad (5)$$

The residue of observation is a stochastic zero mean white process. i.e.,

$$E\{\tilde{Z}(n+1)\} = 0$$

$$E\{\tilde{Z}(n_1)\tilde{Z}(n_2)^T\} = \Re\delta(n_1 - n_2)$$

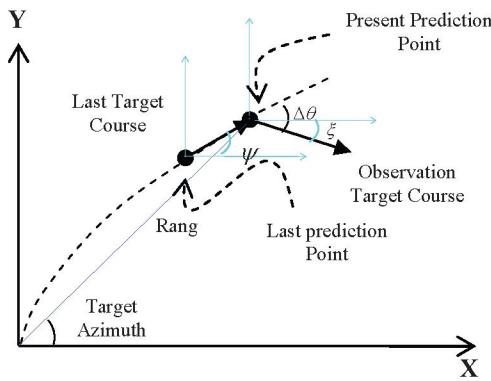


Figure 1. Target movement geometry [14].

Therefore, for non-maneuvering targets, the mean of this sequence ( $\tilde{Z}(n+1)$ ) is zero. But, for the case of maneuvering target, this sequence is no longer zero and contains more information.

This fact was used as another fuzzy rule in fuzzy acceleration estimator system.

#### 4.3. Intelligent error covariance matrix resetting

The proposed method is illustrated in Fig .2. In this figure, block 1, calculates  $\Delta\theta$  and R. Block 2 is a fuzzy controller. The fuzzy system has two inputs and one output. The input variables of fuzzy system are  $|\Delta\theta|$  and R. Input and output fuzzy sets all have three Gaussian membership functions with the following membership grade  $u_i^j(x_i)$ .

$$u_i^j(x_i) = \exp\left[-\frac{1}{2}\left(\frac{x_i - c_i^j}{\sigma_i^j}\right)^2\right] \quad (6)$$

Where,  $c_i^j$  and  $\sigma_i^j$  are the center value and the standard deviation of Gaussian membership function for  $i^{\text{th}}$  input variable of  $j^{\text{th}}$  fuzzy rule, respectively. The output of the fuzzy logic controller determines the estimated acceleration value of target deviation  $a_t$  from its last Target course based on  $\Delta\theta$  and R inputs. Fuzzy inference rules support mentioned information. Block 3 is a simple low pass filter. The main purpose of using a low pass filter is that the output of block 2 ( $a_t$ ) is a noisy signal. To vivify,  $a_t$  is the target acceleration signal added with high frequency noise. After passing this noisy signal through a low pass filter, the real value of target acceleration will be achieved. Block 4 is another fuzzy system. This fuzzy system has two inputs and one output. The inputs are target acceleration (output of Block 3) and the sum of the diagonal elements of  $\Sigma(n|n)$  (Trace). In practice the conventional Kalman filters have a fast convergence rate at the beginning. Therefore, after some iteration the Kalman gain which updates proportional to the estimation, becomes very small. To overcome this defect and to make use of Kalman filter ability, the matrix covariance resetting idea is used.

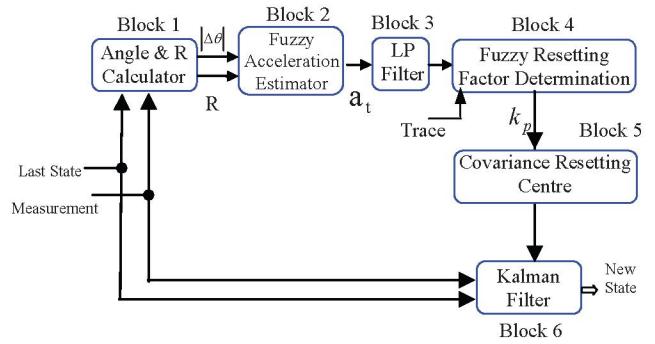


Figure 2. The proposed method.

The matrix covariance resetting is usually used to improve the tracking algorithm specially for high maneuvering targets. The trace norm is usually used as a matrix measure. So after some iterations which the  $\text{Trace}[\sum(n|n)]$ , as the covariance matrix norm becomes smaller than the specified value, the following presetting procedure is done:

$$\sum(n+1|n+1) = k_p \sum(n|n) \quad (7)$$

The main drawback of traditional resetting covariance matrix is on the estimation of the unknown constant coefficient ( $k_p$ ) at the presence of target maneuvering.

