An integer programming-based local search for the covering salesman problem

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ABSTRACT

We consider a generalized version of the well known Traveling Salesman Problem called Covering Salesman Problem. In this problem, we are given a set of vertices while each vertex can cover a subset of vertices within its predetermined covering distance r. The goal is to construct a minimum length Hamiltonian cycle over a subset of vertices in which those vertices not visited by the tour has to be within the covering distance of at least one vertex visited on the tour. The paper proposes an Integer Linear Programming based heuristic method which takes advantage of Integer Linear Programming techniques and heuristic search to improve the quality of the solutions. Extensive computational tests on the standard benchmark instances and on a new set of large sized datasets show the effectiveness of the proposed approach.

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1. Introduction

The Covering Salesman Problem (CSP) is a generalization of the Traveling Salesman Problem (TSP) in which the assumption of visiting all the vertices by the tour is not valid. Here, we are given a set of vertices while each vertex can cover a subset of vertices within its predetermined covering distance r. The goal of the CSP is to construct a minimum length Hamiltonian cycle over a subset of vertices in which those not visited by the tour must be within the covering distance of at least one visited vertex [5].

The CSP was introduced by Current and Schilling [5] and they proposed a heuristic method to solve this problem. In their approach, the optimal TSP tour is constructed over the minimum number of vertices that makes the solution feasible. In other words, to cover all the demands, they proposed to solve the corresponding Set Covering Problem (SCP). Then, the minimum length Hamiltonian cycle is constructed over these vertices by solving the corresponding TSP to optimality. Since the associated SCP may have multiple optimal solutions with the same number of vertices, they suggested to take the minimum length tour found by applying a TSP solver over all of the optimal solutions of the corresponding SCP. They also referred to some applications of this problem in the routing of a rural healthcare delivery team in which visiting all villages is not necessary and people living in unvisited regions could travel to their nearest village, visited by the tour, to receive humanitarian functions.

Arkin and Hassin [1] proposed a geometric version of the CSP. Unlike the CSP, in this problem each neighborhood is a compact set in the plane in such a way that by intersecting that set all the vertices in that neighborhood will be covered. Starting from a vertex in a neighborhood set, the goal is to construct a minimum length tour obtained by intersecting all the neighborhoods and returning to the initial vertex. They proposed several simple heuristic methods for this problem which are able to generate solutions within a constant factor from the optimal ones [1].

Golden et al. [12] proposed two heuristic approaches for the CSP which outperform the Current and Schilling’s method in almost all of the tested instances. The proposed heuristics take advantage of different extraction and reinsertion moves to improve the tour length. In particular, they proposed two heuristics called LS1 and LS2. In LS1 the goal is to improve the tour length using two major phases. In the first phase, the algorithm extracts a subset of vertices, P, visited by the tour and in the next step it continues to maintain the feasibility of the solution by adding some new ones to the tour, using the vertices not visited by the tour and those belonging to P. LS2 combines more sophisticated procedures to improve the tour length. In particular, it applies a simple and fast Extraction-Reinsertion heuristic procedure to find a good subset of vertices to be visited by the tour. It also uses the Lin-Kernighan heuristic [19] to find a good TSP tour over the vertices of the solution. Finally, the performance of LS2 has been improved by applying a perturbation procedure to try to escape from the local optimum solutions.

The concept of covering has many real world applications and has been used by many researchers in a wide area of combinatorial optimization problems. Gendreau et al. [11] proposed a generalization of the CSP called the Covering Tour Problem (CTP). Suppose \( G = (V_1 \cup V_2, E) \) be an undirected graph in which the set of vertices \( V_1 \cup V_2 \) is partitioned into three groups. Let \( V_1 \) be the set of vertices that can be visited, \( T \subseteq V_1 \) the set of vertices that must be visited and \( V_2 \) the set of vertices that must be covered. The goal of the CTP is to construct a minimum length Hamiltonian cycle over a subset of vertices \( V_1 \), in which the tour must contain all vertices in \( T \), and...