Contributing Paper

FREQUENCY ANALYSIS OF ATOMIC FORCE MICROSCOPY CANTILEVERS IN A DYNAMIC MODE CONSIDERING TIP MASS AND MOMENT OF INERTIA

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ABSTRACT

In this paper, the high frequency analysis of a non-contact atomic force microscopy micro-cantilever has been discussed. In modeling and simulation of micro-cantilever in previous investigations, mass and moment of inertia of the tip has been neglected, but in our new model is considered. In an exact analytical solution, the governing equation an AFM micro-cantilever has been derived and also the effects of lateral and vertical interaction stiffness and interaction damping have been considered. The values of the first five natural frequencies for a tilted cantilever are calculated. By comparison between results of our new model and previous ones in a case study on two different conventional micro-cantilevers, the effects of mass and moment of inertia of the tip is shown to be considerable. The results show that the effects of mass and moment of inertia of the tip on the resonance frequencies are different, as effects of moment of inertia in comparison with mass could be neglected.

1. INTRODUCTION

In recent years, researchers have become increasingly interested in imaging and manipulating structures at the nanometer scales. One of the powerful and useful tools in nanoscale science and technologies is Atomic Force Microscopy (AFM) with applications from surface characterization in material science, to the study of living biological systems in their natural environment, to nanolithography. After Binnig et al [1] from IBM first offered the contact mode AFM in 1986 many enhanced AFM systems have been presented [2–4].

In all these types of instruments, a sharp probe interacting locally with the specimen is scanned by a piezoelectric scanner, providing three-dimensional information about the surface. An AFM consists of four main components (A) a cantilever deflection sensor, (B) a micro-cantilever-mounted tip, (C) a piezoelectric micro-positioner, and (D) a digital control system.
The heart of an AFM is the micro-cantilever and tip since this is the part which interacts with the sample. The force between the tip and the sample varies as the sample is scanned beneath the tip. Changes in force are sensed by the tip, which is attached to the flexible cantilever. The deflection of the cantilever is a measurement of the forces sensed by the tip [5].

Typically, these micro-cantilever systems are operated in three modes: (I) contact mode, (II) non-contact mode, and (III) tapping mode. In the contact mode, the tip is brought into physical contact with the sample (the tip-sample interaction is repulsive in nature). The non-contact mode is utilized by moving the cantilever slightly away from the sample surface and oscillating the cantilever at or near its natural resonance frequency. By measuring the shift from its natural resonance frequency due to sample attractive interactions (the tip-sample interaction is in attractive regime), topographical information of the sample can be extracted. The tapping mode (or intermittent mode) combines qualities of both contact and non-contact modes while the tip is in contact with surface for a short time [5-8].

The non-contact mode is taken into account in this paper. As described above the calculation and prediction of the resonance frequencies of the micro-cantilever and measuring the shift from the natural frequencies as a measurement signal is very important, so an emphasis is placed on the problem of high frequency response of AFM cantilever.

Recently, many researchers have become interested in the field of frequency response of AFM cantilevers and there are many readily available papers containing abundant modeling and simulation of micro-cantilever operation [9-17].

Rabe et al [12] have examined the flexural vibration amplitude and frequency of free and surface-coupled AFM cantilevers. High frequency response of AFM cantilevers have been studied by Turner et al [13] by considering damping effects between tip and specimen. Turner
and Wiehn [14] have focused on the sensitivity of vibration modes of AFM cantilevers to surface stiffness in both flexural and torsional vibration.

