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The Effects of Irrigation Efficiency and Uniformity Coefficient on Relative Yield and Profit for Deficit Irrigation

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To avoid constructing expensive hydraulic structures for implementing new water resources, a deficit irrigation project may be designed to optimise the use of the available water resources. Previously, irrigation efficiency and uniformity coefficient have not been considered quantitatively. In fact, efficiency is a very significant factor in optimisation analysis, and the potential for increasing irrigation efficiency is one of the key reasons for deficit irrigation. In this paper, the influences of irrigation efficiency under full irrigation condition η_f on the performance of deficit irrigation, and the effects of deficit irrigation on improving the irrigation efficiency under deficit irrigation η_d have been considered for four crops of winter wheat, spring barley, maize, and sorghum in an arid region of Iran. Furthermore, the combined effects of irrigation efficiency and uniformity coefficient on deficit irrigation were investigated. Results showed that considerable improvements can be achieved in water use efficiency under this concept. Some mathematical relationships were derived to show the quantitative increase in irrigation efficiency under deficit irrigation. These results showed that the performance of deficit irrigation is highly dependent on η_f, such that lower η_f values result in higher allowable water reduction level and also more field income. By combined analysis, water reductions for sorghum and barley were found to be economically feasible at values of irrigation efficiency lower than 1·0, while water reduction for maize was not economically feasible at irrigation efficiencies greater than 0·6. Water reduction for wheat was economically feasible at irrigation efficiencies of 0·6 or lower.

1. Introduction

As population increases, the available water per capita decreases accordingly. A large amount of investment is needed to construct new hydraulic structures and provide new water resources. This imposes a serious obstacle, especially for developing countries with limited financial resources. However, there exist an alternative way to combat with low available water in the agricultural sector, *i.e.*, intentional deficit irrigation. There are many scientific papers on implementing deficit irrigation for a single crop (*e.g.* Rao *et al.*, 1988b) or a crop mix (*e.g.* Ghahraman & Sepaskhah, 1996). Two different optimisation techniques of dynamic programming and non-linear programming are available in the literature to derive the optimal distribution of a limited water supply. It is believed that deficit irrigation can improve the water use efficiency, by eliminating the least productive irrigation or reducing irrigation adequacy (English *et al.*, 1990). However, the available literature has not addressed the problems of how much irrigation efficiency can be improved by deficit irrigation and how variable irrigation efficiency can increase the economic performance of deficit irrigation. The dependency between irrigation efficiency and deficit irrigation for drip irrigation has been analysed by Wu (1988) and Anyoji and Wu (1994) where the water losses are mainly deep percolation. However, it has not been analysed for surface irrigation where the water losses are deep percolation and surface runoff. Furthermore, Wu (1988) and Anyoji and Wu (1994) used a seasonal water production function in their analysis instead of multiple-stage water production function.

Notation

b	coefficient	Y_p	potential yield, $\mathrm{t\,ha^{-1}}$
B	gross revenue per unit land under full irrigation	Z	relative net benefit
C	gross cost per unit land under full irrigation	α	ratio of deficit irrigation efficiency to full irrigation efficiency
C_{ud}	uniformity coefficient under deficit irrigation	β	ratio of actual uniformity coefficient to potential uniformity coefficient
C_{uf}	uniformity coefficient under full irrigation	γ	ratio of actual uniformity coefficient times actual irrigation efficiency to potential uniformity times potential irrigation efficiency ($\gamma = \alpha\beta$)
E_{ta}	actual crop evapotranspiration, mm	ε	small amount to guarantee a non-zero denominator
E_{tp}	potential crop evapotranspiration, mm	η_d	deficit irrigation efficiency
i	an index for crop growth stage	η_f	full irrigation efficiency
I	integer value	λ	relative sensitivity of crop to water stress
K_{yi}	a yield response factor to water	Π	multiplier symbol
N	number of growth stage		
W_a	actual water application		
W_p	potential crop irrigation requirement		
x	fraction of water reduction		
x'	nominal fraction of water reduction		
Y_a	actual yield, $\mathrm{t\,ha^{-1}}$		

In this paper, the effects of irrigation efficiency and combined effects of irrigation efficiency and uniformity coefficient on deficit irrigation for four different crops are analysed using multiple-stage water production function in an arid region of Iran.

2. Theory

2.1. *Analysis using irrigation efficiency*

The water production function for a particular crop, which is needed to implement deficit irrigation quantitatively, is a relationship between relative crop evapotranspiration and relative sensitivity of crop to water stress (as inputs) and relative crop yield (as output). There are two general categories of water production function, *i.e.* additive and multiplicative (*e.g.* Vaux & Pruitt, 1983). Furthermore, numerous types of such a function have been reported in literature (Hall & Butcher, 1968; Minhas *et al*., 1974; Bielorai & Yaron, 1978). Although every function, theoretically, may respond correctly in every location, choosing unsuitable relative sensitivity of crop to water stress is a point that may lead to unrealistic results for deficit irrigation implementation (Ghahraman & Sepaskhah, 1997).

A well-known water production function is attributed to Jensen (1968) which has the following form:

$$Y_a/Y_p = \prod_{i=1,\ldots,N} [(E_{ta}/E_{tp})_i^{\lambda i}] \tag{1}$$

where: Y_a and Y_p are actual and potential (maximum) yield in $\mathrm{t\,ha^{-1}}$, respectively; E_{ta} and E_{tp} are actual and potential crop evapotranspiration, respectively; N is the number of growth stages; i is an index for crop growth stage; λ_i is the relative sensitivity of crop to water stress during the ith stage of growth and $\prod$ is a multiplier symbol. Nairizi and Rydzewski (1977) and Meyer *et al*. (1993) have approximated the ratio of E_{ta}/E_{tp} with W_a/W_p in which W_a and W_p are applied water and crop irrigation requirement, respectively, as follows:

$$Y_a/Y_p = \prod_{i=1,\ldots,N} [(W_a/W_p)_i^{\lambda i}] \tag{2}$$

Although such a simplification has provided an interesting non-linear irrigation optimisation algorithm (Ghahraman & Sepaskhah, 1997), it may now be reconsidered.

It is logical to substitute E_{ta} for $\eta_d W_a$ and E_{tp} for $\eta_f W_p$, where η_d and η_f are irrigation application efficiencies under deficit and full irrigation, respectively. The application efficiency is the ratio of water stored in the root zone to that applied in the field. In general, $\eta_d \geqslant \eta_f$ is based on a premise that, ignoring surface runoff, there is a less chance for deep percolation, and hence, higher irrigation efficiency, under deficit irrigation. In fact, efficiency is a very significant factor in optimisation analysis, and the potential for increasing irrigation efficiency is one of the key reasons for deficit irrigation. Therefore, by equating $E_{ta}/E_{tp} = \alpha(W_a/W_p)$, an appropriate form of Eqn (1) may be read as follows:

$$Y_a/Y_p = \prod_{i=1,\ldots,N} [(\alpha W_a/W_p)_i^{\lambda i}] \tag{3}$$

where α is the ratio of E_a/E_p. The value of α is variable. Its minimum is one, coincident with full irrigation, but it gradually increases as deficit irrigation progresses. Therefore, while Eqn (2) (Nairizi & Rydzwski, 1977; Meyer *et al.*, 1993) may be nearly sound at low water reductions ($\alpha = 1$), it becomes more tenuous at greater water reduction.

