

Optimal design of sandwich composite laminates for minimum cost and maximum frequency using simulated annealing

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F. Kolahan, M. Tahani and A. Sarhadi

*Mechanical Engineering Department
Ferdowsi University, Mashhad, Iran
Corresponding Author E-mail: alisarhadi2002@yahoo.com*

◆ Abstract

Multi objective optimal design of sandwich composite laminates consisting of high-stiffness and expensive surface and low-stiffness and inexpensive core layers with respect to cost and frequency is addressed in this paper. Discrete ply angles and number of low-stiffness core layers are considered as design variables and simulated annealing algorithm is used for simultaneous cost minimization and frequency maximization. The proposed model is applied to a graphite-epoxy/glass-epoxy laminate and results are obtained for various aspect ratios and number of layers.

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F. Kolahan¹, M.Tahani² and A.Sarhadi³
Mechanical Engineering Department
Frdowsi University, Mashhad, Iran
Corresponding Author E-mail:
alisarhadi2002@yahoo.com

Abstract

Multi objective optimal design of sandwich composite laminates consisting of high-stiffness and expensive surface and low-stiffness and inexpensive core layers with respect to cost and frequency is addressed in this paper. Discrete ply angles and number of low-stiffness core layers are considered as design variables and simulated annealing algorithm is used for simultaneous cost minimization and frequency maximization. The proposed model is applied to a graphite-epoxy/glass-epoxy laminate and results are obtained for various aspect ratios and number of layers.

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1 Introduction

Recently, the composite materials have been used extensively because of their high strength-to-weight ratio and their potential for specific design by selecting the fiber materials and orientations. Composite laminates usually are used in aerospace, defence, marine and automotive industries. Composite structures often have components that may be modelled as rectangular plates. A procedure for reducing the cost is using of the sandwich effect which employs the high-stiffness and expensive material in the surface layers and the low-stiffness and inexpensive material in the core layers. This idea combines the advantages of two materials and is used in this study for optimal design of symmetric hybrid laminates. The present study is aimed at using the above ideas in the optimal design of symmetric laminates subjected to free vibrations. In the design of laminates, maximum frequency problems are of practical importance.

Various discrete programming approaches are surveyed in Haftka and Gürdal [1] such as graphical design, penalty function method, integer programming and probabilistic search methods. Various integer programming techniques are employed in Haftka and Walsh [2], Nagendra et al. [3], Le Riche and Haftka [4], Gürdal et al. [5], and Kogiso et al. [6], Boyang et al. [8], to determine the optimal stacking

¹ - Assistant Professor of Applied Mechanics

² - Assistant Professor of Applied Mechanics

³ - M.S Student of Mechanical Engineering



sequences of laminates under buckling loads. Adali et.al.[12] used an integer programming approach with boolean variables for frequency maximization of composite laminates undergoing free vibrations. A procedure was formulated by Tong and Liu [7] to optimize truss structures with discrete design variables and frequency constraints.

In this work, a multi-objective optimal design is considered to design a sandwich composite laminate by using a simulating annealing (SA) algorithm. The design objective is to achieve the maximum frequency with minimum cost. The paper has been organized as follows: in the next section a brief description about simulating annealing algorithm is given. Section three contains the definition of the basic problem. Optimization process of this basic problem is illustrated in section four. Some numerical results are provided and discussed in section five to validate the proposed method of optimal design.

2 Simulated annealing algorithm

As proposed by Kirkpatrick et al. [9] for the first time, simulated annealing (SA) is a powerful stochastic search technique. The method name induces the physical process whereby the temperature of a solid is raised to a melting point, where the atoms can move freely and then slowly cooled. This method models the behavior of solid material atoms during annealing in forming arrangements. There is an analogy between an optimization process and the physical annealing process. Different configurations of the problem correspond to different arrangements of the atoms. The cost of a configuration corresponds to the energy of the system. Optimal solution corresponds to the lowest energy state. In annealing process the atoms find their way to build a perfect crystal structure through random movements; similarly the global optimum is reached through a search within randomly generated configurations.

