Stress Analysis Around Crack Tip under Elasto-Plasto-Creep Condition Using Element-Free Galerkin Method

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Abstract

In this paper Element Free Galerkin Method is used and applied to the stress analysis problems where material has a tendency towards both elastoplastic and creep behavior. In doing so, following a brief description of the nonlinear constitutive formulation of elastoplastic and creep, a new technique for the numerical analysis of nonlinear problems has been constructed. The method has been examined in two different plates with and without a crack. The value of C*-integral has been used as a base for comparison of the creep results. A rather close agreement is seen between results of this work and the others.

Key words: Element Free Galerkin Method; Elastoplastic, Creep; Crack tip stress analysis; Plane stress

Introduction

Because of its advantages and applications, different aspects of the well-known method of Finite Elements have been expanded in the last half a century. One of the main steps in FEM is to discretize the domain into a number of finite elements, which is called mesh generation phase. The other important task in this technique is to use a piecewise approximate solution for sub-domains, which does not ensure the continuity of the derivatives throughout the domain. In the last decades different mesh-less methods were introduced. Basically in these methods not only there is no need to discretize problem geometry into finite elements but also the continuity of the derivatives of the approximate solution is ensured. Element Free Galerkin Method is one of these methods, which introduced for the first time by Belytschko et al [1] in 1994. The paper of Nayroles et al [2] is a close work prior to the former one and this work by itself seems to be inspired by another work, which is in the area of Moving Least Square interpolants (MLS) [3].

After introducing of the EFGM, this method has been used in a wide range of different subjects such as dynamic fracture [4,5], crack growth [6,7], elastic plates and shells [8], 3-D problems [9,10] and non-elastic stress analysis [11]. Since then the theoretical foundations of this method have been reconsidered, developed or revised for several times [12-16].

In this paper a new mixed method of Elastic-Plastic-Creep Element Free Galerkin Method (EPC-EFGM) is constructed. The composed method is intrinsically based on nonlinear relations; therefore a section is devoted to present its solution algorithm. The solution requirements for two solved examples and comparison of their results with other similar works have been proposed in another section.

The EFG method

Similar to FEM, the first step in EFGM modeling is to assume an appropriate mathematical form for

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The EFG method

Similar to FEM, the first step in EFGM modeling is to assume an appropriate mathematical form for the approximate function. Once an approximate function is assumed the shape functions can be calculated. It should be pointed out that the basic difference between the EFGM and FEM is that in EFGM, the approximate function is an approximation through the nodal values while in FEM it is an interpolation between nodal values. Where, the approximation and interpolation have the same explanation as, for example, in [١٧].

Calculated shape functions should be inserted in the integral forms of the governing equations of the system and convert them to a new system of nonlinear equations. Solving this system of equations results in some primary unknowns, which in solid mechanics, are generally displacement field components.

Moving Least Square interpolants (MLS)

To construct the matrix form of a variational principle in terms of the scalar function U(**x**), an approximate function u(**x**) must be chosen such that,

$$u = \varphi^{T}\,\delta \qquad (١)$$

in which δ is the vector of nodal unknown parameters and φ(**x**) is the vector of shape functions for n different number of nodes.

As it is presumed, approximate function will not pass through all base point magnitudes. Hence, to obtain the best approximation, a criterion should be chosen and then minimized. In MLS this criteria is as follows [١]:

$$R = W_I\,[\,u(\mathbf{x}_I) - \delta_I\,]^{٢} \qquad (٢)$$

in which, $W_I = W(\mathbf{x} - \mathbf{x}_I)$ is the weight function and $u(\mathbf{x}_I)$ is the magnitude of approximate scalar function in point $\mathbf{x}_I$. The approximate function is chosen as,

$$u = \mathbf{p}^{T}\,\mathbf{c} \qquad (٣)$$

where **c** is a vector with unknown variable coefficients, which ought to be calculated, and **p** is called the base vector. In this paper the base vector is chosen as, $\mathbf{p}^{T} = \lfloor ١,\ x,\ y \rfloor$. Some other kinds of base vectors are introduced in [١٢].