In this analysis, it was assumed that with deficit irrigation surface runoff may be ignored which is a common practice in small-scale farm irrigation systems, especially with dead end furrows and that there is a less chance for deep percolation. In fact, where total amount of irrigation water is reduced, the depth of water for each irrigation application is also reduced and this may result in no runoff and less deep percolation. In small-scale farm irrigation system, the flow rate, length of irrigation plots and advance time is also small which end up to no runoff and less deep percolation.

Unfortunately, the correct quantitative behaviour of η_d, and therefore α, is not known. For the purpose of illustration, it was assumed that η_d takes a value of 1 at half full irrigation [$x = 0{\cdot}5$, where x is fraction of water reduction which is defined as $(W_p - W_a)/W_p$)]. Therefore,

$$\alpha = 1 \quad \{x = 0\} \tag{4a}$$

$$\alpha = 1/\eta_f \quad \{x \geqslant 0{\cdot}5\} \tag{4b}$$

For practical purposes, it was assumed that intermediate water reduction make linear changes in α. Therefore, one may easily derive the following:

$$\alpha = 1 + 2(1/\eta_f - 1)x \quad \{0{\cdot}5 > x > 0\} \tag{5a}$$

$$\alpha = 1/\eta_f \quad \{x \geqslant 0{\cdot}5\} \tag{5b}$$

These two different segments can be put into one equation, as follows:

$$\alpha = [1 + 2\{1/\eta_f - 1\}x](0{\cdot}5 - b) + [1/\eta_f](0{\cdot}5 + b) \tag{6}$$

where the coefficient b is given by

$$b = 0{\cdot}5 \text{ abs } (x - 0{\cdot}5)/(x - 0{\cdot}5 + \varepsilon) \tag{7}$$

where abs stands for absolute value for $(x - 0{\cdot}5)$ and ε is a small amount to guarantee a non-zero denominator. Therefore, α increases as the water reduction increases and decreases with an increase in the value for η_f.

Rao *et al.* (1988a) have expanded a single-stage water production function of Doorenbos and Kassam (1979) and proposed another useful water production function, which is similar to Eqn (1) and has the following form:

$$Y_a/Y_p = \prod_{i=1,\ldots,N} [1 - K_{yi}(1 - E_{ta}/E_{tp})_i] \tag{8}$$

where K_{yi} is a yield response factor to water. Ghahraman *et al.* (2001), without considering the changes of irrigation efficiency, approximated the above equation (similar to the approach of Nairizi & Rydzewski, 1977; Meyer *et al.*, 1993) and used it for studying deficit irrigation of maize in a semi-arid region of Iran. However, introducing the concept of variable irrigation efficiency, changes Eqn (8) as follows:

$$Y_a/Y_p = \prod_{i=1,\ldots,N} [1 - K_{yi}(1 - \alpha W_a/W_p)_i] \tag{9}$$

To implement an optimal deficit irrigation, it is necessary to determine optimal allocation of a limited seasonal water supply ($\sum W_a$) among different growth stages of a crop. Reducing total crop irrigation water needs ($\sum W_p$) by the fraction x, imposes the following constraint to an optimisation algorithm in which the objective function should maximise crop relative grain yield [Eqn (3) or (9)]:

$$\sum W_a = (1 - x) \sum W_p \tag{10}$$

Since α is equal or greater than one, therefore the multiplication of α and $(W_a)_j$ should be smaller than $(W_p)_j$. Therefore, two sets of constraints are also necessary as follows:

$$\alpha(W_a)_j < (W_p)_j \quad j \in I \quad I = \{1, 2, \ldots, N\} \tag{11}$$

$$\alpha(W_a)_j \geqslant (W_a)_j \quad j \in I \quad I = \{1, 2, \ldots, N\} \tag{12}$$

where I is an integer value between 1 and N. Constraint [12] is on a premise that commonly reported water sensitivity indices are valid only for water reductions of less than 0·5 (Doorenbos & Kassam, 1979). Ghahraman and Sepaskhah (1997) have addressed the mathematical solution of such a non-linear optimization algorithm, excluding α, based on Lagrangian multiplier with Kuhn–Tucher conditions (Luenberger, 1984).

At this point it seems useful to investigate the concept of water reduction level, from two different standpoints, in more detail. As far as crop growth is concerned, only that part of applied water that can be translated to crop evapotranspiration is effective. Therefore, the nominal water reduction level (say x') can be computed by a comparison between actual and potential crop evapotranspiration (E_{ta} and E_{tp}, respectively):

$$x' = (E_{tp} - E_{ta})/E_{tp} = 1 - \alpha W_a/W_p \tag{13}$$

On the other hand, from standpoint of a farmer any saved water that has not been delivered to a unit land, is important. The farmer can cultivate more lands by implementing this saved water to other lands. Therefore, an actual water reduction level (say x) can be defined by using actual and potential water applied to a unit land:

$$x = 1 - W_a/W_p \tag{14}$$

Manipulating Eqns (13) and (14) results in the following equation:

$$\alpha = (1 - x')/(1 - x) \tag{15}$$

where $x \geqslant x'$. Substituting for α from Eqn (5a) into Eqn (15) would result in

$$x = 1 - (1 - x')/\{1 + 2x'(1/\eta_f - 1)\} \tag{16}$$

To choose the optimal water reduction level (*i.e.* x), a relative net benefit function Z has been proposed by Ghahraman and Sepaskhah (1991):

$$Z = [(B/C)(Y_a/Y_p) - 1]/\{(1 - x')[(B/C) - 1]\} \tag{17}$$

where: B and C are gross revenue and cost per unit land under full irrigation, respectively; and $1/\{x'-1)$ is total area farmed under deficit irrigation compared with the initial unit land considered under full irrigation. Therefore, the ratio of B/C is considered to be constant for a specific crop and location. As deficit irrigation proceeds, it is assumed that the benefit per unit area decreases only according to a reduction in relative yield, and reduction in cost per unit area due to deficit irrigation is in accordance with the reduction in cost per unit area due to the reduction in benefit. However, the above equation was derived without considering irrigation efficiency. Therefore, instead of $1/(x'-1)$, $1/(x-1)$ should be interpreted as 'area cultivation under deficit irrigation'. Hence, Eqn (17) should be changed as follows:

$$Z = [(B/C)(Y_a/Y_p) - 1]/\{(1 - x)[(B/C) - 1]\} \tag{18}$$

2.2. *Analysis with combined irrigation efficiency and uniformity coefficient*

In surface irrigation, the water application is spatially non-uniform. This non-uniformity may be systematic down the length of the field. Furthermore, the non-uniformity will be more pronounced under deficit irrigation and higher application efficiency.