In the SA (Simulating Annealing) algorithm, a random initial point is selected and systematically updated until a stopping criterion is satisfied. Updating is an iterative procedure. A random point is generated in the neighborhood of the current configuration, iteratively. The new point is accepted, if the point has a smaller value of cost function compared to that of the current record. This point replaces the old one. On the other hand, if the new cost function has a larger value, the acceptability of the point is decided according to the probability of Boltzman distribution. The probability of accepting a new solution is given as follows:

$$p = \begin{cases} 1 & \text{if } \Delta < 0 \\ e^{-\frac{\Delta}{T}} & \text{if } \Delta \geq 0 \end{cases} \quad (1)$$

The calculation of this probability relies on a temperature parameter, T , which is referred to as temperature, since it plays a similar role as the temperature in the physical annealing process. The temperature parameter is kept constant for a number of trials and then reduced. To avoid getting trapped at a local minimum point, the rate of reduction should be slow. For instance, we can propose the following method to reduce the temperature is as follows:

$$T_{i+1} = cT_i \quad i = 0, 1, \dots \quad (2)$$

$$0.9 \leq c < 1 \quad (3)$$



At initial stages of the algorithm (at high temperatures), the probability of accepting worse designs is higher but at low temperatures, this probability becomes smaller and smaller so that in the end the designs having higher cost are almost never accepted.

3 Basic Problem

Consider a simply supported hybrid laminated plate of length a , width b and thickness h in the x , y and z directions, respectively. The laminate is composed of an even number of orthotropic layers made of different materials. The surface layers are made of composite with high stiffness fiber reinforcements and the core layers of a composite with low stiffness reinforcements. Each layer has a constant thickness t so that $h = N \times t$ where N is the total number of layers. Note that the total thickness of the laminate is kept constant as the number of layers is changed in order to compare the performance of equal thickness designs.

The hybrid laminates are made of N_i inner plies and N_o outer plies such that $N = N_i + N_o$. The equation governing the free vibrations of these laminates is given by

$$D_{11} \frac{\partial^4 w}{\partial x^4} + 2(D_{12} + 2D_{66}) \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_{22} \frac{\partial^4 w}{\partial y^4} = \rho h \frac{\partial^2 w}{\partial t^2} \quad (4)$$

In equation (4), w denotes the deflection in the z direction, ρ is the mass density, and h is the total thickness of laminate. The bending stiffness D_{ij} are computed from

$$D_{ij} = \frac{1}{3} \sum_{k=1}^N (\bar{Q}_{ij})_k (z_k^3 - z_{k-1}^3) \quad (5)$$

Where z_k is the distance from the middle plane of the laminate to the top of the k th layer and \bar{Q}_{ij} is the plane stress reduced stiffness component of the k th layer which can be computed as a function of fiber orientations and material properties using standard transformation relations.

$$\begin{aligned} \bar{Q}_{11} &= Q_{11} \cos^4 \theta + 2(Q_{12} + 2Q_{66}) \sin^2 \theta \cos^2 \theta + Q_{22} \sin^4 \theta \\ \bar{Q}_{12} &= (Q_{11} + Q_{22} - 4Q_{66}) \sin^2 \theta \cos^2 \theta + Q_{12} (\sin^4 \theta + \cos^4 \theta) \\ \bar{Q}_{22} &= Q_{11} \sin^4 \theta + 2(Q_{12} + 2Q_{66}) \sin^2 \theta \cos^2 \theta + Q_{22} \cos^4 \theta \\ \bar{Q}_{16} &= (Q_{11} - Q_{12} - 2Q_{66}) \sin \theta \cos^3 \theta + (Q_{12} - Q_{22} + 2Q_{66}) \sin^3 \theta \cos \theta \\ \bar{Q}_{26} &= (Q_{11} - Q_{12} - 2Q_{66}) \sin^3 \theta \cos \theta + (Q_{12} - Q_{22} + 2Q_{66}) \sin \theta \cos^3 \theta \\ \bar{Q}_{66} &= (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66}) \sin^2 \theta \cos^2 \theta + Q_{66} (\sin^4 \theta + \cos^4 \theta) \end{aligned} \quad (6)$$

The mass density of a hybrid laminate is computed as a thickness weighted average given by

$$\rho = h^{-1} \int_{-h/2}^{h/2} \rho^{(k)} dz \quad (7)$$

Where $\rho^{(k)}$ denotes the mass density of the material in the k th layer.



