New Concepts of Control Structure Design in Determination of PSS Location

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Abstract

In a multi-machine power system, it is important to determine the best location for the application of power system stabilizers. A number of techniques have been proposed to perform this selection. In this paper a new selection measure based on relative gain array and singular value decomposition is proposed. A comparison is made between the performance of the new measure and the older methods. The proposed methodology is based on use of system transfer function.

Key words--Power system stabilizer, relative gain array, singular value decomposition, controllability and observability measures.

I Introduction

Most available control design techniques assume that a control structure is given at the outset, but one of the first steps for an engineer is to determine the variables that should be controlled, the variables that should be measured, and the inputs that should be manipulated. An important task in the design of a control system is the specification of the control structure, referred to as control structure design. This leads to closing the gap between theory and application in this area [1-2]. Relative gain array (RGA) and singular value decomposition (SVD) [3], partial relative gains (PRG) [4] and closed loop interaction number (CIN) [5] have been introduced as measures of control structure design.

In power systems, several techniques have been proposed in the literature to determine the best location for applying power system stabilizers [6-18]. Participation factor method is one of the methods in detecting the contribution of various generators in each mode and for suitable location of the PSSs [6]. Some authors have proposed sensitivity of PSS effect (SPE) as a criterion for determining the suitable location of PSSs [7]. Modal controllability and modal observability is also considered as a measure for determining the suitable location of PSSs and suitable signal as input of PSSs [8-9]. Relation among the most popular techniques for identifying suitable location for applying PSSs is given in [10]. For selection of best position for applying static var compensators, residue has been used as a screening index and final selection have done with relative gain array (RGA) [11]. RGA uses as a measure for selection of suitable location of stabilizers [12].

Singular value decomposition (SVD) has been proposed as a measure of the distance of a controllability and observability matrix from singularity in a state space model [8-9]. This distance is used as an index to compare the ability of inputs to control an oscillation mode.

The main drawback of the above approaches is the determination of linear models from the state space models that usually have large dimensions. Some authors have used transfer
function approach to locate the PSSs. In [13] peak of transfer function elements in frequency domain is used as an index. Transfer function residues are used in [14] to locate PSSs. Singular value is used as a measure to determine the suitable signal for PSSs and its locations in [15-18].

Surprisingly, the use of RGA and SVD in conjunction with power systems seems to be new, or at least has not been clearly exploited. In this paper mathematical consideration of RGA and SVD is used for control structure design in power systems. Relation between measures based on SVD and methods based on controllability measure, observability measure and residue are introduced as well. This paper uses only transfer function of a system so there is a large reduction in the size of the involved matrices.

The paper is organized as follows: RGA and SVD are described in Section II and III respectively; application of the new measures in power system is described in Section IV; relationship of results based on SVD and controllability, observability and residue methods is given in Section V; and the application of RGA and SVD in multi-machine power system is described in Section VI.

II   Relative gain array (RGA)

The relative gain array of an \( l \times m \) matrix \( G \) is defined as

\[
\text{RGA}(G) = \Lambda(G) = G \times (G^T)^{-1}
\]  

(1)

where \((\cdot)^{-1}\) is pseudo inverse, \((\cdot)^T\) means transpose and \(\times\) denotes element by element multiplication[3]. The RGA has a number of interesting control properties, of which the most important ones are[3]:

a. For a non-singular square matrix \( G \), RGA(G) is independent of input and output scaling. For a full row rank matrix, it is independent of output scaling and for a full column rank matrix, it is independent of input scaling.

b. The sum norm of the RGA matrix is very close to the minimized condition number \( \gamma \). This means that plants with large RGA elements are always ill conditioned.

c. The RGA of a matrix can be used to measure diagonal dominance, by the simple equality

\[
\text{RGA}_{-\text{no}} = \| \text{H}(G) - I \|_{\text{max}}
\]

(2)

For decentralized control to avoid instability caused by interaction in the crossover, one should prefer pairing for which the RGA number at crossover frequency is close to zero[3]. And also to avoid instability caused by interactions at low frequency, one should avoid pairing with negative steady state RGA elements.