In bounded surface irrigation or surface irrigation with no tail water, crop water use in deficit and full irrigation conditions may be written as follows:

$$E_{ta} = C_{ud}\eta_d W_a \tag{19}$$

$$E_{tp} = C_{uf}\eta_f W_p \tag{20}$$

where C_{ud} and C_{uf} are uniformity coefficients under deficit and full irrigation conditions, respectively. The ratio of $[(C_{ud})(\eta_d)]/[(C_{uf})(\eta_f)]$ is considered as γ:

$$\gamma = [(C_{ud})(\eta_d)]/[(C_{uf})(\eta_f)] \tag{21}$$

or

$$\gamma = \beta(E_a/E_p) \tag{22}$$

or

$$\gamma = \alpha\beta \tag{22a}$$

where β is the ratio of C_{ud} to $C_{uf}(\beta = C_{ud}/C_{uf})$. The value of γ is one at full irrigation where $C_{ud} = C_{uf}$ and $\eta_d = \eta_f$, or where $\beta = 1$. The value of β generally increases as irrigation efficiency decreases. For purpose of illustration, it was assumed that η_d takes a value of 1 at half full irrigation ($x = 0{\cdot}5$, where x is fraction of water reduction). Therefore

$$\gamma = \beta = 1 \quad \text{for } x = 0 \tag{23}$$

$$\gamma = \beta(1/\eta_f) \quad \text{for } x \geqslant 0{\cdot}5 \tag{24}$$

If intermediate water reductions make linear changes in γ, one may easily derive the following:

$$\gamma = \beta(1 + 2(1/\eta_f - 1)x \quad \text{for } 0{\cdot}5 > x > 0 \tag{25}$$

$$\gamma = \beta(1/\eta_f) \quad \text{for } x \geqslant 0{\cdot}5 \tag{26}$$

Again for purpose of illustration, it was assumed that the value of C_{ud}/C_{uf} is 0·9 at deficit irrigation ($x = 0$) and 0·45 at deficit irrigation ($x = 0{\cdot}5$, where x is fraction of water reduction), therefore:

$$C_{ud}/C_{uf} = \beta = 0{\cdot}9(1 - x) \tag{27}$$

and

$$\gamma = 0{\cdot}9(1 - x)(1 + 2(1/\eta_f - 1)x) \quad \text{for } 0{\cdot}5 > x > 0 \tag{28}$$

$$\gamma = 0{\cdot}9(1 - x)(1/\eta_f) \quad \text{for } x \geqslant 0{\cdot}5 \tag{29}$$

Equations (28) and (29) are combined to obtain similar equations to Eqns (6) and (7) for further analysis.

3. Materials and methods

The proposed theory was applied for four crops of winter wheat (*Triticum aestivum* L. Adle cv.), spring barley (*Hordeum vulgare* L.), maize (*Zea mays* L.) and sorghum (*Sorghum dura* L.) at Bajgah, 16 km north of Shiraz (Fars province, I.R. of Iran) at 29°32′N and 52°35′ E, 1819 m elevation under an arid environment (Malek, 1981). The necessary data were summarised from our previous papers, and therefore, original sources of these data are not referred in here.

Crop water requirement and relative sensitivity to water stress at the various growth stages of winter wheat and spring barley (adapted from Ghahraman & Sepaskhah, 1997) are tabulated in Table 1. The relevant characteristics of maize and sorghum (Sepaskhah *et al.*, 2001) are presented in Tables 2 and 3, respectively. It is

Table 1
Crop water requirements and sensitivity indices at the various growth stages of winter wheat and spring barley at Bajgah (Fars province)

Growth stage	*Winter wheat*		*Spring barley*	
	Crop water requirement, $m^3\ ha^{-1}$	*Relative sensitivity to water stress*	*Crop water requirement, $m^3\ ha^{-1}$*	*Relative sensitivity to water stress*
Planting				
	102·0	0·00	63·4	0·00
Emergence				
	104·4	0·12	111·2	0·12
Tillering				
	591·6	0·15	215·5	0·15
Stem elongation				
	598·2	2·10	156·6	1·50
Heading				
	811·8	0·33	472·1	0·40
Milking				
	1330·0	0·20	1016·7	0·14
Maturity				

Table 2
Some characteristics of maize at Bajgah

Physiological stage	*Relative sensitivity to water stress*	*Crop potential evapotranspiration, mm*
Establishment	0·01	71·4
Early vegetation	1·42	248·1
Late vegetation	5·81	178·7
Flowering	0·55	145·0
Yield formation	0·45	270·9
Ripening	0·20	23·1

Table 3
Some characteristics of sorghum at Bajgah

Physiological stage	*Yield response factor to water*	*Crop potential evapotranspiration, mm*
Establishment	0·01	114·1
Vegetation	0·25	189·0
Flowering	0·55	145·0
Yield formation	0·45	270·9
Ripening	0·20	23·1

important to mention that relative sensitivities to water stress of wheat, barley, and maize [based on Jensen's Eqn (3)] were measured directly in the field, but there is no any local field measured relative sensitivities to water stress for sorghum. Therefore, they were considered from Doorenbos and Kassam (1979), and hence Eqn (9) was utilised for the analysis.

4. Results and discussion

4.1. *General behaviour of model variables*

Based on Eqn (16), there exist a non-linear relationship between actual and nominal water reduction levels (x and x', respectively). A portrayal of this relationship at different values of η_f may be found in *Fig. 1*. In general, at $\eta_f = 1$, $x = x'$ (straight line), but as η_f decreases $x : x'$ relationship deviates more from this straight line and becomes more curvilinear with $x > x'$.

While $\eta_d : x'$ and $\alpha : x'$ relationships (*Fig. 2*) are linear [$\eta_d = \eta_f + 2x'(1 - \eta_f)$ and Eqn (5), respectively], $\eta_d : x$ and $\alpha : x$ (*Fig. 3*) are non-linear in nature, which is because of non-linearity in $x : x'$ [Eqn (16)]. Both *Figs 2 and 3* depict the dependency of deficit irrigation on irrigation efficiency.

4.2. *Effects of irrigation efficiency on deficit irrigation*

As irrigation efficiency under full irrigation η_f changes, actual and nominal water reduction levels (x and x', respectively) deviate more among each other (*Fig. 1*). Therefore, although the relationship of relative yield-nominal water reduction level ($[Y_a/Y_p] : x'$) does not depend on η_f, the relationship between Y_a/Y_p and x does depend on η_f (*Fig. 4*). However, such a dependency is not constant for different crops (*Fig. 4*). The

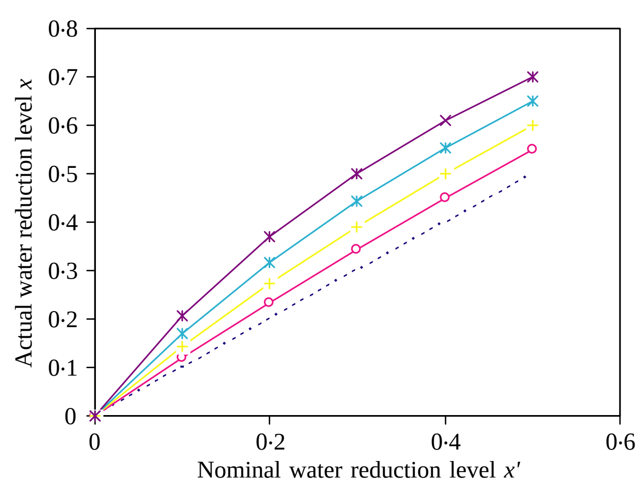

Fig. 1. Mutual relationships between actual and nominal water reduction levels for various values of the full irrigation efficiency (ηf) ▪▪▪▪▪, $\eta_f = 1{\cdot}0$; —○—, $\eta_f = 0{\cdot}9$; —+—, $\eta_f = 0{\cdot}8$; —✳—, $\eta_f = 0{\cdot}7$; —✕—, $\eta_f = 0{\cdot}6$