To overcome this drawback the Intelligent Matrix Covariance Resetting is suggested in this paper. Therefore, our fuzzy system in block 4 is utilized to support this fact.

Input and output fuzzy sets all have two Gaussian membership functions. As mentioned,  $\sum(n|n)$  should

be reset when the target is maneuvering and  $\Sigma(n|n)$  is low.

Block 5 is the covariance resetting centre. The system in this block, gets  $k_p$  and uses it to reset error covariance matrix using relation 7.

## 5. Simulation result

The estimation improvements obtained by the proposed method is illustrated by the following examples.

In experiments reported in this section, the following assumptions and parameter values are used. In this simulation, the sampling time is  $T=0.015$  (sec). Covariance elements generated for  $R$  and  $\theta$  axis are both Gaussian random variables; in addition, the measurement noise vector in Cartesian coordinates is related to the measurement noise vector in polar coordinates by the following equation [14].

$$\begin{bmatrix} \delta_x^2 \\ \delta_y^2 \end{bmatrix} = \begin{bmatrix} \cos^2 \theta_0 & R_0^2 \sin^2 \theta_0 \\ \sin^2 \theta_0 & R_0^2 \cos^2 \theta_0 \end{bmatrix} \begin{bmatrix} \delta_R^2 \\ \delta_\theta^2 \end{bmatrix} \quad (8)$$

Where,

$$\delta_R = 200, \quad \delta_\theta = 1$$

$$R_0 = 5000 \text{ (m)} \text{ and } \theta_0 = 30 \text{ (deg)}$$

In order to evaluate and compare the new tracking scheme with two existing augmented Kalman filter methods [6], two scenarios were considered as follows.

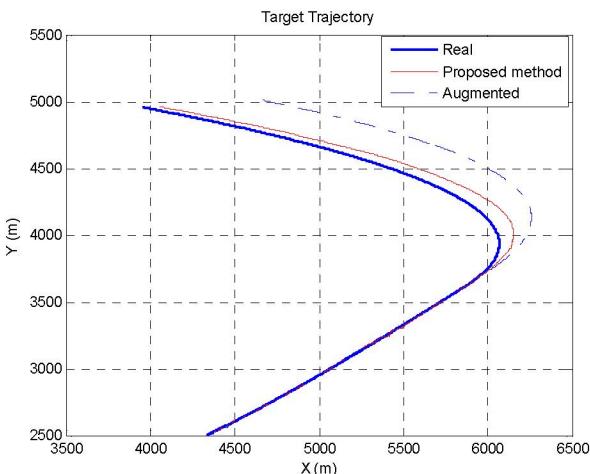


Figure 3. Trajectory of the maneuvering target in Cartesian coordinates and tracking result of the proposed method and augmented Kalman filter in the first scenario.

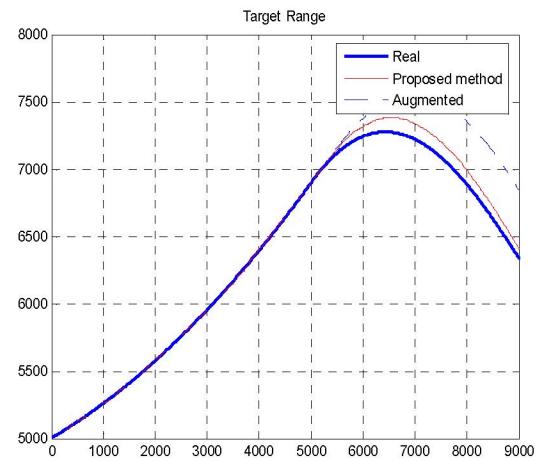


Figure 4. Range of the maneuvering target and estimation result of the proposed method and augmented Kalman filter in the first scenario.