In all the above papers for simplicity, it is assumed that the cantilever is parallel to the sample surface, whereas in commercial AFM a tilted cantilever is utilized which cause to more complicated analysis. Chang [15] have made an analysis on the sensitivity of the flexural vibration modes of the AFM cantilever by taking into account the angle between the cantilever and the surface, including vertical and lateral interaction forces, however, the tip-sample damping localized to the end of the beam has been neglected in his analysis. These damping effects have been considered by Rabe et al [16] which have been examined using the elastic beam model and compared with solutions from the point-mass model. Besides tip-sample damping, the damping of the cantilevers is also very significant. The damping of the cantilevers is caused by two very different effects. System damping, which is caused by internal losses in the cantilever and by the surrounding air, affects all length elements of the beam in the same way. Mahdavi et al [17] have examined the high-frequency response of AFM cantilevers using the elastic beam model and three different lumped models considering the damping of the cantilever. With lumped models it is possible to model this damping in addition to tip-sample damping and derive analytical solution which is not simply possible for distributed model. All of the models mentioned above have been successful in advancing material property measurement techniques. Despite the importance of tip, little attention has been paid to the effects of dimensions, mass and rotary inertia of the tip which is in direct interaction with the sample.

In this paper an attempt is made to formulate the response of the cantilever by vibrating the clamped end with a sinusoidal excitation.

The purpose of this paper is to give a more comprehensive analytical model than the prior ones by considering the effects of dimensions, mass and rotary inertia of the tip. First an
exact analytical solution for flexural vibration of the cantilever will be derived in section 2 and then results of section 2 will be applied to a case study. It can be shown that these effects may be sizeable and should be considered.

2. THEORY

A conventional AFM cantilever which is investigated in this study has a rectangular uniform cross section with an arbitrary rigid tip at its end. As illustrated in figure 1, the cantilever which is tilted as $\alpha$, relative to the sample surface has a length $L$, thickness $t_b$, width $w$ and cross-sectional area $A = t_b w$ and the area moment of inertia $I$. Its mass density and modulus of elasticity are $\rho$ and $E$. The tip of mass $m_t$ and rotary inertia $J_t$ about the axis through the C.G. (centre of gravity) of the tip normal to page is attached to the end of the cantilever which has a total height $h_t$. Because of arbitrary shape of the tip, it is assumed that the C.G. of the tip has a distance $h_y$ from the tip end and $h_x$ from the conjunction point to the cantilever.

Two different coordinate systems are considered in this formulation (figure 1), the first one is $\{x,y\}$, in which $x$ is the coordinate along the cantilever and $y(x,t)$ is the deflection of a length element from its rest position, and the other one is $\{\xi,\psi\}$ which is a rotation of $\{x,y\}$ with $\alpha_\xi$ where $\xi$ is parallel to the sample surface.

Also the interaction between the sensor tip and the surface for very small tip-sample vibration amplitudes can be approximated by linear springs $k_{ver}$ and $k_{lat}$ for vertical and lateral interactions, respectively, shown in figure 1. The effects of damping are modeled here by two dashpots with constant damping coefficients $\gamma_{ver}$ and $\gamma_{lat}$ [16].

The equation of motion for flexural vibrations of a uniform cross-section beam which is a linear partial differential equation of fourth-order is expressed as [18].
\[ \frac{\partial^4 y}{\partial x^4} + \frac{1}{c^2} \frac{\partial^2 y}{\partial t^2} = 0 \]  
(1)

Where \( c = EI/\rho A \). One can obtain a characteristic equation [18]:

\[ \omega_n = (\beta_n L)^2 \left[ \frac{EI}{\rho AL^2} \right] \]  
(2)

Where \( \beta_n \) and \( n = (1, 2, \ldots) \) are the wave number and mode number respectively.

Sinusoidal vibrations of the base of the cantilever \( x = 0 \) in the \( y \) direction are assumed to excite the system with the angular frequency \( \omega \) and the amplitude \( A \). When the time dependencies are omitted the boundary conditions for the tilted cantilever, taking into account both vertical and lateral interaction and also tip properties are:

\[ y(0, t) = A \exp(i \omega t) \]  
(3)

\[ \frac{\partial y(0, t)}{\partial x} = 0 \]  
(4)

\[ EI \frac{\partial^2 y(L, t)}{\partial x^2} = -EI \frac{\partial^3 y(L, t)}{\partial x^3} h_x - J_t \frac{\partial^3 y(L, t)}{\partial t^2 \partial x} + F_\psi \sin \alpha_0 h_y - F_\xi \cos \alpha_0 h_y \]  
(5)

\[ EI \frac{\partial^3 y(L, t)}{\partial x^3} = F_\xi \sin \alpha_0 + F_\psi \cos \alpha_0 \]  
(6)

\[ + m_\gamma \frac{\partial^2 y(L, t)}{\partial t^2} \]