Inserting (٣) in (٢), and forcing R to be stationary with respect to **c** leads to the following linear relation between **c** and δ.

$$\mathbf{A}\ \mathbf{c} = \mathbf{B}\ \delta \qquad (٤)$$

Where, **A** and **B** are matrices as follows,

$$\mathbf{A} = W_I\ \boldsymbol{p}_I\ \boldsymbol{p}_I^{T} \qquad (٥a)$$

$$\mathbf{B} = [\mathbf{b}_١ ,..., \mathbf{b}_I\ ,..., \mathbf{b}_n\ ] \qquad (٥b)$$

in which

$$\mathbf{b}_I = W_I\ \boldsymbol{p}_I \qquad (٦)$$

and $\boldsymbol{p}_I$ is the magnitude of base vector **p** in nodal point $\boldsymbol{x}_I$. After solving (٤) for **c**, back substituting in (٣) and combining the result with (١) we obtain,

$$\varphi_I = \mathbf{p}^T \mathbf{A}^{-1} \mathbf{b}_I \qquad (7)$$

Usually a bell type function is chosen as weight function and the resulting shape function is also nearly a bell type function. As shown in [4,5], exponential type of weight function causes more accurate solution. Therefore, in this paper, this type of weight function is used. i.e.,

$$W(\rho) = \frac{e^{(\beta.\rho)^2} - e^{-\beta^2}}{1 - e^{-\beta^2}} \qquad (8)$$

where, in this relation $\rho = r / r_m$ *is* the dimensionless radius, r is the radial distance, r_m is the radius of the support domain for the weight function and β is a parameter that controls the bell-type shape of weight function and in this work has the value of 4.

Discretized variational formulation

In the field of solid mechanics the equilibrium equation for a continuous media under small displacements is given as,

$$\nabla.\,\boldsymbol{\sigma} + \mathbf{v} = 0 \quad \text{in } \Omega \qquad (9)$$

with essential and natural boundary conditions as follows.

$$\mathbf{u} = \bar{\mathbf{u}} \quad \text{on } \Gamma_u \qquad (10a)$$

$$\boldsymbol{\sigma}.\mathbf{n} = \bar{\mathbf{t}} \quad \text{on } \Gamma_t \qquad (10b)$$

In these relations, $\boldsymbol{\sigma}$ is the stress tensor, $\mathbf{v}$ is the body force vector, $\mathbf{u}$ is the displacement vector, ∇ is the gradient operator, $\mathbf{t}$ is the traction force and $\mathbf{n}$ is the unit normal vector to the boundary. Also the superscripted bar denotes a prescribed boundary value.

As in [6], the weak form of equilibrium equation is given as follows,

$$\int \delta(\nabla_s \mathbf{u}) : \boldsymbol{\sigma}\, d\Omega - \int \delta\mathbf{u} \,.\, \mathbf{b}\, d\Omega - \int \delta\mathbf{u} \,.\, \bar{\mathbf{t}}\, d\Gamma_t - \int \delta\mathbf{l} \,.\, (\mathbf{u} - \bar{\mathbf{u}})\, d\Gamma_u - \int \delta\mathbf{u} \,.\, \mathbf{l}\, d\Gamma_u = 0 \qquad (11)$$

In this relation $\nabla_s\mathbf{u}$ is the symmetric part of $\nabla\mathbf{u}$ term. Double dot product (**:**) represents the dyadic scalar product and **l** is the vector of Lagrange multipliers. **l** and $\delta\mathbf{l}$ belong to Sobolov space of degree 0 whereas **u** and $\delta\mathbf{u}$ belong to Sobolov space of degree 1.

Note that in this method in the absence of Lagrange multipliers, it will be impossible to obtain a solution, which can satisfy essential boundary conditions.