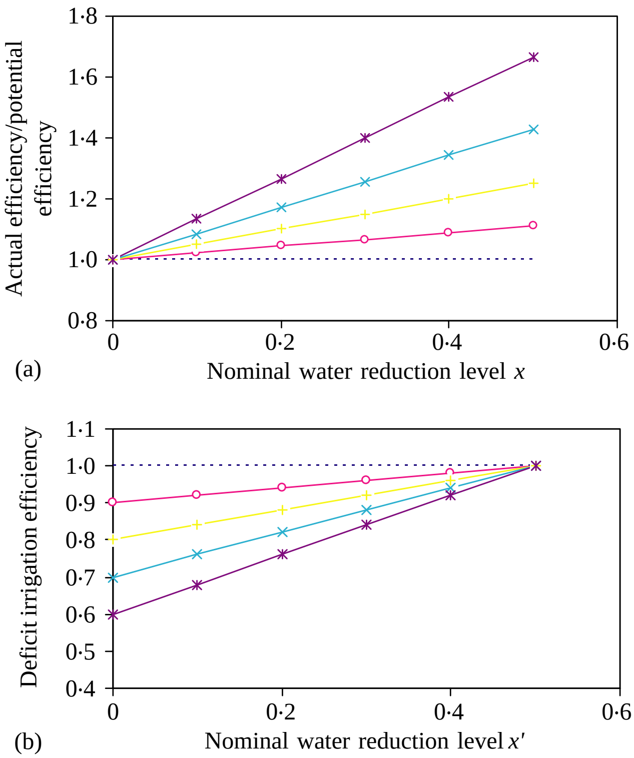

Fig. 2. (a) Variation in the ratio of deficit and full irrigation efficiency (η_d/η_f) and (b) deficit irrigation efficiency (η_d) with nominal water reduction (x'): ▪▪▪▪▪, $\eta_f = 1{\cdot}0$; —○—, $\eta_f = 0{\cdot}9$; —+—, $\eta_f = 0{\cdot}8$; —×—, $\eta_f = 0{\cdot}7$; —✳—, $\eta_f = 0{\cdot}6$

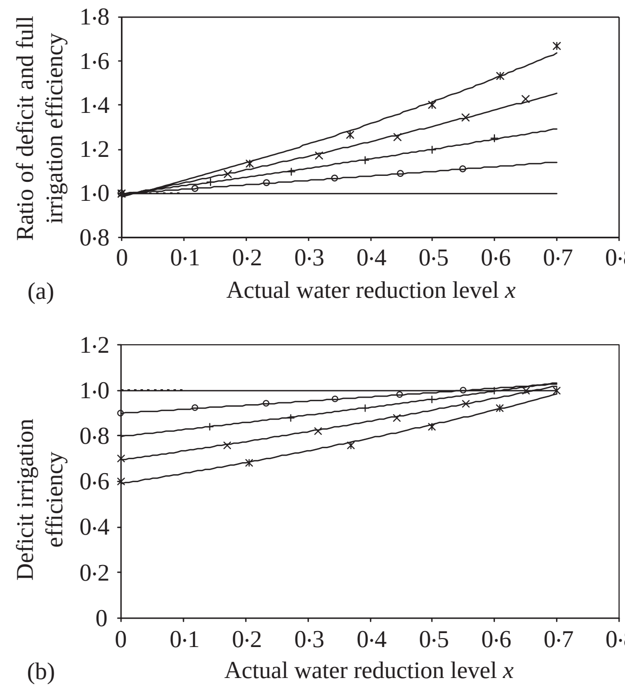

Fig. 3. (a) Variation in the ratio of deficit and full irrigation efficiency (η_d/η_f) and (b) deficit irrigation efficiency (η_f) with actual water reduction (x): —·—·—, $\eta_f = 1{\cdot}0$; ▪▪▪▪▪, $\eta_f = 0{\cdot}9$; —○—, $\eta_f = 0{\cdot}8$; —+—, $\eta_f = 0{\cdot}7$; —×—, $\eta_f = 0{\cdot}6$

differences are probably due to different relative sensitivities to water stress for different crops.

Curves in *Fig. 4* are extensions to only one curve as it was shown in our previous papers (Ghahraman & Sepaskhah, 1997; Ghahraman *et al.*, 2001). In fact specific curves corresponding to a value for η_f of 1 are identical to the previous published ones. The main contribution of *Fig. 4* is that it clearly shows the effects of irrigation efficiency on relative yield. Table 4 shows a more quantitative comparison. The necessary actual water reduction levels for two different relative yields of 99 and 95% and for different crops and irrigation efficiencies η_f are summarised in this table. Meanwhile, on the average, the limit for x below which reduction in Y_a/Y_p is more pronounced is highly dependent on η_f. For example, at a value for η_f of 0·6, the value of x is about 40% while the other η_f values, except for η_f of 1, show relevant x values.

That the curves in *Fig. 4* have increasing trends with decreasing η_f is interesting. Such a finding may explain the high Y_a/Y_p under utilisation of relative sensitivities to water stress of Nairizi and Rydzewski (1977), as opposed to field measured relative sensitivities (*cf.*, with *Figs 1* and *4* of Ghahraman & Sepaskhah, 1997). This indicated that Nairizi and Rydzewski (1977) might have probably gathered their data from experiments with low irrigation efficiency.

Differentiating x and x', has provided a media to analyse relative yield per unit of applied water, as was originally proposed by Ghahraman and Sepaskhah (1996), in more details. *Figure 5* depicts the relationship between $\{(Y_a/Y_p)/(1-x)\}$ and x, as opposed to the relationship between $\{(Y_a/Y_p)/(1-x')\}$ and x' of Ghahraman and Sepaskhah (1996), for four crops under study. The lower curve in each subplot (*i.e.* $\eta_f = 1$) is identical to that proposed by Ghahraman and Sepaskhah (1996). In general, the curve trend is similar to that in *Fig. 4*, although an ever-increasing trend is observed for some crops and low η_f values. The incomplete curves are due to restrictive constraint of Eqn (12). The x' is not allowed to be greater than 0·5, and therefore, x has also an upper band, though variable.

