MORE ON REVERSE TRIANGLE INEQUALITY IN INNER PRODUCT SPACES

A. H. ANSARI AND M. S. MOSLEHIAN

Received 8 February 2005 and in revised form 17 May 2005

Refining some results of Dragomir, several new reverses of the generalized triangle inequality in inner product spaces are given. Among several results, we establish some reverses for the Schwarz inequality. In particular, it is proved that if a is a unit vector in a real or complex inner product space $(H;\langle\cdot,\cdot\rangle)$, $r,s>0$, $p\in(0,s]$, $D=\{x\in H, \|rx-sa\|\leq p\}$, $x_1,x_2\in D-\{0\}$, and $\alpha_{r,s}=\min\{(r^2\|x_k\|^2-p^2+s^2)/2rs\|x_k\|:1\leq k\leq 2\}$, then $(\|x_1\|\|x_2\|-\mathrm{Re}\langle x_1,x_2\rangle)/(\|x_1\|+\|x_2\|)^2\leq\alpha_{r,s}$.

1. Introduction

It is interesting to know under which conditions the triangle inequality went the other way in a normed space X; in other words, we would like to know if there is a positive constant c with the property that $c\sum_{k=1}^{n}\|x_k\|\leq\|\sum_{k=1}^{n}x_k\|$ for any finite set $x_1,\dots,x_n\in X$. Nakai and Tada [7] proved that the normed spaces with this property are precisely those of finite dimensional.

The first authors investigating reverse of the triangle inequality in inner product spaces were Diaz and Metcalf [2] by establishing the following result as an extension of an inequality given by Petrovich [8] for complex numbers.

Theorem 1.1 (Diaz-Metcalf theorem). *Let a be a unit vector in an inner product space $(H;\langle\cdot,\cdot\rangle)$. Suppose the vectors $x_k\in H$, $k\in\{1,\dots,n\}$ satisfy*

$$0\leq r\leq\frac{\mathrm{Re}\,\langle x_k,a\rangle}{||x_k||},\quad k\in\{1,\dots,n\}. \tag{1.1}$$

Then

$$r\sum_{k=1}^{n}||x_k||\leq\left\|\sum_{k=1}^{n}x_k\right\|, \tag{1.2}$$

International Journal of Mathematics and Mathematical Sciences 2005:18 (2005) 2883–2893
DOI: 10.1155/IJMMS.2005.2883