First scenario: The initial position of the target is given by  $(x,y)=(4330,2500)$  with an initial speed of  $(v_x, v_y)=(13,7.5)$ . Target moves with constant acceleration  $u_x=0.2 \text{ m/s}^2$ ,  $u_y=0.02 \text{ m/s}^2$  until  $t= 75\text{s}$ , then it starts to maneuver with acceleration value  $u_x=-2 \text{ m/s}^2$ ,  $u_y= 0 \text{ m/s}^2$ . Target moves with this acceleration until the end of this simulation at  $t=135\text{s}$ . Fig.3 shows, target trajectory estimation by proposed method and augmented Kalman filter in this scenario. Fig.4 shows, target range estimation by two methods. Fig.5 shows, target azimuth estimation by two methods. Fig.6 shows, target speed estimation by two methods.

In order to compare the proposed method with augmented Kalman filter, a Mont Carlo simulation of 50 runs was performed. The STD of estimation error of range, azimuth, course and speed of all three methods in this scenario is compared in table1.

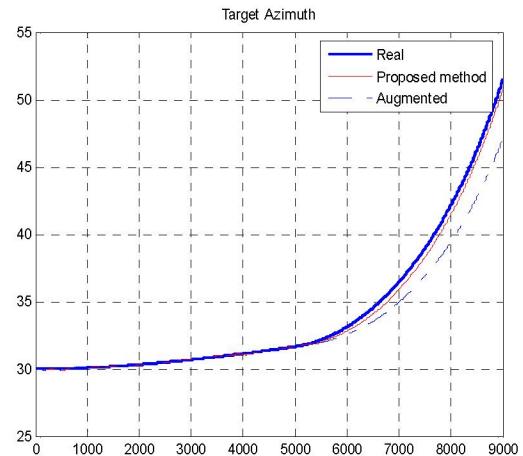


Figure 5. Azimuth of the maneuvering target and estimation result of the proposed method and augmented Kalman filter in the first scenario.

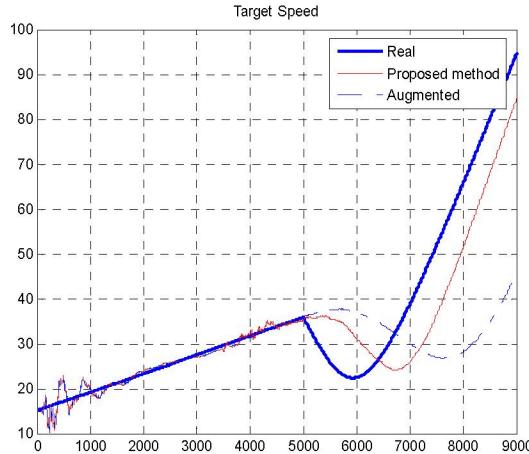


Figure 6. Speed of the maneuvering target and estimation result of the proposed method and augmented Kalman filter in the first scenario.

TABLE I. RADAR FILTER ESTIMATION ERROR IN THE FIRST SCENARIO (std)

	Range	Azimuth	Course	Speed
Proposed method	47.73	0.28	39.75	7.11
Augmented Method	188.43	1.20	48.59	16.83
Percentage Improvement	75	76.5	18	58

Second scenario: The initial position of the target is given by  $(x,y)=(4330,2500)$  with an initial speed of  $(v_x, v_y)=(13, 7.5)$ . Target moves with constant acceleration  $u_x=0.2 \text{ m/s}^2$ ,  $u_y=0.02 \text{ m/s}^2$  until  $t= 52 \text{ s}$ , then it starts to maneuver with acceleration value  $u_x=-1 \text{ m/s}^2$ ,  $u_y=-1 \text{ m/s}^2$ . This acceleration continue to  $t=104 \text{ s}$  at that moment target starts another maneuver with acceleration of  $u_x=1 \text{ m/s}^2$ ,  $u_y=1 \text{ m/s}^2$ . Target moves with this acceleration until the end of this simulation at  $t=157 \text{ s}$ . Fig.7 shows, target trajectory estimation by the proposed method and augmented Kalman filter in this scenario. Fig.8, Fig.9 show, target range and azimuth estimation by two methods. Fig.10 shows, target speed estimation by two methods.

