Fuzzy versions of Hyers–Ulam–Rassias theorem

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Dedicated to the memory of Professor M. Yassi in admiration

Abstract

We introduce three reasonable versions of fuzzy approximately additive functions in fuzzy normed spaces. More precisely, we show under some suitable conditions that an approximately additive function can be approximated by an additive mapping in a fuzzy sense.

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1. Introduction and Preliminaries

In 1984, Katsaras [13] defined a fuzzy norm on a linear space to construct a fuzzy vector topological structure on the space. Later, some mathematicians have defined fuzzy norms on a linear space from different points of view [8,15,21]. In particular, in 2003, Bag and Samanta [4], following Cheng and Mordeson [6], gave an idea of a fuzzy norm in such a manner that the corresponding fuzzy metric is of Kramosil and Michalek type [14]. They also established a decomposition theorem of a fuzzy norm into a family of crisp norms and investigated some properties of fuzzy normed linear spaces [5].

Defining, in some way, the class of approximate solutions of the given functional equation one can ask whether each mapping from this class can be somehow approximated by an exact solution of the considered equation. Such a problem was formulated by Ulam in 1940 (cf. [20]) and solved in the next year for the Cauchy functional equation by Hyers [10]. In 1950, Aoki [2] and in 1978, Rassias [19] proved a generalization of Hyers’ theorem for additive and linear mappings, respectively:

Theorem 1.1. Let $f$ be an approximately additive mapping from a normed vector space $E$ into a Banach space $E'$, i.e. $f$ satisfies the inequality

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