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HAHN-BANACH THEOREM IN GENERALIZED 2-NORMED SPACES

Z. LEWANDOWSKA, M. S. MOSLEHIAN, AND A. SAADATPOUR

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Abstract. In this paper we prove an extension Hahn-Banach theorem in the context of generalized 2-normed spaces.

1. Introduction.

In [4] Z. Lewandowska introduced a generalization of Gähler 2-normed space (see [2]) as follows.

Definition 1.1. Let X and Y be real linear spaces. Denote by D a non-empty subset of $X \times Y$ such that for every $x \in X, y \in Y$ the sets $D_x = \{y \in Y; (x,y) \in D\}$ and $D^y = \{x \in X; (x,y) \in D\}$ are linear subspaces of the spaces Y and X, respectively.
A function $\|.,.\| \colon D \to [0,\infty)$ will be called a generalized 2-norm on D if it satisfies the following conditions:
(1) $\|x, \alpha y\| = |\alpha| \cdot \|x,y\| = \|\alpha x, y\|$ for any real number α and all $(x,y) \in D$;
(2)$\|x, y+z\| \leq \|x,y\| + \|x,z\|$ for $x \in X, y,z \in Y$ with $(x,y),(x,z) \in D$;
(3)$\|x+y, z\| \leq \|x,z\| + \|y,z\|$ for $x,y \in X, z \in Y$ with $(x,z),(y,z) \in D$.
The set D is called a 2-normed set. In particular, if $D = X \times Y$, the function $\|.,.\|$ is said to be a generalized 2-norm on $X \times Y$ and the pair $(X \times Y, \|.,.\|)$ is called a generalized 2-normed space. If $X = Y$, then the generalized 2-normed space $(X \times X, \|.,.\|)$ is denoted by $(X, \|.,.\|)$. In the case that $X = Y$, $D = D^{-1}$, where $D^{-1} = \{(y,x) : (x,y) \in D\}$, and $\|x,y\| = \|y,x\|$ for all $(x,y) \in D$, we call $\|.,.\|$ a generalized symmetric 2-norm and D a symmetric 2-normed set.

Recall that in Gähler definition of a 2-norm $\|x,y\| = 0$ if and only if x and y are linearly dependent, and this is a crucial difference between Gähler's approach and Lewandowska's one.

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