The boundary conditions for the simply supported plate are given by

$$\begin{aligned} w = 0, M_x = 0 \text{ at } x = 0, a \\ w = 0, M_y = 0 \text{ at } y = 0, b \end{aligned} \quad (8)$$

Where M_x and M_y denote the bending moments in the x and y directions, respectively. In the analysis, the influence of bending-twisting coupling stiffness D_{16} and D_{26} will be neglected. The error induced by this assumption is negligible if the following non-dimensional ratios

$$\gamma = D_{16}(D_{11}^3 D_{22})^{-1/4}, \quad \delta = D_{26}(D_{11} D_{22}^3)^{-1/4} \quad (9)$$

Satisfy the constraints

$$\gamma \leq 0.2, \quad \delta \leq 0.2 \quad (10)$$

A detail discussion of this condition and its implications is given in Nemeth [10], where it is shown that for buckling problems the constraints (10) are effective in reducing bending-twisting coupling to a negligible level. Due to similarity of expressions for buckling load and frequencies, the same constraints are used to reduce the error introduced by neglecting D_{16} and D_{26} .

The solution of the eigenvalue problem (4) subject to the boundary conditions (8) is obtained by taking the deflection w for the vibration mode (m,n) as

$$w(x, y, t) = W(x, y) e^{i\Omega t} \quad (11)$$

$$W(x, y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \quad (12)$$

By substituting equation (12) into (4), we compute the eigen-frequency ω_{mn} as

$$\omega_{mn}^2 = \frac{\pi^4}{\rho h} \left\{ D_{11} \left(\frac{m}{a} \right)^4 + 2(D_{12} + 2D_{66}) \left(\frac{m}{a} \right)^2 \left(\frac{n}{b} \right)^2 + D_{22} \left(\frac{n}{b} \right)^4 \right\} \quad (13)$$

Where the various frequencies ω_{mn} correspond to different mode shapes (different values of m and n in equation (13)). The fundamental frequency is obtained when m and n are both one.

4 Optimization Process

Optimal design of a multi-objective composite laminate includes selection of a sequence of ply angles and number of low-stiffness and less expensive layers for maximization of natural frequency and minimization of structure cost. Therefore, an initial solution of SA algorithm can be a sequence of ply angles and number of layers of each material. Ply angles can vary between -90 to 90 degrees with increments of 15 degrees. Therefore, there are 13 possible fiber orientations for each layer. Since the composite laminate is symmetric, we can consider half of the sequence as initial sequence.



in the next step, a general objective function will be defined for the multi-objective optimization problem. The purpose is to minimize the general objective function. This function is the sum of two objectives; material cost and frequency. Constraints are the equations (10) that should be satisfied. These constraints can be included as a penalty in the general objective function. A penalty factor will be added to the objective function if a constraint violates. The general form of objective function is as follows:

$$F = k_1 f_1 + k_2 f_2 + c_1 g_1^2 + c_2 g_2^2 \quad (14)$$

$$f_1 = \frac{1}{\omega_{\text{fundamental}}} \quad , \quad g_1 = \delta \quad , \quad f_2 = \text{material cost} \quad , \quad g_2 = \gamma \quad (15)$$

Where the material cost function is

$$\text{cost} = ab \frac{h}{N_t} g(\alpha_o \rho_o N_o + \beta_i \rho_i N_i) \quad (16)$$

Where h is the total thickness of laminate, N_t is total number of layers, ρ_o is the density of high-stiff layer material, N_o is the number of high-stiffness layers, α_o is the material cost factor of high-stiffness layer, ρ_i is density of low-stiffness layer material, N_i is number of low-stiffness layers, β_i is the material cost factor of low-stiffness layer, a is the length of the plates and b is the width of the plate.

k_1 and k_2 coefficients represent the relative importance of frequency and material cost objective functions. For instance, $k_1 < k_2$ imply that a lower material cost is more important than a higher frequency for designer. In this work, k_1 and k_2 are the same and coefficients c_1 and c_2 should be chosen so that any violation in the constraints imposes a considerable penalty in the general objective function. Therefore the probability of acceptance of solutions which can't satisfy the constraints will decrease.

