ICSV15
15th International Congress on Sound and Vibration
6-10 July 2008, Daejeon, Korea

NONLINEAR MODELING OF HYDRAULIC ENGINE MOUNTS USING COUPLED MOMENTUM EQUATIONS OF DECOUPLER AND INERTIA TRACK

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Abstract:

This paper focuses on the development of a complete nonlinear model of a hydraulic engine mount. The model is capable of capturing both the low- and high-frequency behaviour of hydraulic mounts. The results presented here provide a significant improvement over existing models by considering all nonlinear aspects of a hydraulic engine mount. Current laboratory findings have been used to evaluate nonlinear parameters and hyper-elastic modelling in ANSYS has been used to find complex stiffness. Also switching mechanism of decoupler has been applied. The comprehensive transfer function of linear system has been obtained, by a simultaneous defining of state parameters, scalar output variable and state space matrices.

Results indicate two zones of unsatisfactory in system behaviour which correspond to the real system performance and experimental data.

The measured responses of the mounts to loading at various frequencies and amplitudes are compared to the predictions of the mathematical model. The comparisons generally show a very good agreement, which corroborate the nonlinear model of the mount. It is felt that this work will help engineers in reducing mount design time, by providing insight into the effects of various parameters within the mount.

1-INTRODUCTION

The modern engine mounting systems have been successfully used to isolate the driver and passenger from both noise and vibration generated by the engine and the road inconsistencies.

Different kinds of engine mounting system, from elastomeric to hydraulic, and from passive to active, have been develop to improve the mount performance. Many recent studies have focused on studying and designing of hydraulic engine mounts. A comprehensive design and study on vibration isolating of engine and chassis has been established in a extensive range of amplitudes and frequencies by Haddow and Brach [1]. Adiguna *et al.* [2] in their lastest research have focused on transient response of a typical hydraulic engine mount (HEM) in both analytical and experimental approaches leading to well-developed modelling of switching mechanism of decoupler. Geisberger *et al.* [3] experiments on a typical HEM show that in high frequencies the system has a strong nonlinear behaviour which is not predicted with the theorical result. They have established the momentum equations for decoupler and inertia track fluids with one-orifice in both cases of excitations; high and low amplitudes. Therefore, it is not appropriate to apply their model for a simultaneous performance of components and both orifices. They also have mentioned that the new theoretical model is not suitable to explain the high frequency behaviour (>250 Hz). The numerical analyses performed by Golnaraghi and Nakhaie [4] shows that a simple nonlinear model clarify the switching mechanism of the decoupler. Nevertheless this study can not also lead to an accurate prediction of the decoupler behaviour. Redefining the nonlinearities in a theoretical model developed by Farshidian-Far and Yazdani[5], it has been possible to derive acceptable results in agreement with experimental data, by means of simultaneous considering of both orifices' performances. They have emerged the dynamic response of a typical HEM using the coupled equations of momentums.

This paper studies the nonlinear parameters of a comprehensive HEM model and tries to well-organized simultaneous models of decoupler and inertia track equations. The outcomes resulted from the model are valid for a broad range of amplitudes and frequencies. Qualitative and quantitative results have been discussed. Constitutive equations of fluids have been applied in both orifices [6]. Two nonlinear domains in frequencies higher than 250 Hz have been visualized that are in agreement with the experimental results of the reference [2] and the theoretical results of reference [3]. The experimental data have been used for evaluating nonlinear parameters [3] and the former hyper-elastic analysis of the authors in ANSYS [5] has been used to calculate complex stiffness of rubber component. The switching equations of decoupler system have been properly applied.

2- MODELING OF ENGINE MOUNT

Hydraulic engine mounts, generally, consist of two chambers, main rubber and hydraulic part where create dynamic behaviour in the system. In low amplitude and high frequency excitation fluid flow through the decoupler, due to minimum of decoupler resistance this system acts as normal mount. In high amplitude excitations the decoupler stick to engine and flow go through inertia track. The mathematical model of a hydraulic engine mount with decoupler and inertia track shown in Figure 1(a), is illustrated in Figure 1(b). Equations of this model consist of continuity equations of the upper and lower chambers and coupled momentum equations of two orifices of decoupler and inertia track, which are relatively indicated by equations (1) to (3):

$$C_1\dot{P}_1 = A_P(\dot{X} - \dot{X}_T) - Q_i - Q_d \tag{1}$$

$$C_2\dot{P}_2 = Q_i + Q_d \tag{2}$$

$$P_1 - P_2 = I_d\dot{Q}_d + (R_d + R_d'|Q_d| + R_{add})Q_d + I_i\dot{Q}_i + (R_i + \dot{R}_i|Q_i|)Q_i \tag{3}$$