Where,

\[ F_\psi = \delta_\psi k_{\text{ver}} + \frac{d\delta_\psi}{dt} \gamma_{\text{ver}} \]  
(7)

\[ F_\xi = \delta_\xi k_{\text{lat}} + \frac{d\delta_\xi}{dt} \gamma_{\text{lat}} \]  
(8)
And $\delta_x$ and $\delta_y$ which are displacement of tip end parallel and normal to the surface will be as
follow,

$$\delta_x = y(L,t) \sin \alpha_0 + h_t \frac{\partial y(L,t)}{\partial x} \cos \alpha_0$$  \hspace{2cm} (9)$$

$$\delta_y = y(L,t) \cos \alpha_0 - h_t \frac{\partial y(L,t)}{\partial x} \sin \alpha_0$$  \hspace{2cm} (10)$$

Assuming $y(x,t) = Y(x) \exp(i\omega t)$, Equation (3) to (6) will conclude to Equations (11) to (14)
respectively,

$$Y(0) = A$$  \hspace{2cm} (11)$$

$$\frac{dY(0)}{dx} = 0$$  \hspace{2cm} (12)$$

$$\frac{d^2Y(L)}{dx^2} = \left( M(\beta)Y(L) + N(\beta) \frac{dY}{dx}(L) \right)$$  \hspace{2cm} (13)$$

$$\frac{d^3Y(L)}{dx^3} = \left( U(\beta)Y(L) + X(\beta) \frac{dY}{dx}(L) \right)$$  \hspace{2cm} (14)$$

In which,

$$M(\beta) = h_t U(\beta) + \frac{h_y}{h_t} X(\beta)$$  \hspace{2cm} (15)$$

$$N(\beta) = T(\beta) + h_x X(\beta)$$  \hspace{2cm} (16)$$

$$T(\beta) = \frac{h_t h_y}{EI} \left(-\frac{\alpha^2 J_t}{h_t h_y} + \lambda_{ver} \sin^2 \alpha_0 + \lambda_{lat} \cos^2 \alpha_0 \right)$$  \hspace{2cm} (17)$$

$$X(\beta) = \frac{h_t}{EI} (\lambda_{lat} - \lambda_{ver}) \sin \alpha_0 \cos \alpha_0$$  \hspace{2cm} (18)$$
\[ U(\beta) = \frac{L}{EI} \left( -m\omega^2 + \lambda_{ver} \cos^2 \alpha_0 + \lambda_{lat} \sin^2 \alpha_0 \right) \]  

(19)

Where \( \beta = (\omega/c)^2 \), \( \lambda_{ver} \) and \( \lambda_{lat} \) are functions containing the vertical and lateral contact stiffness and the vertical and lateral interaction damping, respectively;

\[ \lambda_{ver} = k_{ver} + i\omega\gamma_{ver} \]  

(20)

\[ \lambda_{lat} = k_{lat} + i\omega\gamma_{lat} \]  

(21)

A general solution of the equation of motion (1) can be expressed as

\[ y(x, t) = (c_1 \sin \beta x + c_2 \cos \beta x + c_3 \sinh \beta x + c_4 \cosh \beta x) \exp(i\omega t) \]  

(22)

Where \( \gamma_{ver}, c_1, c_2, c_3 \) and \( c_4 \) are constants determined from the boundary conditions (11) to (14). After lengthy manipulation, the results are the following analytical expressions for amplitude and the slope at the end of the beam:

\[ y(L, \beta) = \frac{AB^2}{Z(\beta)} \times \left[ \beta^2 (\cos \beta L + \cosh \beta L) \right. \\
+ \beta N(\beta) (\sin \beta L + \sinh \beta L) \\
+ \left. X(\beta) (\cosh \beta L - \cos \beta L) \right] \]  