5 Numerical results

A multilayer hybrid laminate consisting of glass-epoxy in inner layers and graphite epoxy in other layers with geometrical dimensions of $b=0.25m$, $h=0.002m$ is considered. The properties of materials are taken from Tsai and Hahn [11] as follows:

Graphite Epoxy (T300/5280) : $E_1 = 181Gpa$, $E_2 = 10.3Gpa$, $G_{12} = 7.17Gpa$
 $\nu_{12} = 0.28$, $\rho = 1600kg/m^3$

Glass Epoxy (Scotchply1002): $E_1 = 38.6Gpa$, $E_2 = 8.27Gpa$, $G_{12} = 4.14Gpa$
 $\nu_{12} = 0.26$, $\rho = 1800kg/m^3$

The idea of locating expensive material in the inner layer and inexpensive material in the outer layer can reduce the material costs while satisfies the design specifications. The material cost per unit weight of graphite-epoxy is eight times more expensive than that of glass epoxy and in the material cost function the cost factors α_o , α_i are eight and one, respectively. However the stiffness-to-weight ratio of graphite epoxy is about four times higher than that of glass epoxy.



We prefer to have a laminate with higher frequency and lower cost. It is known that the more graphite layers, the higher frequency and material cost. Therefore it is necessary to find the best trade off between frequency and material cost.

A computer code has been developed in order to find the best sequence of ply orientations and number of glass layers for various number of layers and aspect ratios.

To express the effects of layer number and aspect ratio on the performance of the laminate, the total thickness of the laminate is kept constant and layer number and aspect ratio are changed. In every case, the computer code was ran for several times to get the optimum angle sequence and layer numbers of each material.

Table 1 contains the best sequence of ply angles and number of glass layers for $N=8$ and different aspect ratios varying from 0.2 to 2. Table 1 implies that the best sequence of ply angles are ± 15 , ± 30 for an aspect ratio of 2, ± 30 for an aspect ratio between 0.4 and 0.6, ± 45 for aspect ratios between 0.8 and 1.2, ± 60 for an aspect ratio between 1.4 and 1.6, ± 90 for an aspect ratio between 1.8 and 2.

This table also shows that the best sequence of ply angles changes from ± 15 to ± 90 if the aspect ratio varies from 0.2 to 2.

The last two columns of the table present the percent of material cost and frequency reduction with respect to the situation which all of layers are made of graphite-epoxy. As implied by Table 1, it is clear that a small decrease in the natural frequency leads to a considerable decrease in material costs. The table also shows that use of glass-epoxy layer in the composite laminate made of graphite-epoxy can decrease material costs considerably while there is a small variation in the fundamental frequency.

Similar results for different number of layers are illustrated in tables 2 and 3.

As it can be seen in Table 1, for a average decrease of %16.1 in the fundamental frequency, there is a %64.5 reduction in material costs.

It is seen in table 2 that when $N=16$ there is a average decrease of %25.5 in frequency while there is a %72 average decrease in material costs and table 3 shows that when $N=28$, a %22.5 average decrease in frequency corresponds to a %70 average decrease in material costs.

Figure 1 illustrates the convergence curve of SA algorithm. Horizontal axis represents the number of iterations and the vertical axis shows the general objective function values. As it implies, convergence of SA algorithm will be achieved approximately after 400 iterations. Therefore it can be shown that an optimal solution can be obtained in a few numbers of iterations and convergence speed is acceptable.

SA algorithm initially shows some oscillations for objective function since the temperature parameter is high. This phenomenon allows the SA algorithm to explore an extended space of solutions. However gradual decreases in the temperature parameter lead to fewer oscillations and finally an optimal solution will be achieved in low temperatures.

Nevertheless, SA algorithm is sensitive to initial parameters. This is a drawback that requires several runs to achieve a valid optimal solution. However, SA algorithm is capable of searching a wide space of solutions in a short time to find the optimal solution.