d. RGA elements imply sensitivity to element-by-element uncertainty. The non-singular and square matrix \( G \) becomes singular if one makes a relative change \( 1/\lambda_j \) in the \( ij \)-th element of \( G \):

\[
g_{ij} = g_{ij}(1 - \frac{1}{\lambda_j})
\]

(3)

So large \( \lambda_j \) means that with small change in \( g_{ij} \), \( G \) becomes singular.

e. The \( i \)-th row sum of the RGA is equal to the square of the \( i \)-th output projection and the \( j \)-th column sum of the RGA is equal to the square of the \( j \)-th input projection as follows

\[
\sum_{i=1}^{l} \lambda_{ij} = \| e_i u_j \|^2 \quad \sum_{j=1}^{m} \lambda_{ij} = \| e_i v_j \|^2
\]

(4)

where \( e_i \) is a vector with a 1 in position \( i \) and zeros elsewhere, \( u_j \) and \( v_j \) respectively are the output and input directions with non-zero gains extracted from singular value decomposition.
III  Singular value decomposition

Consider M is a constant matrix in $C^{m \times n}$. Then M can be decomposed into its singular value decomposition according to the following theorem [8].

**Theorem:** Let $M \in C^{m \times n}$ then there exist $\Sigma \in R^{m \times n}$ and unitary matrices $U \in C^{m \times m}$ and $V \in C^{n \times n}$ such that:

$$M = U \Sigma V^H$$

(5)

where $\Sigma = \begin{bmatrix} S & 0 \\ 0 & 0 \end{bmatrix}$, $S = \text{diag}\{\sigma_1, \sigma_2, \ldots, \sigma_r\}$ with $\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_r$ and $r \leq \min\{m,n\}$ and $V^H$ is complex conjugate transpose of V. The column vector elements of U, identified by $u_i$, are orthogonal and of unit length, and represent the output directions. The column vector elements of V, identified by $v_i$, are orthogonal and of unit length, and represent the input directions [3]. These input and output directions are related through singular values by:

$$M v_i = \sigma_i u_i$$

(6)

It means that if an input is applied in the direction $v_i$, then the output is in the direction $u_i$ and has a gain of $\sigma_i$. Input direction $v_i$ for $i > r$ corresponds to inputs that do not have any influence on outputs and similarly output direction $u_i$ for $i > r$ corresponds to the outputs that cannot be accessed by any input. Since the diagonal elements of $S$ are arranged in a descending order, it can be shown that the largest gain for any input direction is equal to the maximum singular value $\sigma_1$ and so one can write:

$$\sigma_i = \max_{d \in \Omega} \frac{||MD||}{||d||} = \frac{||MV||}{||v||}$$

(7)

where $d$ is any input signal and $||.||$ is the Euclidian norm. Expansion of (5) leads to:

$$M = \sum_{i=1}^{r} \sigma_i u_i v_i^H$$

(8)

IV Application of the new measures in power system

To see the applicability of RGA and SVD in power systems, consider that there is one machine connected to an infinite bus as shown in Fig. 1. Generator is modeled by a seventh order dynamic set of equations. Parameters of generator, exciter, governor and transmission system, and operating point are given in Appendix A. The input candidates are: exciter Ex and governor Gov set points, and the output candidates are: power angle $\delta$, speed of generator $\omega$, terminal voltage $V$, electrical power $P_e$, generator current $I$, and integral of acceleration power $\int (P - P_m) dt$. Power angle is in rad, speed is in rad/s and other outputs are in per unit. Scaling plays an important role in figuring out the best input and output [17]. For scaling variables, the maximum expected deviation from the normal value should be chosen. Here for the exciter and governor inputs, and for current and power 1 p.u., for voltage 0.5 p.u., for
power angle $\pi$ rad, and for speed 0.01 $\omega_0$ is considered as the expected maximum deviation value. Dividing each variable by its maximum value scales all variables. Exciter and Governor set points are two manipulations in order and $\delta$, $\omega$, $V_t$, $P_e$, $I_t$, and $\int(P_e - P_m)dt$ are six candidate measurements in order. Transfer function of the system at steady state, corresponding RGA matrix and row sums of RGA, are:

