Research Article

A New Piecewise-Spectral Homotopy Analysis Method for Solving Chaotic Systems of Initial Value Problems

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An accurate algorithm for solving initial value problems (IVPs) which are highly oscillatory is proposed. The proposed method is based on a novel technique of extending the standard spectral homotopy analysis method (SHAM) and adapting it to a sequence of multiple intervals. In this new application the method is referred to as the piecewise spectral homotopy analysis method (PSHAM). The applicability of the proposed method is examined on the differential equation system modeling HIV infection of CD4+ T cells and the Genesio-Tesi system which is known to be chaotic and highly oscillatory. Also, for the first time, we present here a convergence proof for SHAM. We treat in detail Legendre collocation and Chebyshev collocation. The method is compared to MATLAB's ode45 inbuilt solver as a measure of accuracy and efficiency.

1. Introduction

This paper introduces a new method for solving highly oscillatory and chaotic initial value problems (IVPs). The new method extends, for the first time, the application of the spectral homotopy analysis method [1, 2] to IVPs. The spectral homotopy analysis method was developed by Motsa et al. [1, 2] for solving nonlinear boundary value problems (BVPs) over finite intervals. It has been successfully been applied to other BVPs arising mainly in fluid mechanics-related problems [3–6]. In the previously cited applications the SHAM method was applied to problems which possess smooth solutions over small regions. For rapidly oscillating chaotic systems over very large regions, the SHAM may not give accurate results. The current work seeks to develop a new method that will be valid for rapidly changing solutions over all regions, small, medium-, and large sized. A simple way of ensuring the validity of the approximations for large intervals and for all functions is to determine the solution in a sequence of equal intervals, which are subject to continuity conditions at the end points of each interval.

Recently, in an effort to increase the radius of convergence of some analytical methods of approximations, multistage or piecewise approximations have been developed for solving IVPs over general intervals. This multistage approach seeks to implement the standard approximation method on sequences of subintervals whose union makes up the domain of the underlying problem. The effect of this piece-wise (multistep) approach is to accelerate the convergence of the approximate solution over a large region and to improve the accuracy of the parent approximate method of solution, particularly over rapidly oscillating regions. Examples include the, multistage homotopy analysis method [7], piecewise homotopy perturbation methods [8], multistage Adomian decomposition method [9, 10], multistage differential transformation method, [11, 12], and multistage variational iteration method [13]. Because, these methods attempt to obtain analytical solutions at each interval they involve time-consuming and tedious computational operations and if too many small intervals are considered, as may be the case when dealing with highly oscillatory systems, the analytical integration process will be too much to handle even with the use of