To discretize the variational formulation in Eq. (11) we substitute $\mathbf{u} = \boldsymbol{\varphi}^T \delta$ and $\mathbf{l} = \mathbf{N}^T \lambda$ in which **N** is the vector of boundary node shape functions and λ is the vector of discritized boundary node Lagrange multipliers. Having in mind that the relation between strain and small displacement is $\boldsymbol{\varepsilon} = \nabla_s \mathbf{u}$ and constitutive relation is $\boldsymbol{\sigma} = \mathbf{D}\, \boldsymbol{\varepsilon}$ we obtain,

$$\begin{bmatrix} K & G \\ G^T & 0 \end{bmatrix} \begin{Bmatrix} \delta \\ \lambda \end{Bmatrix} = \begin{Bmatrix} f \\ q \end{Bmatrix} \qquad (12)$$

in which

$$K_{IJ} = \int \mathbf{E}_I^T \mathbf{D}\, \mathbf{E}_J\, d\Omega \qquad (13a)$$

$$G_{IK} = - \int \varphi_I\, N_K\, d\Gamma_u \qquad (13b)$$

$$f_I = \int \varphi_I\, \bar{t}\, d\Gamma_t + \int \varphi_I\, v\, d\Omega \qquad (13c)$$

$$q_K = - \int N_K\, \bar{u}\, d\Gamma_u \qquad (13d)$$

For example in plane problems we have,

$$\mathbf{E}_I = \begin{bmatrix} \phi_{I,x} & 0 \\ 0 & \phi_{I,y} \\ \phi_{I,y} & \phi_{I,x} \end{bmatrix} \qquad (14a)$$

$$\mathbf{N}_K = \begin{bmatrix} N_K & 0 \\ 0 & N_K \end{bmatrix} \qquad (14b)$$

Besides, **D** is property matrix, which relates different components of stress and strain to each other.

Elastic-Plastic-Creep constitutive equation

In the last couple of decades different approaches for the modeling of incremental behavior of plastic deformation especially in the isotropic and homogeneous materials have been introduced. In one of these methods, elastoplastic stiffness matrix is obtained by modifying the elastic stiffness matrix. In this case the elastoplastic stiffness matrix $\mathbf{D}_{ep}$ can be represented as:

$$\mathbf{D}_{ep} = \mathbf{D}_e - \mathbf{D}_p \qquad (15)$$

where $\mathbf{D}_e$ is elastic stiffness matrix and following the formulations in [18, 19] the correction to elastic stiffness or $\mathbf{D}_p$ may be shown to be,

$$\mathbf{D}_p = \mathbf{s}\,\mathbf{s}^T / [(H' + 1.5\,E')\,(2\bar{\sigma} / 3E')^2] \qquad (16)$$

in equation (16) $\mathbf{s}^T = \lfloor s_x, s_y, s_z, s_{xy}, s_{yz}, s_{zx} \rfloor$ is a vector comprising of deviatoric stress components.

Furthermore H′ is the plastic modulus, E′ is the slop of stress-strain curve and $\bar{\sigma}$ is equivalent stress.

In plane stress conditions the out of plane components of stress are zero. As a result the dimensions of $\mathbf{D}_{ep}$ in Eq. (۱۵) can be reduced to ۳×۳, which relates planar stress and strain components.

In order to encounter the effect of creep on the formulation one may consider that creep behavior of material is a function of temperature, time and stress level [۱۸-۲۰]. Nevertheless, if a uniform distribution of temperature exists the following equation may be a proper relationship between the uni-axial stress and creep strain rate [۱۹-۲۲].

$$\dot{\varepsilon}_c = B.\sigma^{(m)} \qquad (۱۷)$$

In which B and m are two material constants. To extend this relation to a more general state of stress and strain, a failure criterion is needed. Assuming an isotropic material with von-Mises failure criteria and using the normality rule of flow the following relation holds [۱۹],

$$(\dot{\varepsilon}_c)_{ij} = s_{ij}\dot{\eta} \qquad (۱۸)$$

In which, $(\dot{\varepsilon}_c)_{ij}$ is the creep strain rate, s_{ij} is the component of deviatoric stress tensor and $\dot{\eta}$ is the proportionality constant. By substituting the components of strain rate in (۱۸) into the equivalent strain rate relation for the von-Mises criteria, $\dot{\bar{\varepsilon}}_c$, and extracting stress components in the form of von-Mises equivalent stress $\bar{\sigma}$, one would get:

$$\dot{\eta} = \frac{3}{2}\frac{\dot{\bar{\varepsilon}}_c}{\bar{\sigma}} \qquad (۱۹)$$

Creep strain - stress relation will be obtained by back substituting Eq. (۱۹) into the Eq. (۱۸). That is,

$$\Delta\varepsilon_c = \mathbf{s}\left(\frac{3}{2}\frac{\dot{\bar{\varepsilon}}_c}{\bar{\sigma}}\right)\Delta t \qquad (۲۰)$$