$$G_{all} = \begin{bmatrix} -1.5735 & 0.2669 \\ 0.0000 & 0.0000 \\ 1.9874 & -0.0077 \\ -0.0000 & 1.0000 \\ -0.7506 & 0.9707 \\ -1.0873 & 0.1844 \end{bmatrix} \quad \Lambda = \begin{bmatrix} 0.3024 & 0.0006 \\ 0.0000 & 0.0000 \\ 0.5425 & -0.0014 \\ -0.0000 & 0.5495 \\ 0.0107 & 0.4510 \\ 0.1444 & 0.0003 \end{bmatrix} \quad \Lambda_2 = \begin{bmatrix} 0.3030 \\ 0.0000 \\ 0.5411 \\ 0.5495 \\ 0.4617 \\ 0.1447 \end{bmatrix}$$

One finds from the row sums of the steady state RGA matrix given in $\Lambda_2$ that four outputs, $\delta$, $P_e$, $I_t$, $V_t$ have largest projections onto the output space of $G_{all}$. Of course for control proposes one must also consider higher frequencies up to crossover. Fig. 2. shows the row sums versus frequency. It shows that speed of generator $\omega$ (which has no steady state effect), integral of acceleration power $\int(P_e - P_m)dt$, Electric power $P_e$ and generator current $I_t$ are effective at crossover frequency whereas power angle $\delta$ and terminal voltage $V_t$ are less effective.

The elements of RGA matrix for any two choices of the set $\{V_t, \delta, I_t, P_e\}$ versus frequency are shown in Fig. 3. Note that, RGA of a $2 \times 2$ matrix is symmetric and diagonal elements are equal, so every subplot of Fig. 3 has two graph that solid one is the diagonal element and dash one is off-diagonal element. RGA matrix near to unitary matrix is preferred(Section II). Fig. 3 shows that the suitable candidates are $\{(Ex, V_t), (Gov, \delta)\}$, $\{(Ex, V_t), (Gov, P_e)\}$ and $\{(Ex, V_t), (Gov, I_t)\}$ and clearly the best one is $\{(Ex, V_t), (Gov, P_e)\}$ since its RGA matrix is the nearest one to a unit matrix. Above pairing is physically expected and practically applied. One of the four outputs $\omega$, $P_e$, $I_t$ or acceleration power $\int(P_e - P_m)dt$ must be use chosen for higher frequency. Power system stabilizer use one of the above signals at higher frequency through a washout block (to act at higher frequency). For choosing the best input-output pair for dynamic improvement of systems one must find the SVD of the system at $s = p + \varepsilon$ where $p$ is the oscillation mode of system and $\varepsilon$ is a small value added to $p$ to make $G_{all}(s)$ analytic. The SVD is:

$$\begin{bmatrix} 10^{-2} & 10^0 & 10^1 & 10^2 \end{bmatrix}$$

![Fig. 2. Row sum of RGA matrix](image)

![Fig. 3. RGA elements for different outputs versus frequency](image)
\[ G_{sd} \equiv 57862 \begin{bmatrix} 0.0848 - 0.0321i \\ 0.0939 + 0.5628i \\ 0.0059 + 0.0080i \\ 0.4271 + 0.0015i \\ 0.4101 - 0.0044i \\ 0.1525 + 0.5406i \end{bmatrix} \]

Absolute value of \( u_1 \) and \( v_1 \) are:

\[ |u_1| = \begin{bmatrix} 0.0907 \\ 0.5706 \\ 0.0100 \\ 0.4271 \\ 0.4101 \\ 0.5617 \end{bmatrix} \]

\[ |v_1| = \begin{bmatrix} 0.9983 \\ 0.5706 \end{bmatrix} \]

Relative magnitude of elements of \( v_1 \) shows that exciter is the most important input and relative magnitude elements of \( u_1 \) shows that \( \omega \) is the best signal, after which comes \( \int (P_e - P_m) dt \), then \( P_e \), and so on. So one must use \( \omega \) or \( \int (P_e - P_m) dt \) as measurement signal and apply suitable signal to exciter.

