An effective variational iteration algorithm for solving Riccati differential equations

Asghar Ghorbani\textsuperscript{a,}\textsuperscript{*}, Shaher Momani\textsuperscript{b}

\textsuperscript{a} Department of Applied Mathematics, School of Mathematical Sciences, Ferdowsi University of Mashhad, Mashhad, Iran
\textsuperscript{b} Department of Mathematics, The University of Jordan, Amman 11942, Jordan

\textbf{ARTICLE INFO}

Article history:
Received 11 July 2009
Received in revised form 10 April 2010
Accepted 10 April 2010

Keywords:
Piecewise-truncated variational iteration method
Truncated variational iteration method
Runge–Kutta method
Riccati differential equations

\textbf{ABSTRACT}

The piecewise variational iteration method (VIM) for solving Riccati differential equations (RDEs) provides a solution as a sequence of iterates. Therefore, its application to RDEs leads to the calculation of terms that are not needed and more time is consumed in repeated calculations for series solutions. In order to overcome these shortcomings, we propose an easy-to-use piecewise-truncated VIM algorithm for solving the RDEs. Some examples are given to demonstrate the simplicity and efficiency of the proposed method. Comparisons with the classical fourth-order Runge–Kutta method (RK4) verify that the new method is very effective and convenient for solving Riccati differential equations.

\textcopyright 2010 Elsevier Ltd. All rights reserved.

1. Introduction

The Riccati differential equations (RDEs) of the following form:

\[
\begin{cases}
  u'(t) = A(t) + B(t)u(t) + C(t)u^2(t), & t_0 \leq t \leq T, \\
  u(t_0) = c,
\end{cases}
\]

where \( A(t), B(t) \) and \( C(t) \) are given functions and \( c \) is an arbitrary constant, are a class of nonlinear differential equations of much importance, and play a significant role in many fields of applied science [1]. For instance, solitary wave solutions of a nonlinear partial differential equation can be expressed as a polynomial in two elementary functions satisfying a projective Riccati equation [2]. Such problems also arise in the optimal control literature. However, deriving an analytical solution in an explicit form seems to be unlikely to be achievable except for certain special situations [3]. Of course, if one knows a particular solution, then the general solution can be easily derived. For general cases, one must appeal to numerical techniques or approximate approaches for getting the solutions. Therefore, the problem has attracted much attention and has been studied by many authors (see e.g. [4–7] and the references cited therein).

The variational iteration method, which was proposed originally by He [8–10], has been proved by many authors to be a powerful mathematical tool for various kinds of linear and nonlinear problems [8–19]. Though the VIM leads to fast converging solutions, there is unnecessary calculation in the solution procedure. He and Wu have suggested an effective technique in [9]. They truncate the approximate solution in each of the iterations of the VIM. Also, in order to accelerate the rate of convergence, various other modifications were suggested, for example, variational iteration—the Padé method [20], variational iteration—the Adomian method [11] and variational iteration—the differential transform method [21]. In this