The resulted relation will be used in the variational or weighted form integral to obtain discritized form of field equations. To this end we note that,

$$d\varepsilon = d\varepsilon_{ep} + d\varepsilon_c \qquad (۲۱)$$

According to the definition of $\mathbf{D}_{ep}$ in Section (۳-۱) we have,

$$d\sigma = \mathbf{D}_{ep}(d\varepsilon - d\varepsilon_c) \qquad (۲۲)$$

Which upon its substitution into the Eq. (۱۱), quasi-nodal creep force is obtained as,

$$\Delta \boldsymbol{f}_c = \int \mathbf{E}\mathbf{D}_{ep}\Delta\varepsilon_c d\Omega \qquad (۲۳)$$

More details are given in the next section that devotes to the solution algorithm.

The EPC-EFGM solution algorithm

If in Equation (۱۲), any non-incremental vector components such as total auxiliary nodal displacement vector is replaced by its respective incremental form and furthermore the property matrix is replaced by elastoplastic property matrix, then a new set of equations will be obtained which describes incremental elastoplastic behavior. The incremental form of Eq. (۱۲), is

$$\begin{bmatrix} \boldsymbol{K}_{ep} & \boldsymbol{G} \\ \boldsymbol{G}^{T} & \boldsymbol{0} \end{bmatrix} \begin{Bmatrix} \Delta\boldsymbol{\delta} \\ \Delta\boldsymbol{\lambda} \end{Bmatrix} = \begin{Bmatrix} \Delta\boldsymbol{f} \\ \Delta\boldsymbol{q} \end{Bmatrix} \qquad (۲٤)$$

Adding the creep incremental nodal force, $\Delta\boldsymbol{f}_c$, in (۲۳) to the incremental nodal force in Eq. (۲٤), $\Delta\boldsymbol{f}_{ep}$, results in the total incremental nodal force, $\Delta\boldsymbol{f}_{epc}$. It should be noted that contrary to the creep phenomenon, the elastoplastic phenomenon is almost an instantaneous process. Hence, due to the difference in the nature of elastoplastic and creep generated incremental forces the method of their application is different.

Equation (۲٤) can be shown in more compact form as,

$$\boldsymbol{S}_{ep}\,\boldsymbol{\delta} = \boldsymbol{f}_{epc} \qquad (۲۵)$$

For the sake of brevity, $\boldsymbol{S}_{ep}$ is called as the stiffness matrix, $\boldsymbol{\delta}$ as the Auxiliary Nodal Displacement vector (AND) and $\boldsymbol{f}_{epc}$ as the force vector which includes both elastoplastic and creep nodal force components. In comparison with Eq. (۱۲), the force and displacement components in Eq. (۲۵) are incremental.

Apart from incremental behavior of Eq. (۲۵), there is still another difference between this equation and Eq. (۱۲). Note that in Eq. (۲۵) the behavior of incremental stiffness matrix is nonlinear, whereas the stiffness matrix $\boldsymbol{S}_{ep}$, which is used to obtain displacement field, depends on material properties. In other words it can easily be verified that the elastoplastic material property matrix $\mathbf{D}_{ep}$ indirectly depends on displacement field. So, in order to obtain the unknown $\Delta\delta$ in Eq. (٢٥) a nonlinear solution technique should be chosen.

The flow chart in Fig. ١ shows briefly this process of solution.