State variables are:
P_1 (upper chamber pressure), P_2 (lower chamber pressure), Q_i (flow through the inertia track), Q_d (flow through the decoupler), the resistance R_{add}, is added to equation (3) to take account of decoupler switching resistance. Momentum equation is formed by considering inertia track and decoupler orifices as control volume simultaneously.

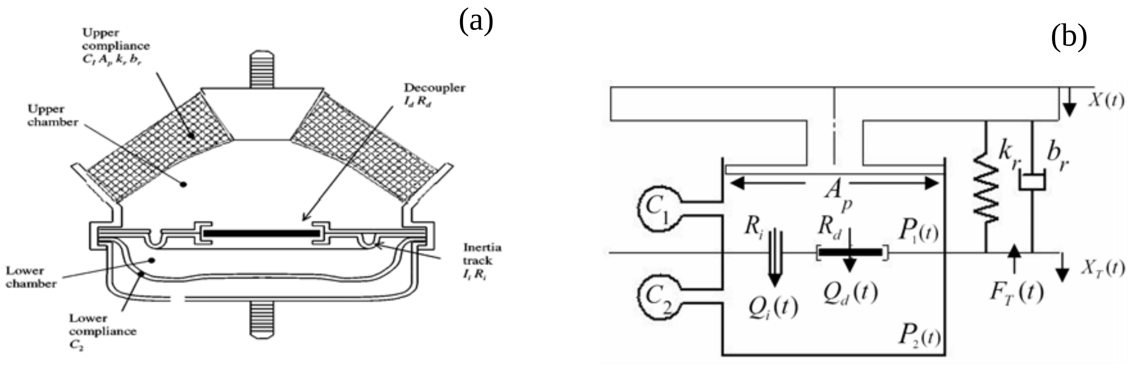

Figure 1. A typical hydraulic engine mount; (a) cross section, (b) mathematical model [3].

The number of dynamic variables of system is 4 (Q_i, Q_d, P_1, P_2) and the number of equations is reduced to 3 because of the coupling of momentum equations for forming state space equations, another independent equation should be derived from internal operation of orifices.
Transmitted force equation as dynamic response of the model is defined as equation (4), in which A_{d-fnc}, the relation of nonlinear cross section of decoupler is added.

$$F_T = k_r X + b_r \dot{X} + (A_p - A_{d-fnc})(P_1 - P_2) + A_p P_2 + A_{d-fnc}(R_d + R'_d |Q_d|) Q_d \qquad (4)$$

3-MODELING OF FLOW THROUGH THE INERTIA TRACK AND THE DECOUPLER

Figure 2 shows the mathematical model of both orifices, for finding the equation of flow through the inertia track and the decoupler, the control volume is considered with following assumptions:

1- Circular cross section.
2- Flat and straight wall with same cross section in thickness.
3- Constant decoupler thickness with flat profile.
4- Newtonian fluid, incompressible and viscous.
5- Laminar flow in thickness.
6- Independing of fluid characteristics to temperature variations due to wall friction.

Main assumptions for deriving aimed equation by detecting and classifying the type of flow through both orifices, can be presented as follows:

