Preconditioned Galerkin and minimal residual methods for solving Sylvester equations

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Abstract

This paper presents preconditioned Galerkin and minimal residual algorithms for the solution of Sylvester equations $AX - XB = C$. Given two good preconditioner matrices $M$ and $N$ for matrices $A$ and $B$, respectively, we solve the Sylvester equations $MAXN - MXBN = MCN$. The algorithms use the Arnoldi process to generate orthonormal bases of certain Krylov subspaces and simultaneously reduce the order of Sylvester equations. Numerical experiments show that the solution of Sylvester equations can be obtained with high accuracy by using the preconditioned versions of Galerkin and minimal residual algorithms and this versions are more robust and more efficient than those without preconditioning.

Keywords: Sylvester matrix equations; Preconditioning; Galerkin method; Minimal residual method; Krylov subspace

1. Introduction

Matrix Sylvester equations are very important in control theory and many other branches of engineering [5,9,11,12,15,20,21]. In this paper, we focus on numerical solution of the Sylvester equations

$$AX - XB = C,$$  \hspace{1cm} (1)

where $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{m \times m}$, $C \in \mathbb{R}^{n \times m}$, and $X \in \mathbb{R}^{n \times m}$ is the solution matrix sought. Without loss of the generality, throughout this paper, we suppose that $m = n$. The necessary and sufficient condition for (1) to have a unique solution is that $S(A) \cap S(B) = \emptyset$,

where $S(A)$ and $S(B)$ are the spectrums of $A$ and $B$, respectively [8,10,14–17,21]. The Bartels and Stewart methods provided the first numerically stable way to systematically solve the Sylvester equation (1). When the matrices $A$ and $B$ are dense, the Bartels–Stewart [1] and Golub–Nash–Van Loan algorithms [10] are attractive. When the matrices $A$ and $B$ are large and sparse, iterative solution of (1) by the alternating-direction-implicit