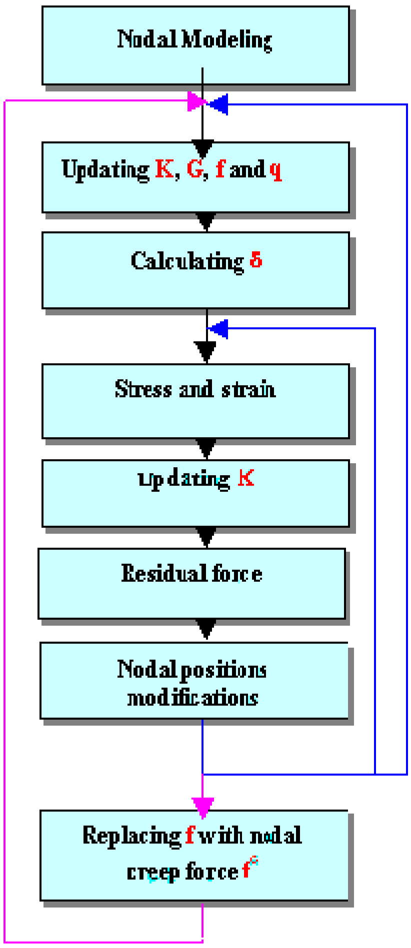

Fig. ١: A simplified flow chart of the nonlinear solution algorithm applied in EPC-EFGM

Numerical solution

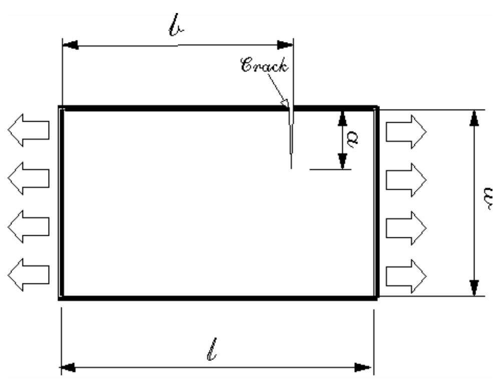

Fig. ٢: A thin cracked plate under planar loading with free upper and lower boundaries and similar loading conditions in two other sides

Based on the outlined method in the previous sections and a self-developed computer code, one can conduct a planar elastoplastic and creep stress analysis in a sheet with or without a simple edge crack. Fig. ٢ illustrates a plate considered for stress analysis using our code. To make sure that the method is working properly, the solution of two well known problems are compared with their respective bench marking analytical solutions in the literature. Both examples devoted to the stress analysis in a thin rectangular plate. The simply supported plate is under uniform tensile traction in each side. In the first example it is assumed that the plate has no crack and should carry a uniform tensile state of stress in its entire field up to a level in the plastic range followed by creep relaxation. In the second example it is assumed that the plate has a middle edge crack and carries a uniform tensile traction over end boundaries (See Fig. ٢).

In the following the detailed solution of two benchmarking problems using our methods are described. In both examples, assumptions are:

- Material is isotropic and homogeneous. Hook's law prevails in the elastic range. Elastic modulus is ٢٠٠ GPa and Poisson's ratio is equal to ٠,٣.
- In elastoplastic region, material yields according to von-Misses criteria and also normality rule prevails.
- Imposed boundary traction and / or extensions and induced deformation are all planar.

Example ١: Uni-axial creep problem

Consider a ٤٠٠×٨٠٠mm rectangular simply supported thin plate under a uniform in-plane tensile traction, which is made up of isotropic and homogeneous material. In addition, assume that material behavior is elastic linearly plastic strain hardening and includes creep deformation. Especially take B and m constants in Eq. (١٧) to be $١٠^{-٢٠}$ and ١٠, respectively.

As it is mentioned before the long-term creep begins just after an instantaneous elastoplastic deformation and based on the defined geometry and loading, it is obvious that the creep strain rate has no reliance on the prior elastoplastic deformations. On the other hand, strain rate is related only to the creep constant parameters indicated in Eq. (١٧) and not on the Elastic and plastic parameters of the body. If a uniform traction of ١٠٠MPa is applied in two opposite sides of the plate, it is clear that the resulted tensile stress should be ١٠٠MPa throughout the plate and based on the assumed creep constants, creep strain rate should be equal to ١ m/m-h.

Obtained results confirm the validity of the method in calculating of stress and strain rate distribution throughout the domain. Furthermore, the results show that as it was expected, in this example elastic and plastic constants of material properties have no apparent effect upon the obtained results.

Example ٢: Capturing the mode (I) of elastoplastic-creep crack tip plane stress singularity

To show the utilization of this method in stress analysis of cracked bodies; numerical solution for plane stress distribution around the tip of a crack is sought. It is assumed that the ١٠٠mm open crack is situated in the middle of a rectangular plate of ٤٠٠×٨٠٠mm size (See Fig. ٢). The plate is loaded uniformly and gradually up to the traction of ١٠٠ MPa in both edges.