1- The first clear assumption is that the laminar flow through both orifices is a forced flow. A flow which is formed due to the pressure difference between upper and lower chambers and its direction varies due to the sign of this pressure difference.
2- Change of decoupler profile has no effect on the pressure difference between chambers $(P_1 - P_2)$, but the noticeable point is that this profile change effects the pressure gradient $(\partial P/\partial X)$, due to which the velocity gradient and, as we will see, the flow of internal current is effected. It's clear that if in flat profile $(\partial P/\partial X)$ is lineared, the convex or concave profile will make this gradient nonlinear. This ineffectiveness is because of that pressure difference only depends on system vibrational conditions, flexibility of upper chamber and dynamic stiffness of the rubber. The dependency of decoupler orifice resistance to profile is clear.
3- The flow (Q_i, Q_d) depends on both internal pressure gradient $(\partial P/\partial X)$ and external pressure difference.
4- Transmitted force, as expected, depends on external pressure and also the flow through decoupler and inertia track. The priority of force function dependency is on decoupler and inertia track resistance which effects the flow.
5- Because of the sudden change of profile in the beginning and end of both orifices, the fluid pressure falls. This pressure falls neglected in equations.
6- Another point is the linear movement of the decoupler wall. In flat type profile this movement does not affect Q_i and Q_d because the pressure gradient is constant and so the mean velocity of the fluid over X is constant. When the profile is not flat and because the other wall is fixed, the decoupler movement causes on alternative change in pressure gradient and equations change in time field. This will make the system behaviour nonlinear. The mean velocity in flat profile type, depends on decoupler wall speed $(\partial X/\partial t)$ more than situation. It's clear that the decoupler speed changes by time.

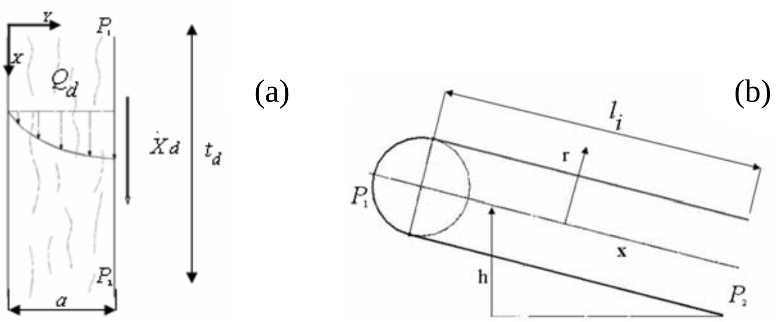

Figure 2. The mathematical model of both orifices; (a) Decoupler, (b) Inertia track.

4-BASIC EQUATIONS FOR INCOMPRESSIBLE NEWTONIAN FLUID THROUGH DECOUPLER

In this section, by considering continuum mechanics contexts and neglecting switching mechanism, equations will be derived. Regarding Figure 2(a) the two dimension field of velocity for laminar flow in decoupler control volume is:

$$v_i = \lfloor v_x \quad v_y \quad v_z \rfloor = [v_x(X, Y) \quad 0 \quad 0] \tag{5}$$

So the continuity equation for the control volume is:

$$\frac{\partial \rho}{\partial t} + v_i \rho_{,i} + \rho v_{i,i} = 0 \tag{6}$$

Considering the assumption of incompressibility of fluid and equilibrium conditions we will have $\dot{\rho} = 0$. So the continuity equation is simplified as:

$$v_{i,i} = \frac{\partial v_x}{\partial X} + \frac{\partial v_y}{\partial Y} + \frac{\partial v_z}{\partial Z} = 0 \tag{7}$$

Finally, considering the boundary conditions defined for velocity field, the continuity equation will become as $(\partial v_x / \partial X) = 0$. Consequently, the velocity field will be:

$$v_i = \lfloor v_x \quad v_y \quad v_z \rfloor = [v_x(Y) \quad 0 \quad 0] \tag{8}$$

Also, we know that the displacement-velocity tensor for incompressible fluid is $D_{ij} = \frac{1}{2}(v_{i,j} + v_{j,i})$.

For incompressible fluid we have $D_{kk} = 0$. Tensor of fluid internal pressure for incompressible fluid is derived as:

$$\sigma_{ij} = -P\delta_{ij} + 2\mu D_{ij} = \begin{bmatrix} -3P & \mu \frac{\partial v_x}{\partial Y} & 0 \\ \mu \frac{\partial v_x}{\partial Y} & -3P & 0 \\ 0 & 0 & -3P \end{bmatrix} \tag{9}$$

In which, δ_{ij} is the Kronecker delta, σ_{ij} is internal stress, P is internal fluid pressure, μ is the viscosity of Newtonian fluid and D is the displacement-velocity tensor. The second order

