Two new conjugate gradient methods based on modified secant equations

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\section{A B S T R A C T}

Following the approach proposed by Dai and Liao, we introduce two nonlinear conjugate gradient methods for unconstrained optimization problems. One of our proposed methods is based on a modified version of the secant equation proposed by Zhang, Deng and Chen, and Zhang and Xu, and the other is based on the modified BFGS update proposed by Yuan. An interesting feature of our methods is their account of both the gradient and function values. Under proper conditions, we show that one of the proposed methods is globally convergent for general functions and that the other is globally convergent for uniformly convex functions. To enhance the performance of the line search procedure, we also propose a new approach for computing the initial steplength to be used for initiating the procedure. We provide a comparison of implementations of our methods with the efficient conjugate gradient methods proposed by Dai and Liao, and Hestenes and Stiefel. Numerical test results show the efficiency of our proposed methods.

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\section{1. Introduction}

Conjugate gradient methods have played special roles in solving large scale nonlinear optimization problems. Although conjugate gradient methods are not the fastest or most robust optimization algorithms for nonlinear problems available today, they remain very popular for engineers and mathematicians engaged with solving large problems. A general conjugate gradient method is designed to solve the following unconstrained optimization problem:

\[
\min_{x \in \mathbb{R}^n} f(x),
\]

where \(f : \mathbb{R}^n \rightarrow \mathbb{R}\) is a smooth nonlinear function.

\textbf{Notation 1.1.} For a sufficiently smooth function \(f\) at \(x_k\), we consider the following notations:

\[
f_k = f(x_k), \quad g_k = \nabla f(x_k), \quad G_k = \nabla^2 f(x_k).
\]

The iterative formula of a conjugate gradient method is given by

\[
x_{k+1} = x_k + s_k, \quad s_k = \alpha d_k,
\]

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