V Relationship between Indices based on SVD and other methods

The state equation of the linearized model of a multi-machine power system can be written as:

\[
\begin{align*}
\dot{X} &= AX + Bu \\
y &= CX
\end{align*}
\]

(9) (10)

where \( A \in \mathbb{R}^{m \times m}, B \in \mathbb{R}^{m \times n}, C \in \mathbb{R}^{p \times m} \), \( m \) is the number of states, \( n \) is the number of possible inputs and \( p \) is the number of possible outputs. If \( \lambda_i \) is the eigenvalue of matrix \( A \), and \( \phi_i \) and \( \psi_i \) are the corresponding right and left eigenvectors of the state matrix \( A \) with respect to \( \lambda_i \), then:

\[
A \phi_i = \lambda_i \phi_i
\]

(11)

\[
\psi_i^T A = \psi_i^T \lambda_i
\]

(12)

Right and left eigenvectors are normalized such that:

\[
\psi_i^T \phi_i = 1
\]

(13)

relative controllability of different inputs on \( \lambda_i \) and relative observability of different outputs on \( \lambda_i \) can be extracted from the corresponding elements of vector \( \bar{b}_i \) and \( \bar{c}_i \) correspondingly:

\[
\bar{b}_i = (\psi_i^T B)^T
\]

(14)

\[
\bar{c}_i = C \phi_i
\]

(15)

where \( \bar{b}_i \) is a vector of dimension \( n \times 1 \), its elements indicate how much i-th mode is excited by different inputs And \( \bar{c}_i \) is a vector of dimension \( p \times 1 \), its elements indicate how much i-th mode is observed by different outputs. So large element of \( \bar{b}_i \) corresponds to the more effective input and large element of \( \bar{c}_i \) corresponds to the more observable output, so one can choose the best measured signal and the best inputs. Residue of \( \lambda_i \) corresponding to different input output pairs could be extracted from residue matrix \( R \):

\[
R = C \phi_i \psi_i^T B = \bar{c}_i \bar{b}_i^T
\]

(16)

\( R \) is an \( p \times n \) matrix that consider both controllability and observability. Largest element of residue matrix, corresponds to the best measurement signal and the best input to apply controller. Note that due to practical problems (decentralized controller) largest diagonal
element preferred. If $A$ is diagonalizable the transfer function of the system described by (9) and (10) can be described by:

$$G(s) = \sum_{i=1}^{N} \frac{1}{s - \lambda_i} \tilde{c}_i \tilde{b}_i^T$$

(17)

If $M$ in (8) substituted by $G(s)$ then (8) can be rewritten as:

$$G(s) = \sum_{i=1}^{N} \sigma_i(s) u_i(s) v_i(s)^H$$

(18)

If $s$ be near to a pole of the system ($p_i$), then transfer function of the system from eq(17) is:

$$G(s) \equiv \frac{1}{s - p_i} \tilde{c}_i \tilde{b}_i^T$$

(19)

since other terms are small enough. For $s$ near to $p_i$, $\sigma_i$ goes to infinity and so the other terms can be omitted, so (18) reduce to:

$$G(s) \equiv \sigma_i(s) u_i(s) v_i(s)^H$$

(20)

As mentioned before, relative magnitude elements of $\tilde{c}_i$ correspond to the relative observability of output candidates, relative magnitude elements of $\tilde{b}_i$ correspond to the relative controllability of input candidates and elements of $\tilde{c}_i \tilde{b}_i^T$ correspond to the residues of $\lambda_i$ with respect to different input output pairs. Following lemma with (19) and (20) shows that $u_i(s)$ has information about observability and $v_i(s)$ has information about controllability and $u_i v_i^H$ gives information about residue of the oscillation mode corresponding to different input output pair.

Lemma: Consider $\tilde{c}$ and $\tilde{u}$ are $p \times 1$ and $\tilde{b}$ and $\tilde{v}$ are $n \times 1$ vectors, and $k_1, k_2, k_3$ and $k_4$ are scalars. Then

$$k_1 \tilde{c} \tilde{b}^T = k_2 \tilde{u} \tilde{v}^H$$

(21)

implies that

$$\tilde{u} = k_3 \tilde{c} \quad \text{and} \quad \tilde{v}^* = k_4 \tilde{b}.$$  

(22)

($\cdot$)* means conjugate. Proof of lemma is in Appendix B.

