An application of a merit function for solving convex programming problems

Alireza Nazemi\textsuperscript{a,}\textsuperscript{*}, Sohrab Effati\textsuperscript{b,1}

\textsuperscript{a}Department of Mathematics, School of Mathematical Sciences, Shahrood University, P.O. Box 3619995161-316, Shahrood, Iran

\textsuperscript{b}Department of Mathematical Sciences, Ferdowsi University of Mashhad, P.O. Box 917751159, Mashhad, Iran

\textbf{A R T I C L E   I N F O}

\textbf{Article history:}
Received 5 November 2012
Received in revised form 14 May 2013
Accepted 18 July 2013
Available online 26 July 2013

\textbf{Keywords:}
Neural network
Convex programming
NCP-function
Merit function
Convergent
Stability

\textbf{A B S T R A C T}

This paper presents a gradient neural network model for solving convex nonlinear programming (CNP) problems. The main idea is to convert the CNP problem into an equivalent unconstrained minimization problem with objective energy function. A gradient model is then defined directly using the derivatives of the energy function. It is also shown that the proposed neural network is stable in the sense of Lyapunov and can converge to an exact optimal solution of the original problem. It is also found that a larger scaling factor leads to a better convergence rate of the trajectory. The validity and transient behavior of the neural network are demonstrated by using various examples.

© 2013 Elsevier Ltd. All rights reserved.

\section{1. Introduction}

Constrained nonlinear optimization has many applications in scientific and engineering areas, such as signal and image processing, manufacturing, optimal control, and pattern recognition (Agrawal & Fabien, 1999; Avriel, 1976; Bazaraa, Sherali, & Shetty, 2006; Bertsekas, 1989; Boyd & Vandenberghe, 2004; Fletcher, 1981). Over the past years, a variety of numerical algorithms have been developed for solving constrained optimization problems, such as the simplex methods for linear programming (Rao, 2009), active set methods (Nocedal & Wright, 2006), and interior point methods (Bazaraa et al., 2006). However, traditional algorithms for digital computers may not be efficient and cannot satisfy real-time requirement such as in signal processing, robotics, function approximation and automatic control. One promising and powerful method to solve the optimization problems in real time is to employ artificial neural networks based on circuit implementation. The essence of neural network approach for mathematical programming problems is to establish an energy function (nonnegative) and a dynamic system which is a representation of an artificial neural network. The dynamic system is normally in the form of first order ordinary differential equations. It is expected that for an initial point, the dynamic system will approach its static state (equilibrium point) which corresponds the solution of the underlying optimization problem. An important requirement is that the energy function decreases monotonically as the dynamic system approaches an equilibrium point. Because of the dynamic nature of optimization and the potential of electronic implementation, neural networks can be implemented physically by designated hardware such as application-specific integrated circuits, where the optimization procedure is truly done in parallel. Therefore, the neural network approach can solve optimization problems in running times that are orders of magnitude much faster than conventional optimization algorithms executed on general-purpose digital computers. It is of great interest to develop some neural network models that could provide a real-time online solution.