As was asserted in Eqn (18), relative net benefit is dependent on B/C, x, and therefore on η_f. *Figures 6–9* show this dependency. The curves corresponding to a value for η_f of 1 are not included in this paper, where they can be found elsewhere (for wheat

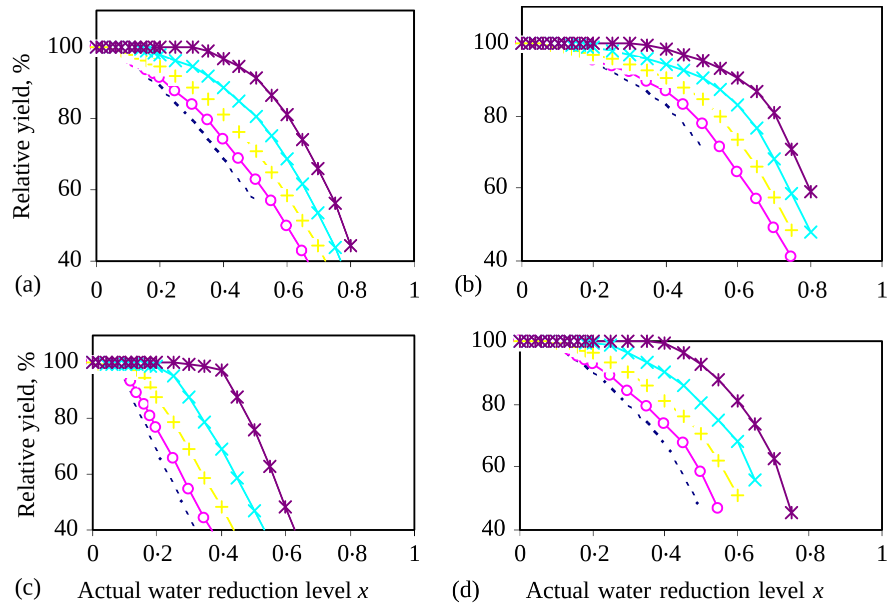

Fig. 4. Relative yield as a function of the actual water reduction level (x) and full irrigation efficiency for different crops, (a) wheat, (b) barley, (c) maize, (d) sorghum: η_f (▪▪▪▪▪, $\eta_f = 1{\cdot}0$; —○—, $\eta_f = 0{\cdot}9$; —×—, $\eta_f = 0{\cdot}8$; —+—, $\eta_f = 0{\cdot}7$; ——, $\eta_f = 0{\cdot}6$*

Table 4
Actual water reduction x obtained for 99 and 95% relative yield Y_a/Y_p* for different crops and irrigation efficiencies η_f

		Water reduction, decimal				
		Irrigation efficiency				
Crop	*Relative yield*	*1·0*	*0·9*	*0·8*	*0·7*	*0·6*
Wheat	0·99	0·040	0·051	0·075	0·149	0·336
	0·95	0·108	0·134	0·183	0·252	0·435
Barley	0·99	0·053	0·069	0·100	0·182	0·362
	0·95	0·175	0·212	0·274	0·376	0·508
Maize	0·99	0·053	0·067	0·098	0·181	0·355
	0·95	0·089	0·112	0·157	0·253	0·412
Sorghum	0·99	0·083	0·104	0·145	0·228	0·404
	0·95	0·135	0·165	0·220	0·321	0·466
Mean	0·99	0·057	0·073	0·105	0·185	0·364
	0·95	0·127	0·156	0·208	0·300	0·455

* Y_a and Y_p, are actual and potential yields, respectively.

and barley from Ghahraman & Sepaskhah, 1997; and for maize from Ghahraman *et al.*, 2001). The values of B/C about 2 and 1·5 reflect the high and low economic feasibility for crop production. After a given value of x, the value of net benefit decreases with decrease in values of B/C. It is interesting to note that as η_f decreases, both the allowable water reduction x, and relative net benefit Z, increase accordingly which brings about a more economic farm irrigation treatment.

4.3. *Combined effects of irrigation efficiency and uniformity coefficient on deficit irrigation*

The relationship between relative yield and actual water reduction depends directly on η_f and indirectly on C_{ud} which is shown in *Fig. 10* for different crops. However, such a dependency is not constant for different crops. Again, on average, the limit for x below which reduction in relative yield is more pronounced is highly dependent on η_f. Furthermore,

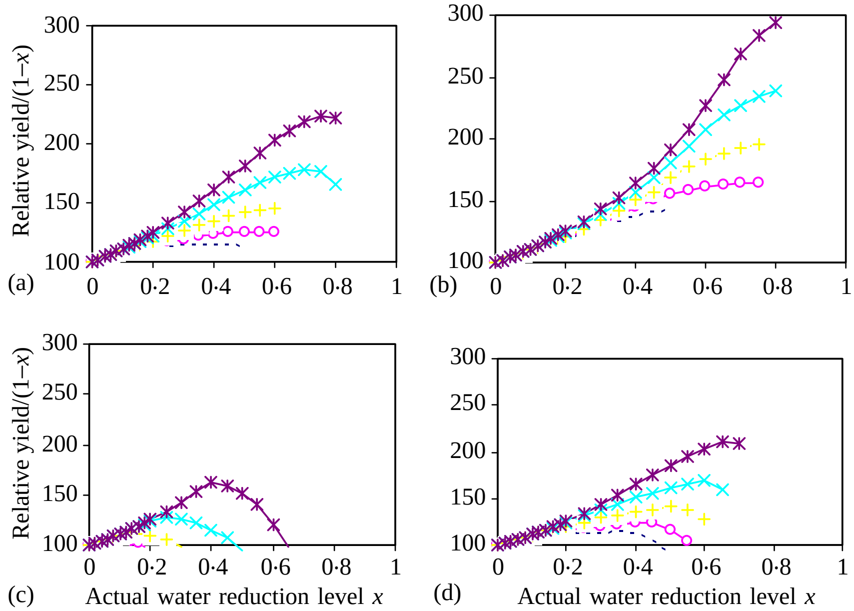

Fig. 5. Relative yield per unit of applied water as a function of the actual water reduction level (x) and the full irrigation efficiency (η_f) for different crops, (a) wheat, (b) barley, (c) maize, (d) sorghum: ▪▪▪▪▪, $\eta_f = 1{\cdot}0$; —○—, $\eta_f = 0{\cdot}9$; —×—, $\eta_f = 0{\cdot}8$; —+—, $\eta_f = 0{\cdot}7$; ——, $\eta_f = 0{\cdot}6$*

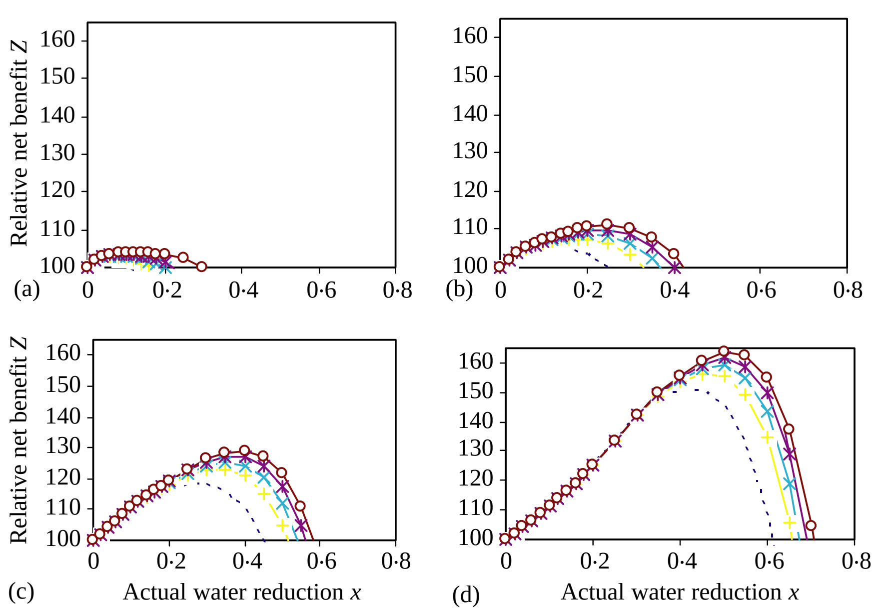

Fig. 6. Relative net benefit of winter wheat as influenced by the actual water reduction (x), the full irrigation efficiency (η_f) and the gross revenue to gross expenditure ratio (B/C): (a) $\eta_f = 0{\cdot}9$; (b) $\eta_f = 0{\cdot}8$; (c) $\eta_f = 0{\cdot}7$; (d) $\eta_f = 0{\cdot}6$; ▪▪▪▪▪, B/C = 1·5; —×—, B/C = 1·7; —+—, B/C = 1·8; ——, B/C = 1·9; —○—, B/C = 2·0*