To simplify the comparison of the results with other works, power law stress hardening model and von Mises criteria have been adopted as material behavior and failure criteria, respectively. In power strain-hardening model the following relation correlates uni-axial plastic strain and stress.

$$\varepsilon_p = A\sigma^{(n)} \qquad (٢٦)$$

in which ε_p and σ are uni-axial plastic strain and stress, respectively. Moreover A and n are two material constants. In our work the magnitude of these two constants are taken as A = $١٠^{-٢٦}$ and n=١٠. Besides, the nodes are arranged in the following manner:

A network of ٨ rows and ٨ columns of nodes devoted to cover the surface of the body (See Fig. ٣). Two other groups of nodal points are used to model the crack. One group of nodes is distributed in a radial pattern around the crack tip comprising of ٦ nodes along ٦ radial directions. The other group of nodes is placed along each line of crack sides comprising of ١٠ nodes per line

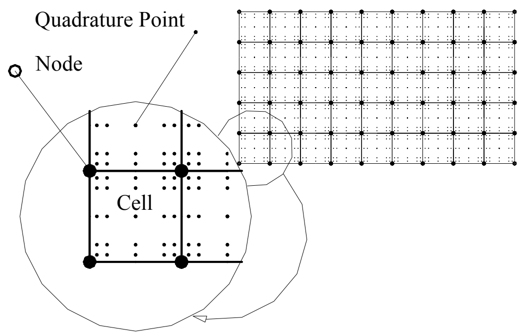

Fig. ٣: Discretized plate of nodes showing, nodes and background integration cells

In this example after applying the external load in a quasi-static manner, a complete elastoplastic analysis is done. When the maximum loading is reached the process is led into the stage of creep deformation. The output results are taken at the start and end of the creep process. To check the validity of this work, C*-integral is used to compare the obtained results with the related published works. C*-integral is a relation similar to the familiar J-integral which is used on capturing the nonlinear singularity of stress distribution around a crack tip [٢٢]. The difference between C* and J-integrals is that in C*-integral definition, implicit displacement dependent terms should be replaced with their respective time rate derivatives. That is [٢٣, ٢٤]:

$$C^* = \int (w^*. n_١ - \mathbf{t} \cdot \frac{\partial \dot{\mathbf{u}}}{\partial x_1}) \, d\Omega \qquad (٢٧)$$

In which

$$w^* = \int_0^{\dot{\varepsilon}_{mn}} \sigma_{ij} d\dot{\varepsilon}_{ij} \qquad (٢٨)$$

Also $\dot{\mathbf{u}}$ is the velocity vector, **t** is the traction vector on Ω contour around the crack tip, $x_١$ represents the coordinate axes which is placed along the crack wall in direction of crack growth and $n_١$ is the $x_١$ component of the unit normal vector to the contour. In this work to make use of this technique, different square shaped contours encircling the tip of the crack are used.

In order to evaluate the results, an analogy between J and C*-integral has been drawn. It has to be mentioned that in elastoplastic crack tip stress analysis it is possible to find few references, which represent the relationship between J-integral and load-geometry parameters. Moreover, it is possible to find some articles such as [٢٥] which analyses stress distributions around the crack tip in creep conditions, but it is not easy to find a relationship between C*-integral and load-geometry parameters in such kind of crack problem. In this work a unique power law relationship is used for both creep and plastic deformations. Therefore the creep strain rate components, $(\dot{\varepsilon}_c)_{ij}$, are proportional to the plastic strain components, $(\varepsilon_p)_{ij}$, [٢٥]. Similarly the correlation between C*-integral and load-geometry parameters remains analogous with the correlation between J-integral and load-geometry parameters. Hence, the J-integral formulation of a problem similar to our work, given in [٢٦], which is based on the power law stress-strain relationship (Eq. (٢٦)), is used to evaluate the C*-integral values. For simplicity, the same values of the material constants used in Eq. (٢٦) are assumed for the material constants in Eq. (١٧), i.e. B= $١٠^{-٢٦}$ and m=١٠.