As expected, the stress tensor is a second order tensor with symmetric matrix. For achieving the equation for fluid displacement, we use the general displacement equation:

$$\sigma_{ij,j} + \rho b_i = \rho(v_{i,t} + v_j v_{i,j}) \tag{10}$$

In this equation, b_i is the body (volume) forces including electromagnetic force or gravity forces due to fluid weight in control volume. For more simplification we assume that the weight is the only body force in displacement equation of fluid in control volume $b_i = (g,0,0)$. By using the displacement equation in stress tensor, 3 obscure differential equations are obtained:

$$-3\frac{\partial P}{\partial X} + \mu\frac{\partial^2 v_x}{\partial^2 Y^2} + \rho g = \rho v_{x,t} \tag{11}$$

$$\mu\frac{\partial^2 v_x}{\partial Y^2} - 3\frac{\partial P}{\partial Y} = 0 \tag{12}$$

$$\frac{\partial P}{\partial Z} = 0 \tag{13}$$

By neglecting the equation 13 and assuming a two dimension range for solving the other two equations, we can conclude that, for solving the mentioned equation in two dimension plane (X,Y) and in time range t, we need 4 boundary conditions for velocity and flow pressure (forced flow, not free) and one initial condition. Defined conditions must explain the behaviour of system in it's displacement bounds correctly. 4 boundary conditions for velocity function $v = v(X,Y,t)$ and pressure function $P = P(X,Y,t)$ are:

Velocity boundary conditions: $v(X,0,t) = 0$ and $v(X,a,t) = \dot{X}_d(t)$,

Pressure boundary conditions: $P(0,Y,t) = P_1$ and $P(t_d,Y,t) = P_2$,

And initial condition is: $v(X,Y,0) = v_0$.

Assuming the equal distribution of pressure in a section, the distribution of pressure in independent from Y, so $(\partial P/\partial Y) = 0$. Consequently, equations (11) to (13) are simplified as:

$$-3\frac{\partial P}{\partial X} + \rho g = \rho v_{x,t} \tag{14}$$

$$\frac{\partial^2 v}{\partial Y^2} = 0 \tag{15}$$

Considering these equations we just use two velocity boundary conditions, one pressure boundary condition and one initial condition for solving equations.
Pay attention that (dX_d / dt) doesn't depend on X and depends only on pressure change of both chambers and viscosity effect of fluid. So (dX_d / dt) is an independent parameter from parameters of the orifice flow and velocity profile of internal flow depends on (dX_d / dt). By applying velocity boundary conditions in second order equation, the velocity field is:

$$v_x(Y,t) = \frac{Y}{a}\dot{X}_d(t) \tag{16}$$

And time derivative of the equation above will be:

$$\frac{\partial v_x(Y,t)}{\partial t} = \frac{Y}{a}\ddot{X}_d(t) \tag{17}$$

By placing this equation in equation (14) we will have:

$$-3\frac{\partial P(X,t)}{\partial X} + \rho g = \frac{Y}{a}\ddot{X}_d(t) \tag{18}$$

Now regardless to time dependency of equations and by applying pressure boundary conditions, pressure change field is:

$$P(X,t) = \frac{1}{3}\rho g X - \rho\frac{Y}{3a}\ddot{X}_d(t).X + P_1(t) \tag{19}$$

Flow through the decoupler is:

$$Q_d(t) = \frac{a\pi D_d}{2}\dot{X}_d(t) \tag{20}$$

In this equation, D_d is the mean diameter of the disc and the free space of decoupler from equation (20) we can understand that, on the other hand, all mentional characteristics of internal flow depend on time changes of disc movement and changes of it's thickness profile, and on the other hand, this equation provides a direct relation between switching mechanism and disk motion. Pay attention that the independency of flow from characteristics change a long decoupler thickness, is a result of flat thickness profile assumption which omits it's second order equations in flow calculation. Flow has the same direction (sign) as decoupler velocity.

5-BASIC EQUATIONS FOR INCOMPRESSIBLE NEWTONIAN FLUID THROUGH THE INERTIA TRACK

Remembering the inertia track model, shown in Figure 2(b), the velocity field in inertia track can be organized as:

$$v_i = \lfloor v_x \quad v_y \quad v_z \rfloor = [v_x(r,\theta,t) \quad 0 \quad 0] \tag{21}$$

With incompressibility assumption, the velocity depends only on r,t. Hegen-poisealle solution for velocity distribution of a laminar incompressible flow of pipes is:

$$v_x = (r,t) = -\frac{R_i^2 - r^2}{4\mu}\frac{\partial}{\partial x}(P + \gamma h) \tag{22}$$