(23)

And

\[ \alpha(L, \beta) = \frac{AB^2}{Z(\beta)} \times \left[ \beta^4 (\sinh \beta L - \sin \beta L) \right. \\
- \beta M(\beta) (\sin \beta L + \sinh \beta L) \\
+ \left. U(\beta) (\cos \beta L - \cosh \beta L) \right] \]  

(24)

With;
\[
Z(\beta) = \beta^3 \left[ 1 + \cos \beta L \cosh \beta L \right] \\
+ \beta^3 N(\beta) \left[ \sin \beta L \cosh \beta L + \sinh \beta L \cos \beta L \right] \\
+ \beta^3 \left[ M(\beta) + X(\beta) \right] \sin \beta L \sinh \beta L \\
+ \beta U(\beta) \left[ \sin \beta L \cosh \beta L - \sinh \beta L \cos \beta L \right] \\
+ \left[ N(\beta) U(\beta) - M(\beta) X(\beta) \right] \left[ 1 - \cos \beta L \cosh \beta L \right] 
\]

(25)

The above equations can be summarized to those obtained by Rabe [16] on the condition that \( h_x = 0, \ h_y = h_z \) and \( m_z = J_z = 0 \), which means that the sensor tip is assumed to be exactly at the end of the beam at \( x = L \) and without mass, though this is not the case for most commercial beams [16].

3. CASE STUDY AND DISCUSSION

To find the effects of dimensions, mass and rotary inertia of the tip on frequency response, results of section 2 is applied to two different commercial micro-cantilevers. Both of them have a uniform rectangular cross-section with a pyramidal tip, but their dimensions are different such that the mass ratio of the tip to the micro-cantilever for the first case is low (\( m_{tip} / m_{beam} = 1.17\% \)) and for the second one is high (\( m_{tip} / m_{beam} = 6.63\% \)). By this, it can be found that how important are the effects of \( m_i \) and \( J_i \) and when they should be considered. Table 1 shows the properties of two different commercial micro-cantilevers.

The sensitivity of modes and resonance frequencies of an AFM cantilever is defined with respect to the change in tip-sample interaction. In this study the oscillating cantilever is assumed to be in linear force interaction with two different surfaces, the first one is a soft surface \( (k_{ver} = 20 \text{ (Nm}^{-1}\text{)}) \) and the other one is a stiff surface \( (k_{ver} = 200 \text{ (Nm}^{-1}\text{)}) \). Also lateral stiffness is supposed to be \( (k_{ver} = 20 \text{ (Nm}^{-1}\text{)}) \) and the inclination of the cantilever \( \alpha_0 = 15^\circ \) [16].
At the first section, the values of the first five resonance frequencies of the oscillating cantilever in the different boundary conditions of its end (free, soft surface, stiff surface) are compared to those of Rabe model [16], as depicted in figures 2-4. The horizontal axis which shows excitation frequency was normalized to $\omega_{n,j}$, i.e. the first natural frequency of the clamped-free undamped beam without tip. The vertical axis is the forced vibration amplitude of the cantilever-end deflection calculated by equation (23). This axis which is normalized to $A$ is displayed in logarithmic scale. The dotted lines show the amplitudes of beam on the condition that $h_x = 0$, $h_y = h_t$ and $m_t = J_t = 0$ (Rabe Model) and full lines show the amplitudes of beam when they are considered (Recent Model). Also, the pale dotted vertical lines are plotted in all figures to show the first five natural frequencies of clamped-free undamped cantilever without tip.

