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OPTIMUM DESIGN OF GRAVITY RETAINING WALLS

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ABSTRCT

This paper is concerned with the optimization of gravity walls that retain a horizontal backfill material. The aim is to demonstrate how the traditional design process can be imitated mathematically to obtain an optimum design, which conforms to the requirements of the wall stability and strength.

The total cost, which must be minimized, includes the cost of concrete, i.e. the volume of the wall per unit length. The optimization problem generally consists of four design variables, which may be reduced to three in special cases. The minimum cost problem is formulated as a non-linear programming problem, which is linearised by geometric programming and then solved, by simplex method using a computer package developed in Standard FORTRAN 77.

Keywords: Optimization, Retaining, Gravity, Wall, Concrete, Design constraints.

1. Introduction

The aim of this paper is to obtain the minimum weight design of three types of gravity retaining walls in conformity with the requirements of ACI Code. The minimum weight design of gravity wall is such that all its appropriate functional states (stability, strength, soil bearing capacity, etc.) are within allowable performance limits.

Gravity walls, generally, are trapezoidal shaped and their dimensions must provide adequate stability against sliding and overturning. The soil pressure beneath the footing of wall, which can be computed from principals of solid mechanics for combined bending and axial stresses, must not exceed allowable bearing capacity of the soil. Moreover, in order the footing to be fully effective in bearing, i.e., to prevent a tensile-state beneath the soil and footing of gravity walls, the vertical resultant of the forces acting downward requires to fall within the middle third of the footing width.

The minimum weight design problem of gravity walls is formulated as a non-linear problem whose solution may be attempted by several techniques [1], namely sequential linear programming (SLP) and sequential convex programming (SCP).

The optimization problem for three common types of gravity retaining walls is formulated in the next sections.

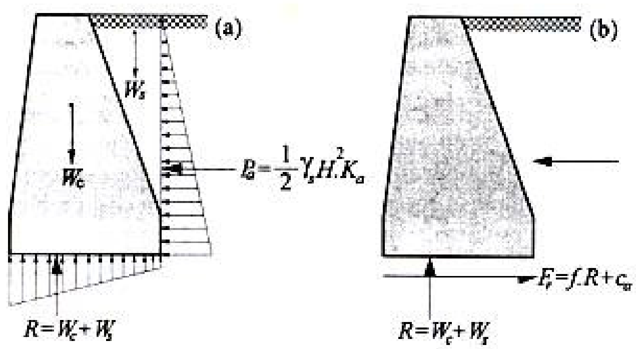

Figure 1: Forces on a gravity wall. (a) Rankin earth pressure. (b) Stability against sliding and overturning.

2. General Considerations

The forces on a gravity wall with a horizontal backfill are as illustrated in Figure 1. The active earth pressure is computed by the Rankin method [2]. In this Figure P_a is the active earth pressure, $P_a=\frac{1}{2}\gamma_s H^2 K_a$.

Sliding and Overturning Stability: The wall must be safe against sliding. That is, sufficient friction $F_r=f.R+c_a.B$ must be developed between the base slab and the soil that a safety factor SF or stability number N_s is

$$SF = N_s = \frac{Fr}{Pa} \geq 1.5 \text{ to } 2$$

all terms are illustrated in Figure 1. Note that the total force R acting on base is

$$R = W_c + W_s$$

The coefficient of friction between the base and the soil may be taken as

$$f = \tan\phi \text{ to } 0.67 \tan\phi$$

and the cohesion between the soil and footing c_a as

$$c_a = 0.5\, c \text{ to } 0.75\, c$$

Where ϕ and c are the angle of internal friction and cohesion of the soil respectively. A retaining wall must also be stable about the center of rotation (the toe) against overturning. We can compute a stability number N_o against overturning as:

$$No = \frac{\Sigma Mr}{\Sigma Mo} \geq 1.5 \text{ to } 2$$

Allowable Bearing Capacity: Stability of the base against a bearing-capacity failure is achieved by using a suitable safety factor with the ultimate bearing pressure:

$$q_a = \frac{q_{ult}}{SF}$$

Where the safety factor, SF, is usually taken as 2.0 for granular soils and 3.0 for cohesive soils [2].