By assuming horizontal inertia track and neglecting it's helix angle, we'll have $(\partial h/\partial x) = 0$. Now the flow the through inertia track can be obtained:

$$Q_i = \int_{r=0}^{r=R_i} -(\frac{R_i^2 - r^2}{4\mu})(\frac{\partial P(x,t)}{\partial X})(2\pi r dr) = -\frac{\pi R_i^4}{8\mu}\frac{\partial P(x,t)}{\partial X} \tag{23}$$

Because of the long length of inertia track in comparison with it's cross section and equal distribution of flow, the last equation can be simplified with a good approximation for medium pressure. The simplified equation is:

$$Q_i = -\frac{\pi R_i^4}{8\mu}\frac{P_2(t)-P_1(t)}{l_i} \tag{24}$$

SIMULATION AND DISCUSSIONS

The equations obtained above are constructed in Simulink according to the nonlinear parameters extracted from experimental data by Giesberger [3] and using complex stiffness for hyper-elastic modelling of rubber component in ANSYS software which has been done by Farshidian-far [5]. Also switching mechanism of decoupler was added to complete the equations and expansion the continuous equations to discontinuous behaviours of the system. Hence, equation A_{d-fnc} has used in the model [3].

The dynamic response of the model against the two standard excitation vibrations amplitude 0.1 mm and 2 mm in the ranges of 0 to 50 Hz (low frequency), 50 to 250 Hz (mean frequency) and 250 to 500 (high frequency) has been investigated

Figure 3 shows the flow through the decoupler and the inertia track in excitation amplitude 0.1 mm and frequency 50 Hz. It can be seen in Figure 3(a) that the switching of decoupler is happen due to zero flow through orifice.

The results show at the excitation amplitude of 0.1 mm, a smaller amount of switching occurs with gradual increase in frequency and also increasing in upper chamber pressure which cause to rigidity of the system, does not happen. This means reduction in transmitted force in low amplitude and high frequency. In this case the flow through the inertia track is approximately zero and the most flow crosses through the decoupler. Figure 3(c) shows the value of transmitted force in this excitation condition.

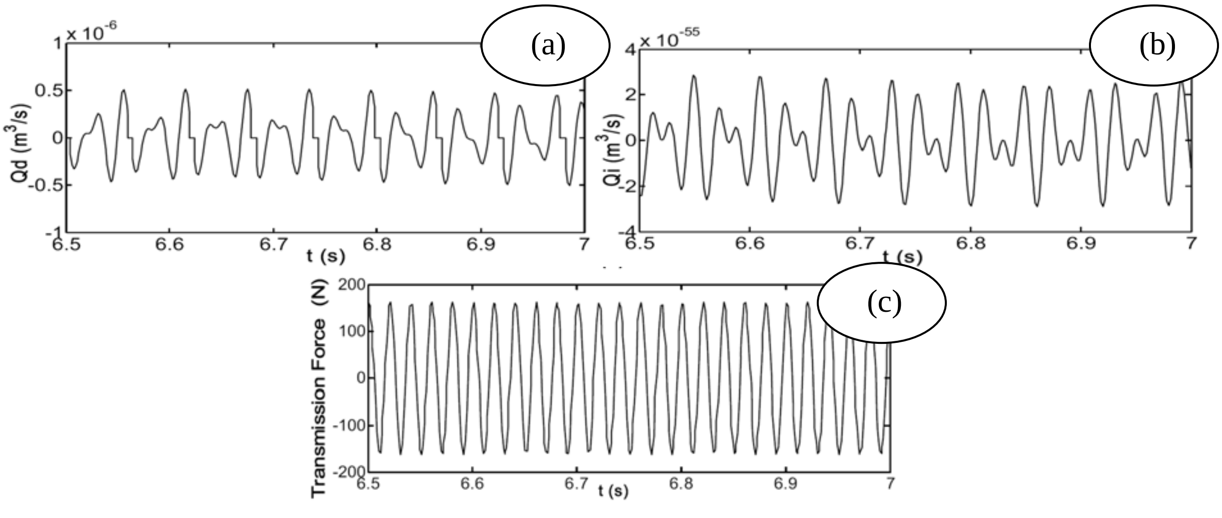

Figure 3. Excitation with amplitude 0.1 mm and frequency 50 Hz; (a)flow through the decoupler, (b)flow through inertia track, (c)transmitted force.