relative yields were, in general, smaller than those obtained in the analysis where only irrigation efficiency was used (*Fig. 4*). For some crops in *Fig. 10*, the relative yields were somewhat lower than 100%. This is due to the fact that at no water reduction the distribution coefficient was considered about 90% and this caused some reduction in the relative yield.

The relative net benefit is dependent on x, and η_f (*Figs 11–14*). Again, as η_f decreases, both the

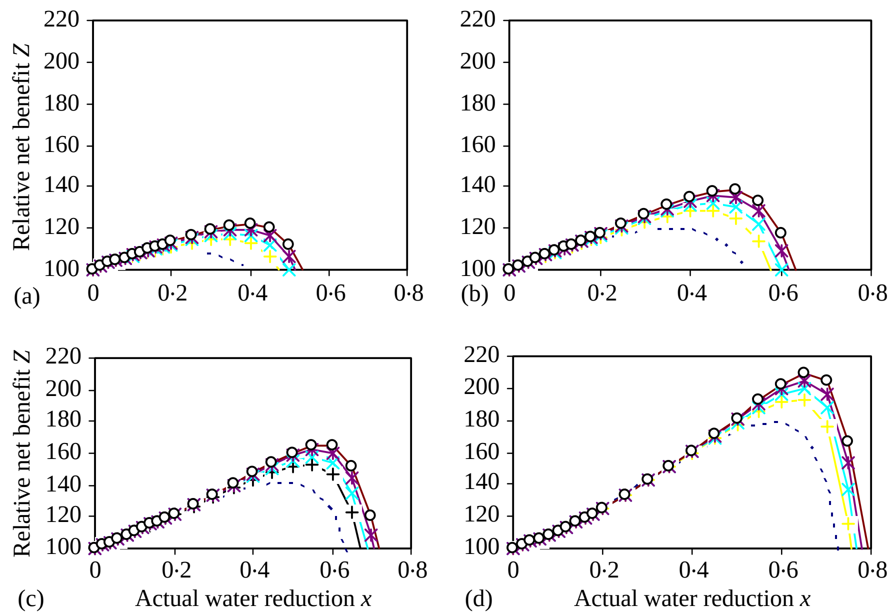

Fig. 7. Relative net benefit of spring barley as influenced by the actual water reduction (x), the full irrigation efficiency (η_f) and the gross revenue to gross expenditure ratio (B/C): (a) $\eta_f = 0{\cdot}9$; (b) $\eta_f = 0{\cdot}8$; (c) $\eta_f = 0{\cdot}7$; (d) $\eta_f = 0{\cdot}6$; ▪▪▪▪▪, B/C = 1·5; —×—, B/C = 1·7; —+—, B/C = 1·8; ——, B/C = 1·9; —○—, B/C = 2·0*

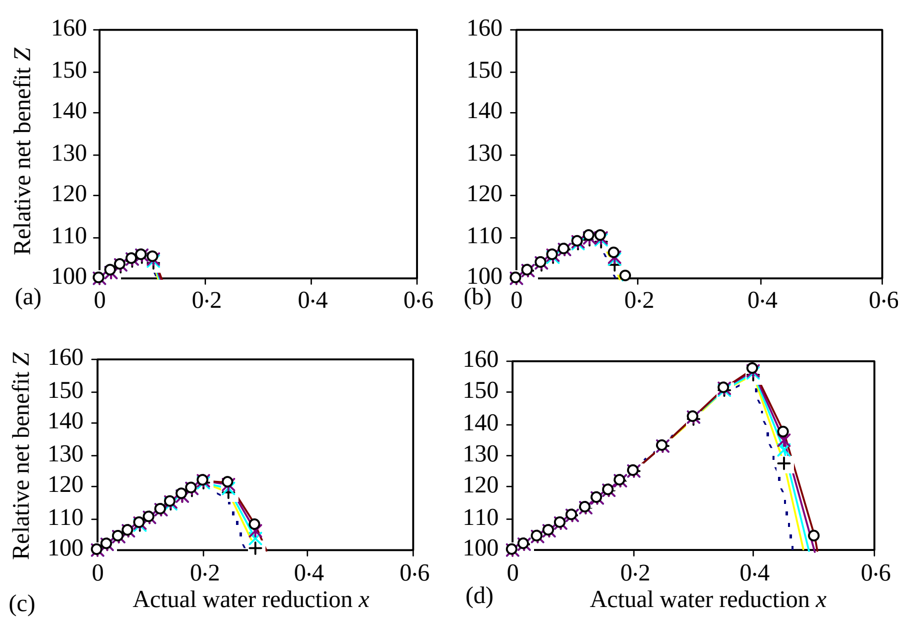

Fig. 8. Relative net benefit of maize as influenced by the actual water reduction (x), the full irrigation efficiency (η_f) and the gross revenue to gross expenditure ratio (B/C): (a) $\eta_f = 0{\cdot}9$; (b) $\eta_f = 0{\cdot}8$; (c) $\eta_f = 0{\cdot}7$; (d) $\eta_f = 0{\cdot}6$; ▪▪▪▪▪, B/C = 1·5; —×—, B/C = 1·7; —+—, B/C = 1·8; ——, B/C = 1·9; —○—, B/C = 2·0*

allowable water reduction x, and relative net benefit Z, increase accordingly which brings about a more economic farm irrigation treatment. However, in analysis with combined effects of irrigation efficiency and uniformity coefficient on deficit irrigation, the effects of η_f on relative net benefit for some crops were economically feasible. Water reductions for sorghum and barley were found to be economically

