Achievable Rate Regions for the Dirty Multiple Access Channel with Partial Side Information at the Transmitters

Elham Bahmani\(^1\) and Ghasem Abed Hoddani\(^2\)

\(^1\)Islamic Azad University, Marvdasht Branch, Marvdasht, Iran, \(^2\)Department of Electrical Engineering, Ferdowsi University of Mashhad, Mashhad, Iran

Bahmani.e@gmail.com, Ghodtani@gmail.com

Abstract— In this paper, we establish achievable rate regions for the multiple access channel (MAC) with side information partially known (estimated or sensed version) at the transmitters. Actually, we extend the lattice strategies used by Philosof-Zamir for the MAC with full side information at the transmitters to the partially known case. We show that the sensed or estimated side information reduces the rate regions, the same as that occurs for Costa Gaussian channel.

Index Terms— achievable rate region; dirty multiple access channel; estimated or sensed or partial side information

I. INTRODUCTION

Nowadays channels with side information are widely studied from both information-theoretic and communications aspects. Side information (SI) can be available at the transmitter (SIT), and/or at the receiver (SIR). Encoding for a single-user with causal SIT was first studied by Shannon [1]. The capacity of a general discrete memoryless channel with non-causal SIT was characterized by Gel’fand and Pinsker in [4]. Costa [5] applied the formula obtained by Gel’fand and Pinsker to the special model of Gaussian channel with additive Gaussian interference, and showed that the channel capacity in the presence of interference known at the transmitter is the same as the case without interference. In Costa’s dirty-paper channel (DPC), Gaussian random binning is able to eliminate the effect of interference which is known at the transmitter, and thus achieves capacity. Cover and Chiang [6] extended the above results and established a general capacity Theorem for the channel with two-sided state information. Gueguen-Sayrac [7] derived the capacity of the DPC with partial side information knowledge. The partial side information models the sensing process approximating the original information. It was shown that the capacity of the DPC with partial SI is reduced compared to the DPC with exact or complete SI.

In the multi-user setting, Das and Narayan [8] provided a multi-letter characterization of the capacity region of time-varying MACs with various degrees of SIT and SIR. In [9], a general framework for the capacity region of MACs with causal and non-causal SI was presented where focused on the MAC with independent side information at the two transmitters. Philosof-Zamir [10], [11], extended Jafar’s work and provided achievable rate regions for the discrete memoryless MAC with correlated side information known non-causally at the encoders using a random binning technique. They also considered the Gaussian doubly dirty MAC in the high-SNR strong interference regime [10-15]. The achievable rates using Costa’s Gaussian binning vanish if both interference signals are strong. In contrast, it is shown that lattice-strategies (lattice pre-coding) can achieve positive rates, independent of the interference power. Furthermore, in some cases (which depend on the noise variance and power constraints) high dimensional lattice strategies are in fact optimal [12].

In this paper, we study the effect of partial SIR knowledge in the Gaussian doubly dirty MAC considered by Philosof-Zamir [12]. We expect that achievable rate regions are reduced just the same as in Costa’s DPC considered by in [7]. It is readily seen that our achievable rates include the achievable rates of the MAC with full side information as special cases.

The rest of the paper is organized as follows. In Section II, we state some basic terminology for lattices. Section III includes related works. In Section IV, we state the system model and our results based on lattice strategies for doubly dirty MAC with partial SIT. In particular, we devote Section IV-A to the imbalanced doubly dirty MAC with partial SIT, and Section IV-B to the nearly balanced doubly dirty MAC with estimated SIT.

II. LATTICES AND NESTED LATTICE CODES

We need some basic terminology for lattices before we can proceed and look at the modulo-lattice modulation. An \( n \)-dimensional lattice \( \Lambda \) is defined by the generator matrix \( \mathbf{G} \in \mathbb{R}^{n \times n} \). A point \( \mathbf{t} \in \mathbb{R}^n \) belongs to the lattice if and only if it can be written as \( \mathbf{t} = i\mathbf{G} \), where \( i \in \mathbb{Z}^n \) and \( \mathbb{Z} = \{0, \pm 1, \pm 2, ...\} \). The nearest neighbor quantizer of a lattice \( \Lambda \) is defined by \( Q_{\Lambda}(x) \equiv \arg\min_{i \in \mathbb{Z}^n} \| x - i \| \) where \( \| . \| \) is the Euclidean norm. The modulo-lattice operation is defined by

\[
x \mod \Lambda = x - Q_{\Lambda}(x).
\]

The modulo-code operation satisfies as follows

\[
\lfloor x \mod \Lambda + y \rfloor \mod \Lambda = \lfloor x + y \rfloor \mod \Lambda.
\]

The fundamental Voronoi region of \( \Lambda \) is the set of all points closer to the origin than to any other lattice point \( \mathbf{v}(\Lambda) = \{ x : Q(x) = 0 \} \) with volume \( V = \text{Vol}(\mathbf{v}(\Lambda)) \). The second moment per dimension of a uniform distribution over \( V \) is...