IMA Journal of Mathematical Control and Information Advance Access published January 10, 2013

IMA Journal of Mathematical Control and Information (2013) Page 1 of 20
doi:10.1093/imamci/dns038

An improvement to the homotopy perturbation method for solving the Hamilton–Jacobi–Bellman equation

SOHRAB EFFATI*, HASSAN SABERI NIK AND MOHAMMAD SHIRAZIAN
Department of Applied Mathematics, School of Mathematical Sciences, Ferdowsi University of Mashhad, Mashhad, Iran
*Corresponding author. Email: s-effati@um.ac.ir

[Received on 19 April 2012; revised on 18 August 2012; accepted on 4 November 2012]

In this paper, the piecewise homotopy perturbation method (PHPM) is employed to solve the Hamilton–Jacobi–Bellman (HJB) equation arising in the optimal control problems. The method is a simple modification of the standard homotopy perturbation method (HPM), in which it is treated as an algorithm in a sequence of small intervals (i.e. time step) for finding accurate approximate solutions to the corresponding HJB equation. Applying the PHPM with He's polynomials reveals that the modified homotopy perturbation is more impressive than the standard HPM. Also, the convergence of the algorithm is discussed in detail. The comparison of the PHPM results with the standard HPM, exact solution, Modal series, multiwavelets spectral method, differential transformations and the measure theory method is made. Simulation examples are employed to test the validity of the PHPM.

Keywords: piecewise homotopy perturbation method; optimal control problems; Hamilton–Jacobi–Bellman equation; He's polynomials.

1. Introduction

In the optimal control theory, the optimal solution of a nonlinear control problem may be computed by solving the Hamilton–Jacobi–Bellman (HJB) equation, which is, in general, a difficult task (Stengel, 1994; Seong & Widrow, 2001). In particular, we will derive the fundamental first-order partial differential equation obeyed by the optimal value function, known as the HJB equation. This shift in our attention, moreover, will lead us to an alternative form for the optimal value of the control vector, namely the feedback or closed-loop form of the control. This form of the optimal control typically gives the optimal value of the control vector as a function of the current time, the current state and the parameters of the control problem. In contrast, the form of the optimal control vector derived via the necessary conditions of optimal control theory is termed open-loop, and in general gives the optimal value of the control vector as a function of the independent variable time, parameters and initial and/or terminal values of the planning horizon and the state vector. Essentially, the feedback form of the optimal control is a decision rule, for it gives the optimal value of the control for any current period and any admissible state in the current period that may arise. In contrast, the open-loop form of the optimal control is a curve, for it gives the optimal values of the control as the independent variable time varies over the planning horizon.

In the literature, there are many techniques that are interested in the approximation of the HJB solution. Kleinman (1968) proposed an iterative method to solve the Riccati equation for nonlinear systems by successively solving a sequence of Lyapunov equations. Saridis & Lee (1979) extended this iterative