The intensity of soil pressure is computed for a rigid footing with the width B and length L= unit. The linear soil pressure distribution is:

$$q = \frac{R}{A}\left(1 \pm 6\frac{e}{B}\right) \leq q_a$$

Where e is the eccentricity, i.e.

$$e = \frac{B}{2} - \frac{\Sigma Mr - \Sigma Mo}{\Sigma Wi}$$

Also the base width should be adjusted until $e \leq B/6$ for maximum efficiency of footing and minimum difference in soil pressure beneath the footing.

The optimization technique can be used to obtain the best shape for the gravity retaining walls. The design process involves the satisfaction of safety factor requirements against sliding, overturning and allowable soil bearing capacity constraints. These requirements, however, must be met so that the volume (i.e. the weight) is minimized.

Strictly speaking if the weight of the gravity wall is the only criterion of design then the following equation should be taken as the objective function:

$$\textit{Minimize:} \quad Z = A_w . L . \gamma_c \tag{1}$$

Where A_w and γ_c are the cross section area of the wall and the specific gravity of concrete respectively. L is the length of the wall which is taken as unit, i.e. L=1. The requirements of wall stability against overturning and stability, eccentricity limitation and finally allowable bearing capacity may be formulated as the following constraints:

$$\frac{\Sigma Mr}{\Sigma Mo} \geq N_o \qquad \text{Overturning stability constraint} \qquad (2)$$

$$\frac{Fr}{Pa} \geq N_s \qquad \text{Sliding stability constraint} \qquad (3)$$

$$e = \frac{B}{2} - \frac{\Sigma Mr - \Sigma Mo}{\Sigma Wi} \leq \frac{B}{6} \qquad \text{Eccentricity constraint} \qquad (4)$$

$$q_{max} = \frac{R}{A}(1 + 6\frac{e}{L}) \leq qa \qquad \text{Soil stress constraint} \qquad (5)$$

The optimization problem for three common types of gravity walls is formulated in the following sections.

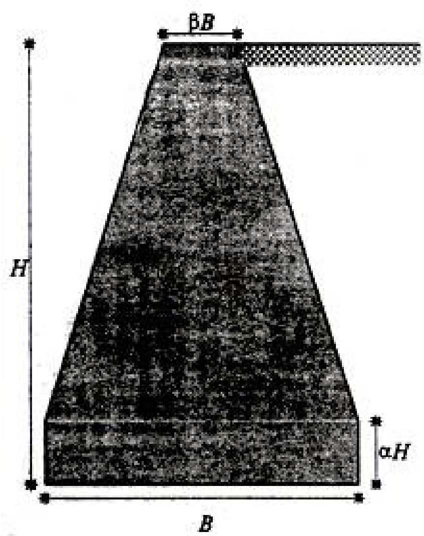

Figure 2: A symmetric trapezoid shape gravity wall

3. Formulation Procedure

Type I: Consider a symmetric trapezoid shape gravity wall shown in Figure 2, which retains a horizontal embankment of height *H*. While the wall is constant in height other dimensions should be adjusted so that all requirements are met and minimum amount of concrete is consumed per unit length of the wall, i.e., the minimum cross section area of the wall is reached. Referring to Figure 2, the minimum area of the wall is, therefore, the objective function of the problem and must be minimized, so

$$Minimize: \quad Z=\frac{1}{2}B.H.(1+\alpha+\beta-\alpha\beta) \qquad (1a)$$

To compute overturning stability it is necessary to evaluate resisting and overturning moments as follow:

$$M_r = B^2.H.\left[\frac{1}{4}\gamma_c.(1+\alpha+\beta-\alpha\beta)+\frac{1}{24}\gamma_s.(1-\alpha).(1-\beta).(5+\beta)\right]$$

$$M_o = \frac{1}{24}B^2.H.\gamma_s.(1-\alpha).(1-\beta).(5+\beta)$$

And then the overturning constraint is

$$\frac{M_r}{M_o}=\frac{B^2}{H^2}.\frac{6\gamma_c(1+\alpha+\beta-\alpha\beta)+\gamma_s(1-\alpha)(1-\beta)(5+\beta)}{4\gamma_s.Ka}\geq No \qquad (2a)$$