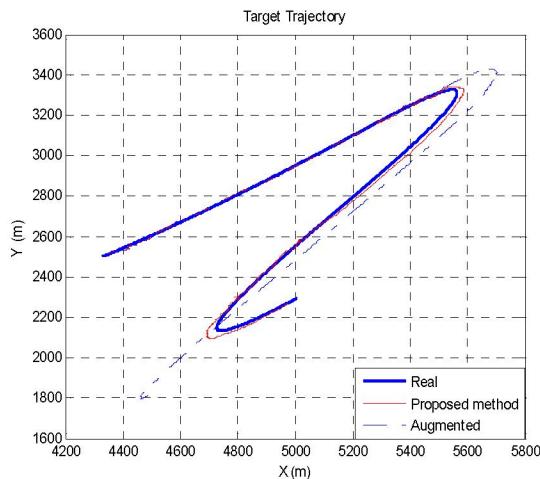


Figure 7. Trajectory of the maneuvering target in Cartesian coordinate and tracking result of the proposed method and augmented Kalman filter in the second scenario.

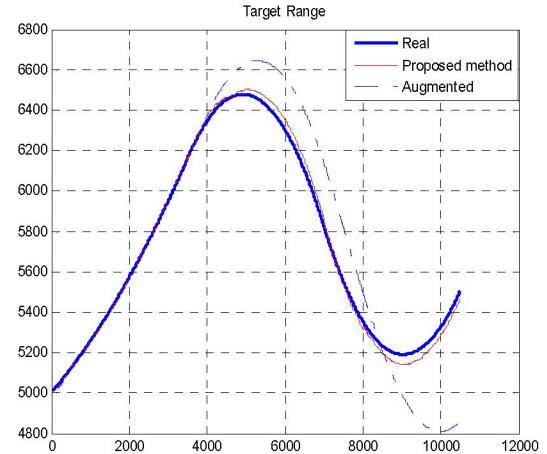


Figure 8. Range of the maneuvering target and estimation result of the proposed method and augmented Kalman filter in the second scenario.

Table 2 shows comparison of the STD of estimation error in second scenario. These results are the mean value over 50 runs.

TABLE 2. RADAR FILTER ESTIMATION ERROR IN THE SECOND SCENARIO (std)

	Range	Azimuth	Course	Speed
Proposed method	29.15	0.09	35.74	7.32
Augmented Method	224.77	0.83	39.78	17.00
Percentage Improvement	87	89	1	57

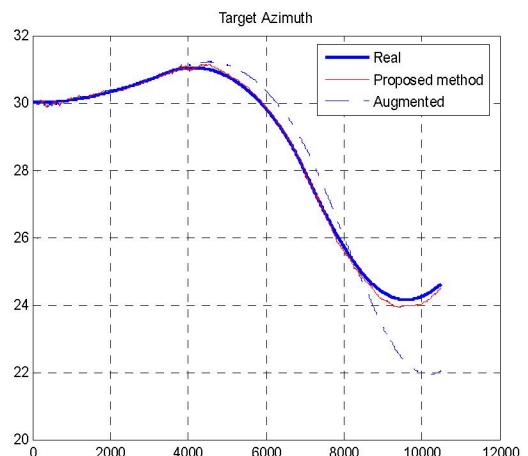


Figure 9. Azimuth of the maneuvering target and estimation result of the proposed method and augmented Kalman filter in the second scenario.

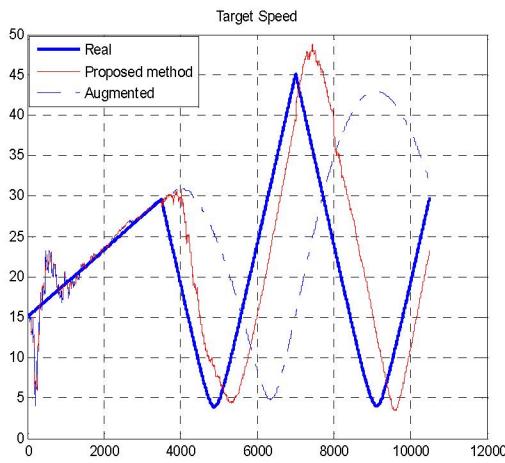


Figure 10. Speed of the maneuvering target and estimation result of the proposed method and augmented Kalman filter in the second scenario.

## 6. Conclusion

In this paper, an innovative technique is presented which presets covariance matrix using fuzzy logic. Simulation results show a high performance of the proposed innovative technique and effectiveness of this scheme specially, in high maneuvering target tracking.

## 7. References

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