After substitution of creep constants in place of plastic power law constants and substitution of other geometrical parameters in a relation given in [٢٦] the magnitude of C*-integral turns to be ٢٩٠٨N/mm-h. Based on the developed method in this paper the value of C*-integral along a square shaped contour with side length equal to ٦٠mm becomes C*=٢٠٠٨N/mm-h.

In this analysis and prior to the creep process it has been noticed that some significant elastoplastic deformation is encountered. By comparison of the obtained values of the C*-integral it seems that the main reason for ٣١٪ difference between our result and the one from reference [٢٦] is due to not taking into account the pre-creep elastoplastic deformation by latter one.

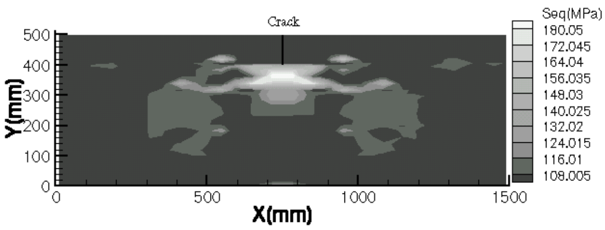

Fig. ٤: Distribution of equivalent stress, $\bar{\sigma}$, after a sudden elastoplastic deformation.

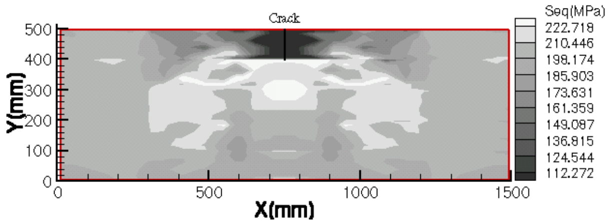

Fig. ٥: Post-creep redistribution of equivalent stress, $\bar{\sigma}$.

Figs. ٤ and ٥ illustrate the flooded contour plots of displacement strain and stress components throughout the body. Fig. ٤ represents the distribution of equivalent stress just after termination of the sudden but incremental elastoplastic deformation prior to the creep process. Fig. ٥ illustrates the distribution of the same field quantity as in Figs. ٤, but one hour after commencement of the creep process. To demonstrate the capabilities of the developed method only one creep increment has been treated and studied. Nevertheless, since some big values are chosen for creep constants in Eq. (١٧), the difference between the field quantities before and after the creep process is appreciable. In other words, the creep process causes the highly concentrated state of stress around the tip of the crack to expand itself into the points far from this region.

Conclusion

The results extracted by this work indicate that by preservation of some difficulties, EFGM can also be used in the field of creep stress analysis in nonlinear media. However and in general one can see that the outcome results have some kinds of unknown dependence on nodes distribution and the more is the complexity of the geometry the less is the accuracy of the results. Definitely the experience of user of this method could overrule some of these difficulties, but not all.

References

١- T. Belytschko, Y.Y. Lu, L. Gu, Element-free Galerkin methods, Int. J. Numer. Methods Engrg. Vol. ٣٧, pp ٢٢٩-٢٥٦, ١٩٩٤.

٢- B. Nayroles, G. Touzot, P. Villon, Generalizing the finite element method: diffuse approximation and diffuse element, Comput. Mech. Vol. ١٠, pp ٣٠٧-٣١٨, ١٩٩٢.

٣- P. Lancaster, K. Salkauskas, Surfaces generated by moving least squares methods, Math. Comput. Vol. ٣٧, pp ١٤١-١٥٨, ١٩٨١.

٤- T. Belytschko, Y.Y. Lu, L. Gu, M. Tabbara, Element-free Galerkin method for static and dynamic fracture. Int. J. Solids Struct. Vol. ٣٢, pp ٢٥٤٧-٢٥٧٠, ١٩٩٥.

٥- T. Belytschko, M. Tabarra, Dynamic fracture using element-free Galerkin methods, Int. J. Numer. Methods Engrg. Vol. ٣٩, pp ٩٢٣-٩٣٨, ١٩٩٦.

٦- T. Belytschko, Y.Y. Lu, L. Gu, Crack propagation by element-free Galerkin methods, Engng Fracture Mech. Vol. ٩٥, pp ٢٩٥-٣١٥, ١٩٩٥.

٧- Y. Xu, S. Saigal, An element-free Galerkin formulation for stable crack growth in an elastic solid, Int. J. Solids Struct. Vol. ١٥٤ (١٩٩٨), pp ٣٣١-٣٤٣, ١٩٩٨.