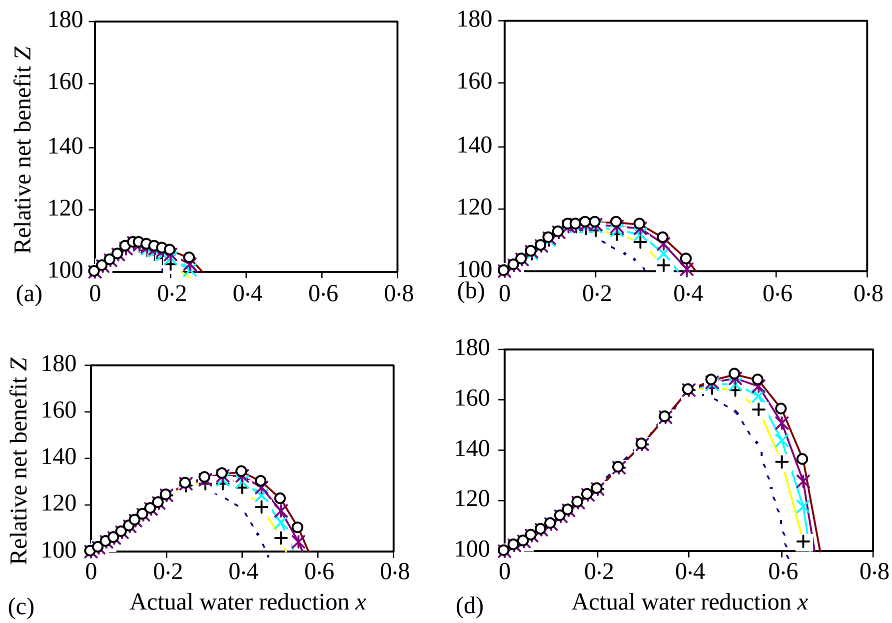

Fig. 9. Relative net benefit of sorghum as influenced by the actual water reduction (x), the full irrigation efficiency (η_f) and the gross revenue to gross expenditure ratio (B/C): (a) $\eta_f = 0.9$; (b) $\eta_f = 0{\cdot}8$; (c) $\eta_f = 0{\cdot}7$; (d) $\eta_f = 0{\cdot}6$; ....., B/C = 1·5; —×—, B/C = 1·7; —+—, B/C = 1·8; ——, B/C = 1·9; —×—, B/C = 2·0*

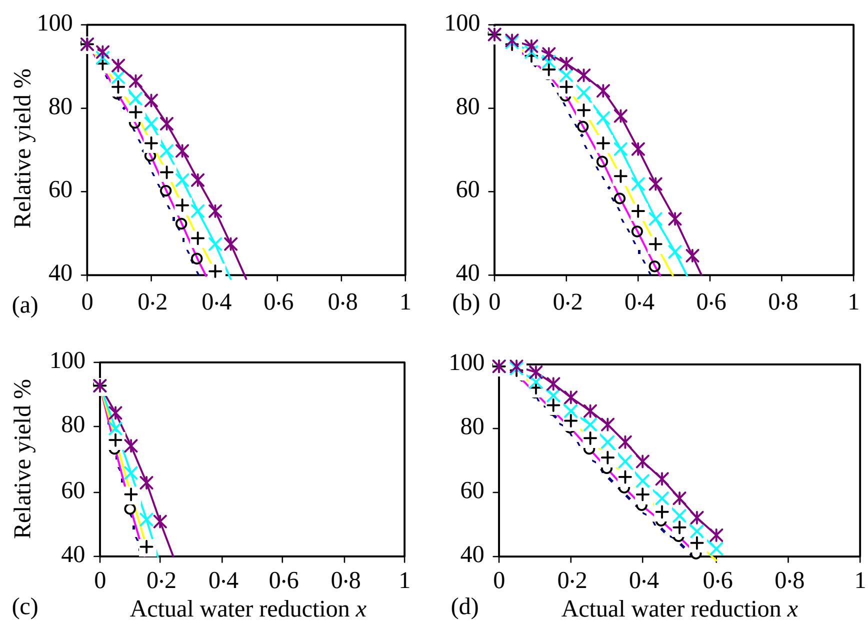

Fig. 10. Relative yield as a function of actual water reduction with combined effects of uniformity coefficient and irrigation efficiency (η_f) for different crops: (a) wheat; (b) barley; (c) maize; (d) sorghum; ....., $\eta_f = 1{\cdot}0$; —○—, $\eta_f = 0{\cdot}9$; —×—, $\eta_f = 0{\cdot}8$; —×—, $\eta_f = 0{\cdot}7$; ——, $\eta_f = 0{\cdot}6$*

feasible at values of η_f smaller than 1·0; water reduction for maize is not economically feasible at all. Water reduction for wheat is economically feasible at η_f values of 0·6 or smaller, especially at higher values of B/C ratios. Therefore, it may be concluded that combined effects of irrigation efficiency and uniformity coefficient on deficit irrigation are more realistic.

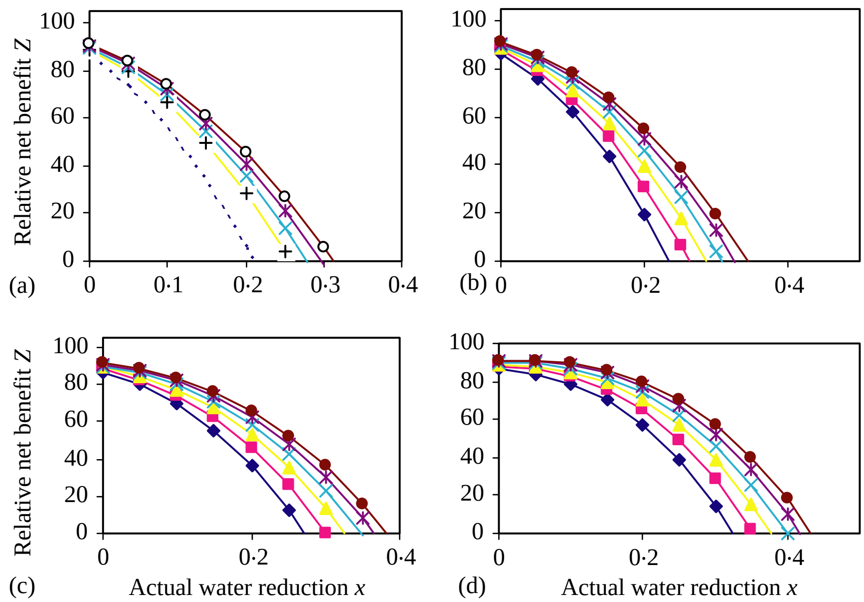

Fig. 11. Relative net benefit of wheat as influenced by actual water reduction (x) with combined effects of uniformity coefficient and full irrigation efficiency (η_f) and the gross revenue to gross expenditure ratio (B/C): (a) $\eta_f = 0{\cdot}9$; (b) $\eta_f = 0{\cdot}8$; (c) $\eta_f = 0{\cdot}7$; (d) $\eta_f = 0{\cdot}6$; ▪▪▪▪▪, B/C = 1·5; —×—, B/C = 1·7; —+—, B/C = 1·8; ——, B/C = 1·9; —○—, B/C = 2·0*