6 Conclusion

In this paper, a multi-objective optimal design of composite laminate for minimum cost and maximum frequency is investigated. Hybrid laminates consisting of low-stiffness and less expensive inner layers and high stiffness and expensive outer



layers were considered. A simulating annealing (SA) algorithm has been developed in order to achieve an optimal design for various numbers of layers and several aspect ratios. Maximum frequency and minimum cost of design have been achieved using SA for a graphite-epoxy/glass-epoxy laminate. Several aspect ratios and number of layers have been considered in this optimal design. Results express that use of glass-epoxy layer in the composite laminate, made of graphite-epoxy, can decrease material costs considerably while there is a small variation in the fundamental frequency. This approach can be useful in decreasing the total production cost of laminates. Moreover, computational results show, SA algorithm is quite capable to solve this problem in reasonable computational times. Future works may contain an optimal design of composite laminate with different boundary conditions and material properties.

Table 1: Optimal sequences of angle ply for maximum frequency and minimum costs vesus the aspect ratio for $N=8$

a/b	b	h	N	θ_{best}	ω_{max} (rad/s)	cost _{min}	n_{glass}	Cost reduction (%)	Frequency reduction (%)
0.2	0.25	0.002	8	$[-15/30/-30/-30]_s$	19322	1.1375	6	64.5	19.1
0.4	0.25	0.002	8	$[-30/-30/30/30]_s$	5142.8	2.275	6	64.5	16.6
0.6	0.25	0.002	8	$[-30/-30/-30/-30]_s$	2519.8	3.4125	6	64.5	10.0
0.8	0.25	0.002	8	$[-45/45/-45/-45]_s$	1642.3	4.55	6	64.5	8.6
1	0.25	0.002	8	$[-45/45/45/-45]_s$	1236.4	5.6875	6	64.5	12.5
1.2	0.25	0.002	8	$[-45/-45/45/45]_s$	1009.6	6.825	6	64.5	14.7
1.4	0.25	0.002	8	$[-60/-60/-45/60]_s$	882.83	7.9625	6	64.5	18.1
1.6	0.25	0.002	8	$[-60/-60/60/60]_s$	816.19	9.1	6	64.5	19.7
1.8	0.25	0.002	8	$[90/90/90/90]_s$	790.07	10.238	6	64.5	21.2
2	0.25	0.002	8	$[90/90/90/90]_s$	784.04	11.3750	6	64.5	21.3

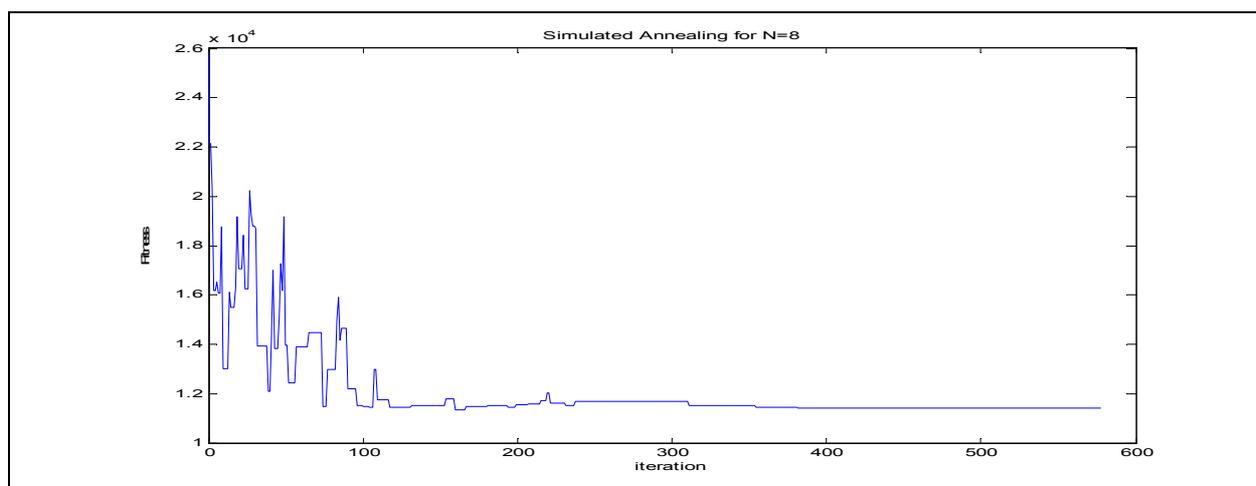


Figure 1: Convergence process of SA algorithm for optimal design. ((a/b) = 0.2)



