Two optimal Dai–Liao conjugate gradient methods
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Two adaptive choices for the parameter of Dai–Liao conjugate gradient (CG) method are suggested. One of which is obtained by minimizing the distance between search directions of Dai–Liao method and a three-term CG method proposed by Zhang et al. and the other one is obtained by minimizing Frobenius condition number of the search direction matrix. Global convergence analyses are made briefly. Numerical results are reported; they demonstrate effectiveness of the suggested adaptive choices.

Keywords: unconstrained optimization; large-scale optimization; conjugate gradient method; condition number; global convergence

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1. Introduction

Conjugate gradient (CG) methods have attracted special attention for solving large-scale unconstrained optimization problems in the form of \( \min_{x \in \mathbb{R}^n} f(x) \) with the smooth nonlinear function \( f : \mathbb{R}^n \rightarrow \mathbb{R} \), because of low memory requirement, simple computations and strong global convergence. Generally, iterations of the CG methods are in the form of

\[
x_0 \in \mathbb{R}^n, \quad x_{k+1} = x_k + s_k, \quad s_k = \alpha_k d_k, \quad k = 0, 1, \ldots,
\]

where \( \alpha_k \) is a steplength to be computed by a line search procedure and \( d_k \) is the search direction defined by

\[
d_0 = -g_0, \quad d_{k+1} = -g_{k+1} + \beta_k d_k, \quad k = 0, 1, \ldots,
\]

in which \( g_k = \nabla f(x_k) \) and \( \beta_k \) is a scalar called the CG (update) parameter. Different CG methods mainly correspond to different choices for the CG parameter for which a nice review has been presented in [1].

One of the essential CG parameters has been suggested by Hestenes and Stiefel [2] (HS), that is,

\[
\beta_{k}^{HS} = \frac{g_{k+1}^T y_k}{d_k^T y_k},
\]

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