Also the sliding stability can be obtained by dividing the resisting forces against sliding to the driving forces. Consequently the sliding constraint is:

$$\frac{F_r}{P_a}=\frac{2B.c_a+B.H.[\gamma_c.(1+\alpha+\beta-\alpha\beta)+0.5\gamma_s.(1-\alpha)(1-\beta)].tag\phi}{H^2.\gamma_s.K_a}\geq Ns \qquad (3a)$$

The maximum eccentricity of the resultant vertical forces is $e=B/6$ which results in the soil-pressure for the entire footing area to be effective with no soil tensile stresses beneath it. For this case the eccentricity constraint is:

$$e=\frac{4H^2.\gamma_s.Ka-B.^2\gamma_s.(1-\alpha)(1-\beta)(2+\beta)}{6B.[2\gamma_c.(1+\alpha+\beta-\alpha\beta)+\gamma_s.(1-\alpha)(1-\beta)]}\leq\frac{B}{6} \qquad (4a)$$

Finally, the maximum intensity of soil pressure beneath the footing must be equal or less than the allowable soil pressure. The stress constraint is, therefore, as:

$$q_{max}=\frac{H^3\gamma_sKa}{B^2}+\frac{1}{2}H\gamma_s(1+\alpha+\beta-\alpha\beta)-H\gamma_c(1-\alpha)(1-\beta)\leq q_a \qquad (5a)$$

In addition the above constraints (2a), (3a), (4a) and (5a) on overturning stability, sliding stability, eccentricity limit and allowable soil pressure, the designer may wish to prescribe constraints on the geometry of the gravity wall for practical purposes and prevention of stress concentration, e.g. the vertical portion of the wall, α (see Figure 1), may be limited to a minimum value K_1. These constraints may be written as

$$\alpha\geq K_1 \qquad (6a)$$

$$\beta\geq K_2 \qquad (7a)$$

For all three cross-sectional shapes gravity walls, described in this and the following sections, the minimization problem consists of three design variables, say B, α and β, and up to six design constraints which is a non-linear programming problem.

If β is assumed to be known, the problem consists of only two variables B and α and thus it is possible to obtain the optimum solution graphically. Figure 3 illustrates a typical solution for this program. Any pair of values for B and α that satisfies constraints in equations (2a) to (7a) are called feasible solutions. Among the various answers, however, we are interested in the one that gives the minimum value for the objective function (Eq. 1a).

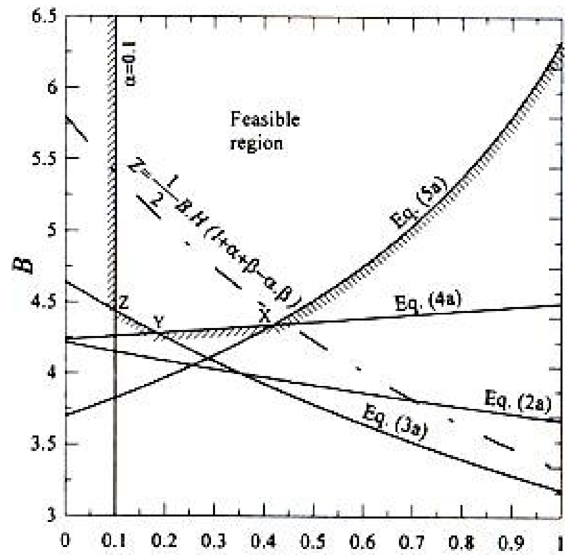

Figure 3: Graphical solution of the non-linear programming problem.

It is worth pointing out that from amongst the various possible points, attention was drawn to nodes X, Y and Z. This is because it can be proved that in any non-linear programming problem, the optimum solution is always given by a node where two or more constraints intersect [2]. As mentioned earlier the original non-linear programming can be solved by first linearising the non-linear functions using various methods and then to be solved by simplex method. Numerical example is discussed later.