Figure 2 shows the vibration spectra for the free cantilever without tip-sample forces $k_{lat} = k_{ver} = \gamma_{lat} = \gamma_{ver} = 0$ for the micro-cantilevers of case 1 and 2. As demonstrated by the figure 2(a) for the case 1, the difference between natural frequencies predicted by our new model and those of Rabe Model, amounts -2.31\% for the first mode up to -2.26\% for the fifth one, whereas these difference for the case 2 in the same condition, as can be seen from figure 2(b) differs from -12.15\% to -13.72\% for the first and fifth mode, respectively. It should be noted that the sign of negative implies that resonance frequencies predicted by recent model is lower than those of Rabe model as it was expected.

From comparison between figures 3(a) and 3(b) in which the tip is in linear interaction with a soft surface, it can be observed that variations of resonance frequencies of the case 1 decrease from -2.37\% to -10.13\%, but for the case 2 these variations are very larger than the other case such that they differ from -14.62\% to -13.72\%. Figures 3(c) and 3(d) reveal that this behavior is qualitatively indefeasible for the condition in which the tip is in interaction with a
stiff surface, although quantitatively different, especially for the case 2. The variations of resonance frequencies predicted by recent models are very different from the Rabe one in a number of modes, in particular at the first mode of the case 2.

In addition, the first flexural mode of both cantilevers is the most sensitive and also the flexural modes of a specific cantilever on a softer specimen are more sensitive than a stiffer one which is the same as the corresponding results obtained by Chang [15]. Furthermore, the sensitivity of the case 1 whose bending stiffness is very lower than the case 2 is vastly greater. These concepts can be seen from figures 3(a) and 3(c) for the case 1 and from 3(b) and 3(d) for the case 2.

At the second section, focus is on the effects of tip parameters. Up to now, the effects of $m_t$ and $J_t$ are examined simultaneously, nevertheless, it remains unclear which one has more influence on the resonance frequencies. Hence, their effects are inspected separately. Figure 4(a) and table 2 illustrate the first five resonance frequency of the case 1 over a soft surface. Two another curves are added in this figure, in which the bold dotted curve show the amplitudes of beam when $m_t = 0$ and $J_t \neq 0$ and the chain curve with data points of star show the amplitudes of beam on the condition that $m_t \neq 0$ and $J_t \neq 0$.

The quantitative data (Table 2) indicate that at lower resonance frequencies, the predicted values are almost the same; however these values have more differences at higher resonance frequencies. It is obvious that the resonance frequencies calculated on the condition that $J_t$ is neglected, are very close to those of both of $m_t$ and $J_t$ are considered in our new model. In other words, the effect of $J_t$ in comparison with $m_t$ is very small and negligible.

In view of the fact that tips are made of various materials such as Nitride, Silicon Nitride, Diamond, metal tips (Nickel, Fe) and etc. which can be occasionally different from
substance of the micro-cantilever, hence, the influence of density of the tip on resonance frequencies is investigated whilst the tip dimensions are assumed to be constant. It is seen from figure 4(b) that the tip density has an obvious effect on the resonance frequencies for modes in higher frequency regions. The resonance frequencies increase as the tip density decrease in all modes so that this increasing behavior has more intensity at higher modes. Also by raising the value of tip density from zero up to 3500 $kg/m^3$ the dependence of the resonance frequencies to the tip density decreases. To be more accurate, the intensity of decrease in resonance frequencies in low density is more.

4. CONCLUSION

In this paper the frequency response of AFM cantilever has been examined. In contrast to the previously utilized modeling methods, a more comprehensive distributed-parameters base modeling approach has been developed considering the effects of dimensions, mass and rotary inertia of the tip. First, an exact analytical solution for flexural vibration of the cantilever-mounted tip has been derived. Then in a case study, this solution has been applied on two different commercial micro-cantilevers and finally its results have been compared with solutions from the previous models.