Figure 4 shows the flow crossing through the decoupler and the inertia track at on excitation amplitude of 2 mm and 50 Hz frequency. In high excitation amplitudes the decoupler had been closed and the fluid will pass through the inertia track. High resistance of inertia track in front of the fluid flow leads to increase in the upper pressure and consequently increasing the transmitted force up to 3000 N.

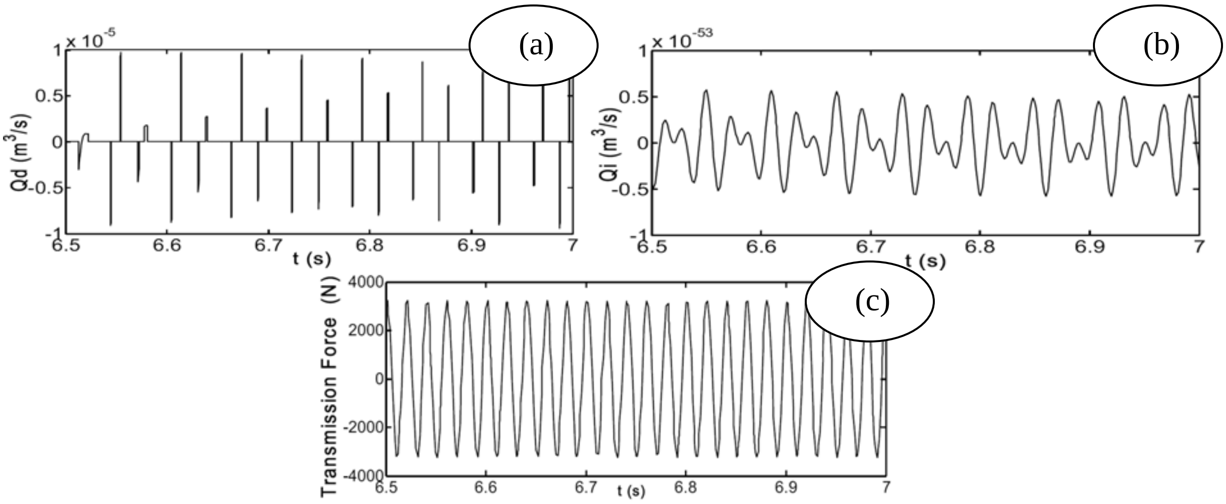

Figure 4. Excitation with amplitude 2 mm and frequency 50 Hz; (a) flow through the decoupler, (b) flow through the inertia track, (c) transmitted force.

CONCLUSION

In this paper, a more comprehensive model of hydraulic engine mount using coupled momentum equations of decoupler and inertia track have been proposed. It was shown that the model developed here provides the appropriate system response over the full range of loading conditions. The results show that at low excitation amplitude (0.1 mm), the transmitted force is minimized (150 N). Moreover, excitation with higher amplitudes (2mm) leads to more switching and caused to flow pass trough the inertia track with higher resistance. In this case, the rigidity of system will increase and the higher transmitted force is resulted (3000 N). Finally, the mathematical model responses of the mounts to various frequencies and amplitudes are compared to the measured responses. The comparisons show a very good agreement, which corroborate the nonlinear model of the mount with coupled equations.

REFERENCES

[1] Brach R. M. and Haddow A.,” On the dynamic response of hydraulic engine mounts”, *SAE Technical Paper Series* 931321(1993).

[2] Adiguna H., Tiwari M., Singh R. and Hovat D.,” Transient Response of a Hydraulic Engine Mount”, *Journal of Sound and Vibration*, **268**, 217-248(2003).

[3] Geisberger A., Khajepour A. and Golnaraghi F., “Non-linear Modeling of Hydraulic Engine Mounts: Theory and Experiment”, *Journal of Sound and Vibration*, **249**, 371-397(2002).

[4] Golnaraghi F. and R. Nakhaie, “Development and Analysis of A Simplified Nonlinear Model of A Hydraulic Engine Mount”, *Journal of Vibration and Control*, **7**, 495-526(2000).

[5] Farshidianfar A.,Yazdani H.”Nonlinear modeling of HEM”,ISME 2007 Tehran,Iran,2547(2007).

[6] G.E.Mase,T.Mace,*Continuum Mechanics for engineers*,Book ISBN 0-8493-1855-6 second edition, CRC Press, Florida (1991).