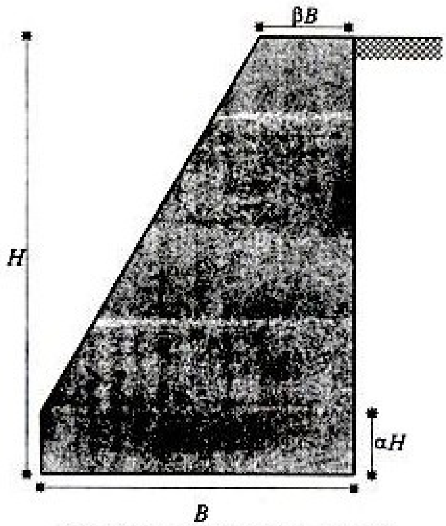

Figure 4: Second type of gravity wall

Type II: The second type of gravity wall considered here is shown in Figure 4. The volume of this type wall whose face in contact with soil is vertical, is the same as the first type wall. The objective function, therefore, can be written as:

Minimize: $Z=\frac{1}{2}B.H.(1+\alpha+\beta-\alpha\beta)$ (1b)

Although the constraints for this type of gravity wall is not the same as the first type but they are similar. Thus we can write:

$$\frac{B^2}{H^2}\cdot\frac{\gamma_c}{\gamma_s}\cdot\frac{[3\alpha+(1-\alpha)(2+2\beta-\beta^2)]}{Ka} \geq No \quad (2b)$$

$$\frac{2B.c_a + B.H.\gamma_c.(1+\alpha+\beta-\alpha\beta).tag\phi}{H^2.\gamma_s.K_a} \geq Ns \quad (3b)$$

$$e=\frac{2H^2.\gamma_s.Ka + B^2\gamma_c.(\alpha-1)(1+\beta-2\beta^2)}{6B.\gamma_c.(1+\alpha+\beta-\alpha\beta)} \leq \frac{B}{6} \quad (4b)$$

$$q=\frac{H^3\gamma_s Ka}{B^2}+H\gamma_c.(\alpha+\beta^2-\alpha\beta^2)\leq q_a \quad (5b)$$

$\alpha \geq K_1$ (6b)

$\beta \geq K_2$ (7b)

Type III: Shown in Figure 5 is the third type of gravity wall. The objective function for this type wall is also similar to the previous ones, i.e.

Minimize: $$Z=\frac{1}{2}B.H.(1+\alpha+\beta-\alpha\beta) \qquad (1c)$$

The constraints for this type of the gravity walls can be driven in a similar way to the other types. These constraints are as follow:

$$\frac{B^2}{H^2}.\frac{3\gamma c+(\gamma s-\gamma c).(1-\alpha).(2-2\beta-\beta^2)}{\gamma s.Ka} \geq No \qquad (2c)$$

$$\frac{2B.c_a+B.H.[\gamma c.(1+\alpha+\beta-\alpha\beta)+\gamma s.(1-\alpha-\beta+\alpha\beta)].tag\phi}{\gamma s.H^2.Ka} \geq Ns \qquad (3c)$$

$$e=\frac{2H^2.\gamma s.Ka-B^2(\gamma c-\gamma s).(1-\alpha)(1+\beta-2\beta^2)}{6B.[\gamma c.(1+\alpha+\beta-\alpha\beta)+\gamma s.(1-\alpha)(1-\beta)]} \leq \frac{B}{6} \qquad (4c)$$

$$q=\frac{H^3.\gamma s.Ka}{B^2}+H.\gamma s\beta.(\alpha-1)(1-\beta)+H.\gamma c.[1+\beta.(1-\alpha)(1-\beta)] \leq qa \qquad (5c)$$

$$\alpha \geq K_1 \qquad (6c)$$

$$\beta \geq K_2 \qquad (7c)$$

4. Numerical Examples

In order to illustrate the application of the algorithm, the following example is discussed.

Consider the design of a gravity wall which is to retain an embankment of H=9 m. The wall is on a soil of ϕ=23° and also the same allowable bearing kg/cm^2. The stability and sliding are specific gravity of practical purposes α to k_1=k_2=0.1

The objective wall is identical and

Minimize:

