CONJUGATE GRAPHS OF FINITE GROUPS

A. ERFANIAN* and B. TOLUE†

Department of Mathematics and Center of Excellence in Analysis on Algebraic Structures Ferdowsi University of Mashhad, Mashhad, Iran
*erfanian@math.um.ac.ir
†b.tolue@gmail.com

Received 23 December 2011
Revised 28 February 2012
Published 9 June 2012

In this paper we introduce the conjugate graph $\Gamma_G^c$ associated to a nonabelian group $G$ with vertex set $G\setminus Z(G)$ such that two distinct vertices join by an edge if they are conjugate. We show if $\Gamma_G^c \cong \Gamma_S^c$, where $S$ is a finite nonabelian simple group which satisfy Thompson’s conjecture, then $G \cong S$. Further, if central factors of two nonabelian groups $H$ and $G$ are isomorphic and $|Z(G)| = |Z(H)|$, then $H$ and $G$ have isomorphic conjugate graphs.

Keywords: Conjugacy class; nonabelian simple group; conjugate graph.

Mathematics Subject Classification 2000: 05C25, 20P05, 20D05

1. Introduction

Suppose $G$ is a finite group. Two elements $a$ and $b$ of $G$ are called conjugate if there exists an element $g \in G$ with $gag^{-1} = b$. The conjugacy is an equivalence relation and therefore partition $G$ into some equivalence classes. This means that every element of the group belongs to precisely one conjugacy class. The equivalence class that contains the element $a \in G$ is $a^G = \{ gag^{-1} : g \in G \}$ and is called the conjugacy class of $a$. The classes $a^G$ and $b^G$ are equal if and only if $a$ and $b$ are conjugate, and disjoint otherwise. The class number of $G$ is the number of distinct (nonequivalent) conjugacy classes and we denote it by $k(G)$. The elements of any group may be partitioned into conjugacy classes. Members of the same conjugacy class share many properties and study of conjugacy classes of nonabelian groups reveals many important features of their structure.

Recently there are tremendous research to construct a graph by a group or semigroup. Since algebraic graph theory has close links with group theory, algebraic methods are applied to problems about graphs or vice versa. One can refer to the work of Bertram et al. in [2]. The current paper is organized in four parts. In