٨- P. Krysl, T. Belytschko, Analysis of thin plates by element-free Galerkin method, Comput. Mech. Vol. ١٧, pp ٢٦-٣٥, ١٩٩٦.

٩- N. Sukumar, B. Moran, T. Black, T. Belytschko, An element-free Galerkin method for three dimensional fracture mechanics, Comput. Mech. Vol. ٢٠, pp ١٧٠-١٧٥, ١٩٩٧.

١٠-P. Krysl, T. Belytschko, The element free Galerkin method for Dynamic propagation of arbitrary ٣-D cracks, Int. J. Numer. Methods Engrg. Vol. ٤٤, pp ٧٦٧-٨٠٠, ١٩٩٩.

١١-S. Barry, S. Saigal, A three-dimensional element-free Galerkin elastic and elastoplastic formulation, Int. J. Numer. Methods Engrg. Vol. ٤٦, pp ٦٧١-٦٩٣, ١٩٩٩.

١٢-M. Fleming, Y.a. Chu, B. Moran, T. Belytschko, Enriched element-free Galerkin methods for crack tip fields, Int. J. Numer. Methods Engrg. Vol. ٤٠, pp ١٤٨٣-١٥٠٤, ١٩٩٧.

١٣-T. Krongauz, T. Belytschko, Enforcement of essential boundary conditions in mesh-less approximations using finite elements, Comput. Methods Appl. Mech. Engrg. Vol. ١٣١, pp ١٣٣-١٤٥, ١٩٩٦.

١٤-J. Dolbow, T. Belytschko, Volumetric locking in the element free Galerkin method, Int. J. Numer. Methods Engrg. Vol. ٤٦, pp ٩٢٥-٩٤٢, ١٩٩٩.

١٥-T. Belytschko, Y. Krongauz, J. Dolbow, C. Gerlach, On the completeness of meshfree particle methods, Int. J. Numer. Methods Engrg. Vol. ٤٣, pp ٧٨٥-٨١٩, ١٩٩٩.

١٦-P. Krysl, T. Belytschko, Element-free Galerkin method: Convergence of the continuous and discontinuous shape functions, Comput. Methods Appl. Mech. Engrg. Vol. ١٤٨, pp ٢٥٧-٢٧٧, ١٩٩٧.

١٧-B. Carnahan, Applied numerical methods, John wiley & son Inc., ١٩٦٩.

١٨-D.R. Owen, E. Hinton, Finite element in plasticity, Pineridge press, U.K., ١٩٨٠.

١٩-T.R. Hsu, The finite element in thermomechanics, Prantice-Hall Inc., Canada, ١٩٨٧.

٢٠- A.G. Atkins, Mai Y. W., Elastic and plastic fracture, Ellis Horwood limited England, ١٩٨٨.

٢١- T. L. Anderson, FRACTURE MECHANICS Fundamentals and applications. CRC Press Inc., ١٩٩٥.

٢٢-J.R. Rice, A path independent integral and the approximate analysis of strain concentration by notches and cracks, J. appl. Mech. Vol. ٣٥, pp ٣٧٩-٣٨٦, ١٩٦٨.

٢٣-B. Moran, C.F. Shih, Crack tip and associated domain integrals from momentum and energy balance, Engng Fracture Mech. Vol. ٢٧, pp ٦١٥-٦٤١, ١٩٨٧.

٢٤-T.Nishioka, Theoretical and computational aspects of the dynamic J-integral in dynamic fracture mechanics for homogeneous and non-homogeneous materials, Computer Modeling and Simulation in Engineering. Vol. ٤, pp ٢٩-٣٦, ١٩٩٩.

٢٥-C.Y. Hui, H. Riedel, The asymptotic stress and strain field near the tip of a growing crack under creep conditions, Int. J. Fracture. Vol. ١٧, pp ٤٠٩-٤٢٥, ١٩٨١.

٢٦-V. Kumar, M.D. German, C.F. Shih, An Engineering Approach for Elastic Plastic Fracture Analysis. EPRI Report NP-١٩٣١, Electric Power Research Institute, Palo Alto, CA, ١٩٨١.