As it was expected, the resonance frequencies predicted by recent model is lower than those of previous models. We have clarified that on the condition where the mass ratio of the tip to the cantilever is high, the differences between the natural frequencies predicted by our new model and those of previous models are much more considerable than the mass ratio is low, especially at the first resonance frequency. However, it depends on not only to the mass ratio, but also to the mode shape of the cantilever and sample stiffness. In addition, the first flexural mode cantilevers is the most sensitive and also the flexural modes of a specific cantilever on a softer
specimen are more sensitive than a stiffer one which is the same as the corresponding results obtained by Chang [15]. Furthermore, the sensitivity of the cantilever which has lower bending stiffness is vastly greater. Also it has been shown that the effect of rotary inertia of the tip on the resonance frequencies in comparison with tip mass is very small and negligible. As a final point, the influence of density of the tip on resonance frequencies has been investigated whilst the tip dimensions are assumed to be constant. As the tip density increase, the resonance frequencies decrease so that this decreasing behavior has more intensity at higher modes and lower densities.

Since the computation of the resonance frequencies of the micro-cantilever and measuring the shift from the natural frequencies as a measurement signal is very important, and also due to the micro-cantilevers examined in this study are of the commercial type, it seems that we should consider the dimensions, mass and rotary inertia of the tip during designing.

REFERENCES


Fig. (1)
Fig. (2-a)

Normalized Amplitude

Normalized Frequency
Fig. (2-b)

Normalized Amplitude vs. Normalized Frequency
Fig. (3-a)
Fig. (3-b)
Fig. (3-c)
Fig. (3-d)
Fig. (4-a)
Fig. (4-b)
Table (1).

<table>
<thead>
<tr>
<th></th>
<th>( L (\mu m) )</th>
<th>( W (\mu m) )</th>
<th>( t_b (\mu m) )</th>
<th>( E (GPa) )</th>
<th>( h_t (\mu m) )</th>
<th>( m_t (kg) )</th>
<th>( J_t (kg.m^2) )</th>
<th>( \frac{m_{ip}}{m_{beam}} )</th>
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<tr>
<td>Case 1</td>
<td>200</td>
<td>20</td>
<td>0.6</td>
<td>169</td>
<td>3.5</td>
<td>9.88 \times 10^{-14}</td>
<td>1.3592 \times 10^{-25}</td>
<td>1.17%</td>
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<tr>
<td>Case 2</td>
<td>108</td>
<td>50</td>
<td>2</td>
<td>169</td>
<td>14.5</td>
<td>4.66 \times 10^{-12}</td>
<td>1.0763 \times 10^{-22}</td>
<td>6.63%</td>
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### Table (2).

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<th>$\omega_2 / \omega_{n,1}$</th>
<th>$\omega_3 / \omega_{n,1}$</th>
<th>$\omega_4 / \omega_{n,1}$</th>
<th>$\omega_5 / \omega_{n,1}$</th>
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<td>4.7423</td>
<td>14.6443</td>
<td>29.4020</td>
<td>47.2137</td>
</tr>
</tbody>
</table>
List of Figures and Tables

Fig. (1) - Schematic of AFM cantilever-mounted tip. The tip has mass \( m_t \) and rotary inertia \( J_t \) with an arbitrary shape whose centre of gravity (C.G.) has a distance \( h_y \) from the tip end and \( h_x \) from the conjunction point to the cantilever. The tip-sample interaction is modeled by vertical and lateral stiffness and dashpots.

Fig. (2) – Variations of resonance frequencies of the cantilever without tip-sample forces \((k_{lat} = k_{ver} = \gamma_{lat} = \gamma_{lat} = 0)\) cantilever of (a) case 1 and (b) case 2.

Fig. (3) – Variations of resonance frequencies in which the tip is in linear interaction with a specimen: cantilever of (a) case 1 on a soft surface, (b) case 2 on a soft surface, (c) case 1 on a stiff surface and (d) case 2 on a stiff surface.

Fig. (4) – (a) Effect of \( m_t \) and \( J_t \) on the resonance frequencies: cantilever of case 1 on a soft surface, (b) Effect of density of the tip on the resonance frequencies.

Table (1)- Properties of beam and tip of two different conventional micro-cantilevers [19].

Table (2)- Effect of \( m_t \) and \( J_t \) on the resonance frequencies, separately.