Table 2: Optimal sequences of angle ply for maximum frequency and minimum costs versus the aspect ratio for $N=16$

a/b	b	h	N	θ_{best}	ω_{max} (rad/s)	$cost_{min}$	n_{glass}	Cost reduction (%)	Frequency reduction (%)
0.2	0.25	0.002	16	$[-15/-15/-30/-30/-30/-30/-30/30]_s$	19322	1.1375	12	64.5	19.1
0.4	0.25	0.002	16	$[-30/-30/-30/-30/-30/30/30/-30]_s$	4326.2	1.5875	14	75.2	29.8
0.6	0.25	0.002	16	$[-30/-30/-30/-30/30/30/-30/30]_s$	2122.7	2.3813	14	75.2	24.2
0.8	0.25	0.002	16	$[-45/45/45/45/-45/45/45/45]_s$	1385.1	3.175	14	75.2	22.9
1	0.25	0.002	16	$[-45/-45/45/-45/45/-45/-45/45]_s$	1043.4	3.9688	14	75.2	26.2
1.2	0.25	0.002	16	$[-45/45/45/45/45/45/45/-45]_s$	853	4.7625	14	75.2	27.9
1.4	0.25	0.002	16	$[-60/-60/-45/-60/60/-60/60/-60]_s$	745.88	5.5563	14	75.2	30.8
1.6	0.25	0.002	16	$[-60/-60/-60/60/-60/-60/60/-60]_s$	689.77	6.35	14	75.2	32.1
1.8	0.25	0.002	16	$[90/90/90/90/90/90/90/90]_s$	790.07	10.238	12	64.5	21.2
2	0.25	0.002	16	$[90/90/90/90/90/90/90/90]_s$	784.04	11.3750	12	64.5	21.3

Table 3: Optimal sequences of angle ply for maximum frequency and minimum costs versus the aspect ratio for $N=28$

a/b	b	h	N	θ_{best}	ω_{max} (rad/s)	$cost_{min}$	n_{glass}	Cost reduction (%)	Frequency reduction (%)
0.2	0.25	0.002	28	$[-15/15/-15/-30/-30/30/-30/-30/30/30/30/30/30/-30]_s$	18571	1.0393	26	67.5	22.2
0.4	0.25	0.002	28	$[-30/-30/30/30/-30/30/-30/30/-30/30/-30/30/-30/30]_s$	4944.4	2.0786	26	67.5	19.8
0.6	0.25	0.002	28	$[-30/-30/-30/-30/-30/30/30/-30/30/-30/-30/30/-30/45]_s$	2190.9	2.5286	24	73.7	21.8
0.8	0.25	0.002	28	$[45/-45/45/45/45/-45/45/-45/-45/-45/-45/-45/-45]_s$	1429.2	3.3714	24	73.7	20.5
1	0.25	0.002	28	$[45/-45/-45/45/45/45/45/-45/-45/-45/45/-45/45/45]_s$	1076.5	4.2143	24	73.7	23.8
1.2	0.25	0.002	28	$[-45/45/45/45/-45/45/45/-45/45/45/-45/45/-45/45]_s$	879.83	5.0571	24	73.7	25.7
1.4	0.25	0.002	28	$[-60/-60/60/-60/-60/60/60/60/-45/-60/60/-45/-60/60]_s$	849.86	7.275	22	67.5	21.2
1.6	0.25	0.002	28	$[-60/-60/-60/-60/-60/-60/-60/60/-60/-60/60/-60/-60/75]_s$	785.42	8.3143	22	67.5	22.7
1.8	0.25	0.002	28	$[90/90/90/90/90/90/90/90/90/90/90/90/90/90]_s$	760.08	9.3536	22	67.5	24.2
2	0.25	0.002	28	$[90/90/90/90/90/90/90/90/90/90/90/90/90/90]_s$	754.01	10.393	22	67.5	24.3



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