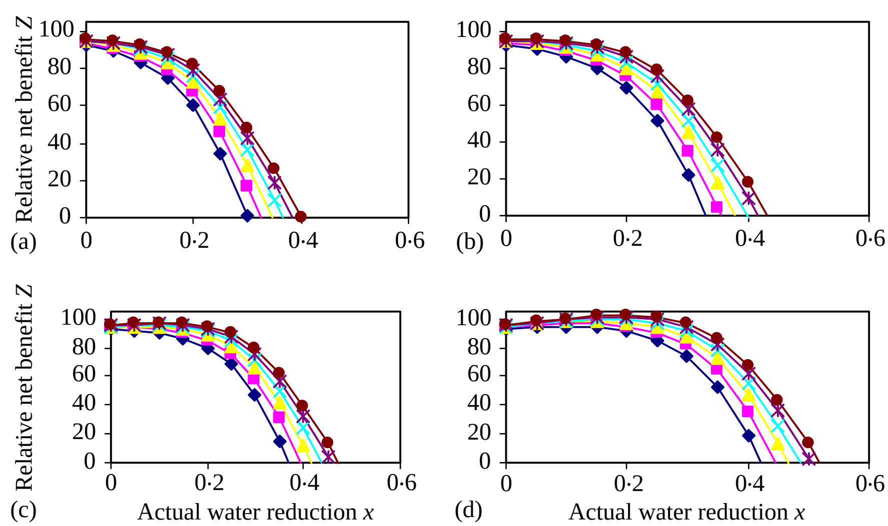

Fig. 12. Relative net benefit of barley as influenced by actual water reduction (x) with combined effects of uniformity coefficient and full irrigation efficiency (η_f) and the gross revenue to gross expenditure ratio (B/C): (a) $\eta_f = 0{\cdot}9$; (b) $\eta_f = 0{\cdot}8$; (c) $\eta_f = 0{\cdot}7$; (d) $\eta_f = 0{\cdot}6$; ▪▪▪▪▪, B/C = 1·5; —×—, B/C = 1·7; —+—, B/C = 1·8; ——, B/C = 1·9; —○—, B/C = 2·0*

5. Conclusion

Incorporating irrigation efficiency in a deficit irrigation analysis makes more sound, understandable and defendable results. For low irrigation efficiency under full irrigation (*i.e.* η_f), more water reduction level is allowed to make benefit from the irrigation water loss for crop water use. This may increase the irrigation efficiency under deficit irrigation (*i.e.* η_d). On the other hand, by no consideration of field irrigation efficiency, it leads to underestimating the allowable water reduction level, and hence low farm net benefit.

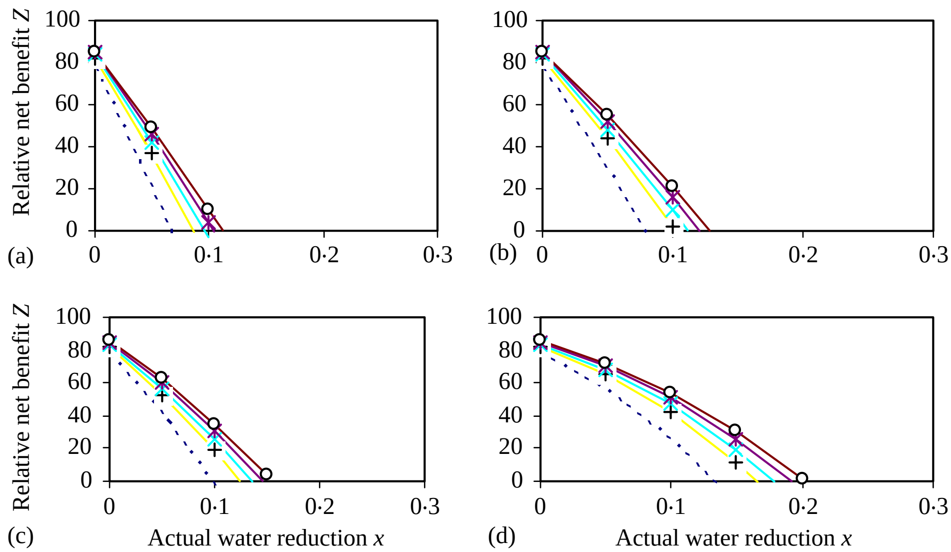

Fig. 13. Relative net benefit of maize as influenced by actual water reduction (x) with combined effects of uniformity coefficient and full irrigation efficiency (η_f) and the gross revenue to gross expenditure ratio (B/C): (a) $\eta_f = 0{\cdot}9$; (b) $\eta_f = 0{\cdot}8$; (c) $\eta_f = 0{\cdot}7$; (d) $\eta_f = 0{\cdot}6$; ▪▪▪▪▪, B/C = 1·5; —×—, B/C = 1·7; —+—, B/C = 1·8; ——, B/C = 1·9; —○—, B/C = 2·0*

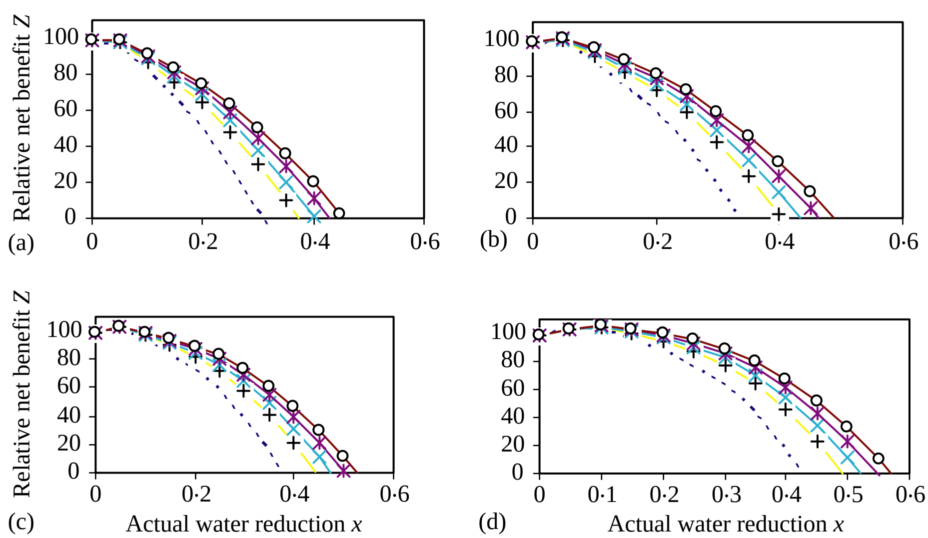

Fig. 14. Relative net benefit of sorghum as influenced by actual water reduction (x) with combined effects of uniformity coefficient and full irrigation efficiency (η_f) and the gross revenue to gross expenditure ratio (B/C): (a) $\eta_f = 0{\cdot}9$; (b) $\eta_f = 0{\cdot}8$; (c) $\eta_f = 0{\cdot}7$; (d) $\eta_f = 0{\cdot}6$; ▪▪▪▪▪, B/C = 1·5; —×—, B/C = 1·7; —+—, B/C = 1·8; ——, B/C = 1·9; —○—, B/C = 2·0*

It is concluded that relative yields in analysis with combined effects of irrigation efficiency and uniform coefficient were lower than those obtained in the analysis where only irrigation efficiency was used. By combined analysis, water reductions for sorghum and barley found to be economically feasible at values of η_f smaller than 1·0, while water reduction for maize was not economically feasible at all irrigation efficiencies. Water reduction for wheat was economically feasible at η_f values of 0·6 or smaller, especially at higher values of the gross cost and revenue ratio. Therefore, combined effects of irrigation efficiency and uniformity coefficient on deficit irrigation are more realistic. Furthermore, considerable reduction in applied water and not significant decrease in yield may result in higher water use efficiency, especially for sorghum, barley and wheat crops.

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