VI Application of RGA and SVD in multi-machine power systems

To use the method based on RGA and SVD to locate PSSs in MIMO power system, RGA elements are considered in frequency range between 1 rad/s and 16 rad/s corresponding to the system modes of oscillation. The system has large sensitivity of element-by-element uncertainty for large elements in RGA matrix. Each mode of oscillation is due to contribution of many generators which have large interaction among them. Large RGA elements are a measure of interaction, so one can find generators contributed in each oscillation mode through RGA elements. After finding generators contributed in each oscillation mode, input and output direction and their products leads to determination of best location according to Section V.

Using the example of a 4-machine system, Fig. 4, it is shown in [19] that this system has two oscillation modes very close to each other. It is also illustrated in [19] that in this case the indices according to eigenvector do not work correctly and one must make minor variation in some system parameters (inertia of all machines of one area in [19]) to find suitable results for applying PSSs according to the eigenvector methods.

A study has been conducted to examine the applicability of the RGA and SVD method to such systems with modal resonance. Exciter of generators considered as input and their speeds considered as output and transfer function of system is extracted. Elements of RGA
versus frequency are shown in Fig. 5. It can be seen that there are two peaks in the RGA elements. Absolute value of RGA at 3.35 red/s and 6.7 rad/s are:

\[
\begin{bmatrix}
0.7169 & 1.0937 & 0.3551 & 0.4968 \\
0.7529 & 0.4311 & 0.3633 & 0.5053 \\
0.3709 & 0.4676 & 5.8485 & 6.1447 \\
0.4002 & 0.5991 & 6.2690 & 5.6513
\end{bmatrix}
\]

Relative values of RGA elements at 3.35 rad/s show that all generators contributed in this mode. Relative values of RGA elements at 6.7 rad/s show that at this frequency there is only interaction between both sets \(\{G_2, G_3\}\) and \(\{G_1, G_4\}\). Thus there are two local oscillation modes corresponding to each set at 6.7 rad/s. For choosing the best input-output pair for improving the damping of inter-area mode \(p\) one must find the SVD of the system at \(s = p + \varepsilon\). The SVD of the system at this frequency is:

\[
G_{ef} \equiv 41.6
\[
\begin{bmatrix}
0.1578 - 0.0578i & 0.4794 \\
0.1117 - 0.0581i & 0.4904 + 0.0239i \\
-0.6039 - 0.3962i & -0.5028 + 0.0004i \\
-0.5487 - 0.3648i & -0.5251 - 0.0242i
\end{bmatrix}^H
\]

Absolute value of \(u_i\), \(v_i\) and \(u_i v_i^H\) as residue of the system are:

\[
\begin{bmatrix}
0.1680 \\
0.1259 \\
0.7223 \\
0.6589
\end{bmatrix}
= \begin{bmatrix}
0.4794 \\
0.4910 \\
0.5028 \\
0.5256
\end{bmatrix}
\]

Relative magnitude of \(v_i\) shows that exciter of generator four is the best input candidate, relative magnitude of \(u_i\) shows that speed of generator three is the best measurement signal. Residue matrix \(R\) considers both controllability and observability and suggest generator three for applying a local PSS.

For study on local oscillation modes, RGA shows that there are two local modes for sets \(\{G_2, G_3\}\) and \(\{G_1, G_4\}\). For local mode of set \(\{G_2, G_3\}\) one must extract transfer function corresponding to this generators. The SVD of this transfer function at \(s = p + \varepsilon\) is:

\[
G_{ef} \equiv 22.51
\]

\[
\begin{bmatrix}
0.6213 - 0.0422i \\
-0.7807 + 0.0522i
\end{bmatrix}
= \begin{bmatrix}
0.7143 \\
-0.6995 - 0.0236i
\end{bmatrix}
\]