It can be seen that which must be design which is

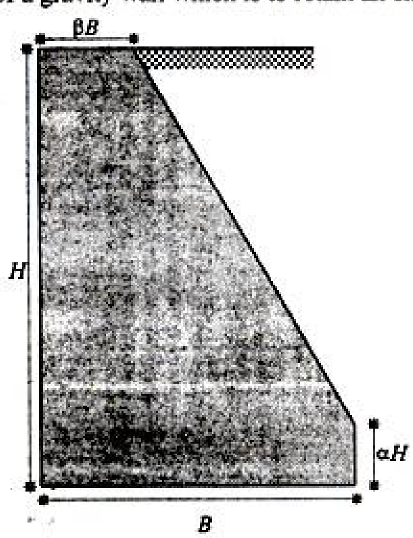

Figure 5: Third type of gravity wall

γ_s=1.8 t/m^3. The backfill soil has properties as the base soil. The capacity of the soil is q_a=3.2 requirements against overturning respectively N_o=2 and N_s=2. The concrete is γ_c=2.4 t/m^3. For and β design variables are limited

function for all three types gravity therefore can be written as:

$$Z=4.5B.(1+\alpha+\beta-\alpha\beta) \qquad (1)$$

the design variables are B, α and β determined so that the feasible minimum in volume, meets all strength and serviceability requirements. All constraints for three types gravity walls may be obtained by substituting the known values in equations (2) to (5).

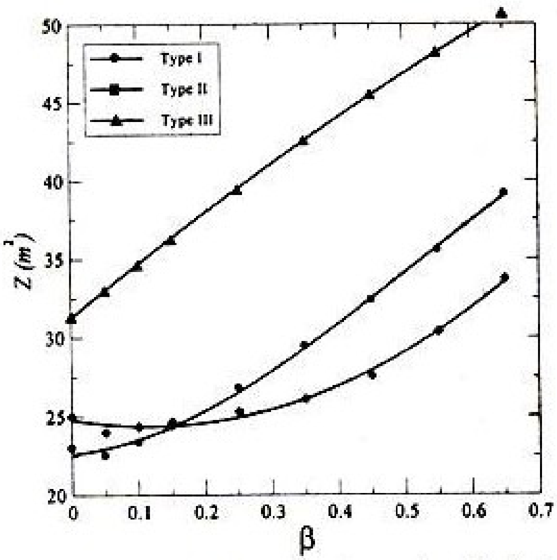

Figure 6: Variation of cross section of the gravity wall with β.

The nature of the minimization problem is non-linear programming problem whose solution is attempted by linearisation of the problem by geometric programming and then solving by simplex method [4]. This has been done by developing a Code in standard FORTRAN 77 by the Author.

The problem, however, may be solved graphically (see Figure 3) if one of the design variables, e.g. β, is assumed to be known. For this purpose different values are assigned to β and then the two other design variables, α and *B*, have been evaluated so that the volume of the wall (per unit length) is minimized for these particular values of α and *B*. Figure 6 illustrates the volume of the wall (i.e. the objective function, Eq. 1) against β. Also summarized in Table 1 is the optimum distribution of the design variables.

Table 1: The optimum values of design variables and minimum cross section area of the wall.

Wall Type	$B\ (m)$	α	β	$Z\ (m^2)$
I	4.359	0.1	0.1	23.342
II	4.495	0.116	0.1	24.362
III	6.466	0.1	0.1	34.625

It can be seen that, except wall type II, the limits on α and β control the design. It means that the cross section area of the wall may even be reduced if it was practically possible to reduce α and β further.

5. Conclusions

Based on the problem formulation and optimality criteria developed in Section 3 of this paper, a numerical approach and a computer Code are developed to solve the minimization problem. The cost to be minimized involves the concrete volume used in unit length of the gravity wall. The design constraints includes limits on safety factor against overturning, sliding, eccentricity and soil bearing capacity.

Following the actual construction practice of this type structure with regard to stress concentration, proper limits are imposed on K_1 and K_2. Practical examples are solved to demonstrate the usefulness of the problem. It is shown that employing optimization approaches leads to a design much more economical than a design obtained by traditional methods.

6. Notation

A_w	Cross section area of gravity retaining wall
B	Width of gravity retaining wall
c	Cohesion of soil
F_r	Resisting forces against sliding
H	Height of gravity retaining wall
K_a	Active earth pressure coefficient
L	Length of gravity retaining wall=unit
M_r	Resisting moment against overturning
M_o	Overturning moment
P_a	Active earth pressure
q_a	Allowable soil pressure
Z	Volume of retaining wall per unit length (objective function)
φ	Angle of internal friction of soil
γ_c	Specific gravity of concrete
γ_s	Specific gravity of backfill soil

7. References

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