Absolute value of \(u_i\), \(v_i\) and \(u_i v_i^H\) as residue of the system are:

\[
\begin{bmatrix}
0.6227 \\
0.7825
\end{bmatrix}
= \begin{bmatrix}
0.7143 \\
0.6999
\end{bmatrix}
\]

\[
R = \begin{bmatrix}
0.4448 & 0.4358 \\
0.5589 & 0.5476
\end{bmatrix}
\]
Relative magnitude of $v_1$ shows that exciter of generator one is the best input candidate, relative magnitude of $u_1$ shows that speed of generator two is the best measurement signal. Residue matrix $R$ suggest generator two for applying a local PSS. The same procedure for the set $\{G_1, G_2\}$ says that exciter of generator four is the best input candidate and speed of generator four is the best measurement signal so generator four is the best candidate for applying a local PSS.

VII Conclusions

In this paper new indices according to RGA and SVD are introduced and the relationship between the SVD and residue measure are investigated. The new index can consider controllability and observability of oscillation modes as well as the residue index. This measure only uses the information of transfer function matrix and can be used to find best location for application of PSS. The advantages of the proposed method are:

- No computation involving the right (left) eigenvectors and eigen sensitivity analysis is required.
- This method requires information about transfer function that commonly find from system simulation or modeling.
- Considerable reduction of the size of involved matrices, since it uses only transfer function of a system and doesn’t consider the states of a system.

Test results on some power system network especially system with resonance mode, show the applicability of the method for systems with resonance modes.

VIII Appendix

Appendix A: Generators parameters: $X_d = 2.673, X_f = 2.434, X_{id} = 2.364, X_q = 2.544, X_{ag} = 2.291, R_n = 0.00528, R_{id} = 0.0179, R_{ag} = 0.0179, R_f = 0.00116, H = 5.683$ and $K_p = 0.025$

Exciter parameters: $K_a = 100$ and $T_a = 0.01$

Governor parameters: $K_g = 0.0796, T_r = 0.1$ and $T_s = 0.3$

Transmission line parameters: $R = 0.06$ and $X = 0.25$

Operating point: $V = 1.025, P = 0.8$ and $Q = 0.0$

All quantities are in p.u. except $H$ and time constants which are in seconds.

Appendix B: Rewrite (21) by its elements leads to

\[
\begin{bmatrix}
  c_1 \\
  c_2 \\
  \vdots \\
  c_p \\
\end{bmatrix}
\begin{bmatrix}
  b_1 \\
  b_2 \\
  \vdots \\
  b_n \\
\end{bmatrix} =
\begin{bmatrix}
  u_1 \\
  u_2 \\
  \vdots \\
  u_p \\
\end{bmatrix}
\begin{bmatrix}
  v_1 \\
  v_2 \\
  \vdots \\
  v_n \\
\end{bmatrix}
\]

(23)

where $c_i, b_i, u_i$ and $v_i$ are elements of $\pi, \bar{e}, \pi$ and $\bar{v}$ respectively. Multiplication of vectors in (23) leads to:

\[
\begin{bmatrix}
  c_1 b_1 \\
  c_2 b_2 \\
  \vdots \\
  c_p b_p \\
\end{bmatrix}
\begin{bmatrix}
  u_1 v_1 \\
  u_2 v_2 \\
  \vdots \\
  u_p v_p \\
\end{bmatrix} =
\begin{bmatrix}
  u_1 v_1^* \\
  u_2 v_2^* \\
  \vdots \\
  u_p v_p^* \\
\end{bmatrix}
\]

(24)

Now comparing elements of first row of (24) shows that:
\[ k_i c_i [b_i \quad b_2 \ldots \quad b_r] = k_i u_i [v_i^* \quad v_2^* \ldots \quad v_u^*] \] (25)

and comparing elements of first column of (24) shows that:
\[ k_i b_i c_i \quad c_1 \ldots \quad c_p]^T = k_i v_i^* [u_i \quad u_2 \ldots \quad u_p]^T \] (26)

(25) and (26) directly lead to (22) with \( k_j = k_i b_i / k_i v_i^* \) and \( k_z = k_i c_i / k_i u_i \)

IX References