Optimal constrained power scheduling in Electricity Market

N. Zendehdel, A. Karimpour

Abstract—An optimal scheduling of units in an electric spot market presents in this paper. Unit commitment is a non-linear and complex combinatorial optimization problem which is difficult to be solved for large-scale power systems so this study addresses a linear expression of the problem. Proposed approach is a mixed-integer linear programming to minimize the total energy dispatch cost in 24 hours of a day. A system as the same structure as Iranian power market is used to demonstrate the linear expression of the problem. Simulation results compared with another approach. The results shows the applicability of the proposed method.

Index Terms-- optimal scheduling, spot market, mixed integer linear programming

I. NOMENCLATURE

\[
\begin{align*}
D_t & \quad \text{Real load demand during period } t \\
DR_t & \quad \text{Ramp-down and shut-down rate limit (MW/min) of generation unit } i \\
DT_t & \quad \text{Minimum down time of generation unit } i \\
p_{\text{max}}(i) & \quad \text{Maximum real power output of generation unit } i \\
p_{\text{min}}(i) & \quad \text{Minimum real power output of generation unit } i \\
Pr & \quad \text{Real power output of generation unit } i \text{ at period } t \\
Pr_{i,b} & \quad \text{Real power of block } b \text{ offered by generation unit } i \text{ at period } t \\
p_{\text{max}}_{i,b} & \quad \text{Maximum real power of block } b \text{ offered by generation unit } i \text{ at period } t \\
p_{\text{off}}(i, b) & \quad \text{Price offered by generation unit } i \text{ at hour } t \text{ for block } b \\
U_t^0 & \quad \text{Time periods that unit } i \text{ has been on or off at the beginning of the planning horizon (end of hour 0)} \\
UR_t & \quad \text{Ramp-up and start-up rate limit (MW/min) of generation unit } i \\
UT & \quad \text{Minimum up time of generation unit } i \\
\mathbf{u}_{i,t} & \quad \text{Binary variable } (0/1) \text{ that represents the commitment state of generation unit } i \text{ at period } t \\
\mathbf{x}_{i,t} & \quad \text{Number of hours that generation unit } i \text{ has been on } (+) \text{ or off } (-) \text{ at the end of hour } t
\end{align*}
\]

Sets

\[
\begin{align*}
T & \quad \text{Set of all period indexes in hours} \\
G & \quad \text{Set of indexes of all generation units} \\
B & \quad \text{Set of indexes of energy sale blocks}
\end{align*}
\]

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II. INTRODUCTION

The unit commitment (UC) has been a subject with increasing interests after 1998. It determines an optimal schedule of units and the amount power generation to be used to meet the demand over a future period. After the liberalization of the electricity industry, in most of countries, the unit commitment problem is solved as a market problem based on bid prices, instead of the cost-based minimization of the classical model. In the simplest formulation, unit commitment can be defined as the problem of finding the best strategy to turn on or switch off generation units in the most economic way taking into account power balance equations and a number of technical limitations. A unit commitment problem is often formulated subject to several constraints that includes minimum up-time and down-time, ramp rate limits, generation constraints, load balances, must-run units, minimum and maximum energy limits, power transmission line capacity and spinning reserve. From the view of mathematics, it is a mixed integer non-linear programming problem to minimize the energy dispatch cost and meet various system constraints.

The presented test system is based on the structure of the Iranian day-ahead pool-based electricity market. Demand-side are not considered in Iran competitive energy market. In Iranian day-ahead energy market, the market participants (producers) submit hourly energy multi-block price bids and the market operator sets the accepted bids. The structure of the energy market is settled in a pay as bid mechanism. Large-scale, mixed-integer, combinatorial, and nonlinear programming problem is an active research topic because of potential savings in operation costs. As a consequence, several solutior techniques have been proposed such as heuristics, Lagrangian relaxation (LR), dynamic programming (DP), mixed-integer linear programming (MILP), Branch and Bound (BB) and Priority List.

Genetic algorithms (GAs) has been employed to solve the unit commitment problem [1]-[6]. However, GAs are time-consuming since it requires binary encoding and decoding to represent each unit operation state and to compute the fitness function, respectively, throughout GA procedures. This huge computation, makes it difficult to apply to large-scale systems. Sishaj in [7] implements movement of arts in the search space and also discusses the accuracy of the solution with respect to the solution time. Simon in [8] has solved UC using ant colony system with its exploration and exploitation ability. Simulated Annealing (SA) is a method to solve the Unit commitment problem in [9] based on the simulation of recrystallizing metal in the
process of slow cooling (annealing). EP has been applied to the
economic load dispatch problem [10]-[11]. This
procedure, need to get a good starting point to converge.
The main draw back of all mentioned methods (GA, ant
colony, SA, EP, ) is that they don't guarantee the
optimality of the solution. Dynamic programming is also
used to solve the problem [12]-[13]. A mixed-integer linear
programming approach [14] allows a rigorous modeling of
non linear minimum up and down time constraints. That
approach is based on the formulation stated by Dillon et al.
[15]. Although DP and MIP can provide an optimal
schedule, they cannot be practically applied even for mid-
scale systems because of the so-called 'combinatorial
explosion' [16]. Branch and Bound solution technique is
used to solve UC problem [17]. A straightforward method to
find a solution to the unit commitment problem is the
Priority List (PL) method [19]-[20]. More complex models
of UC proposed Lagrangian relaxation (LR) [21]-[24]. The
LR method is a very effective method. The basic idea of LR
is relaxing the system constraints with Lagrangian
multipliers and formulating a Lagrangian dual function by
appending the relaxed constraints to the primal objective.
The process of solution includes major and minor iterations.
However, if the problem considered more constraints, the
more multipliers will be introduced, which may lead to slow
convergence especially when the constraints are highly
coupled. Even if the objective function of primal problem is
non-convex and if a solution to the dual problem is found,
the feasibility is not guaranteed due to the non-convexity of
the objective function of the primal (original) problem.

Here a linear expression for UC problem is obtained and
a 0/1 mixed integer linear programming is used to solve the
problem and to find the optimal schedule of units and the
corresponding amount of power generation.

The model has been tested on a system with the same
structure of Iranian power market. The proposed method
results compared with method described in [14]. The model
has been programmed in GAMS, using CPLEX solver to
solve the linear mixed-integer programming problem.

The rest of the paper is organized as follows. Section III
formulates the unit commitment problem. Section IV
described the idea of linearization of nonlinear constraints
and formulated the problem as a mixed-integer linear
programming formula. Section V provides results and
compares it with the result of another approach. Finally,
conclusions are stated in Section VI.

III. PROBLEM FORMULATION

The objective function can be stated as the minimization of:

$$\sum_{i \in G} \sum_{t} \sum_{j \in B} p_{t,j}^i u_{t,j} + \sum_{i \in G} \sum_{j \in B} p_{t,j}^i p_{t,j}$$

(1)

For simplicity, it is assumed that the minimum power of
each unit, $p_{t,j}^\text{min}$, is always offered as the first block and
other blocks contains submitted power of each unit that is
more than $p_{t,j}^\text{min}$.

The objective function will be subjected to the following
constraints:

Real power limits for any blocks:

$$0 \leq p_{t,j}^i \leq p_{t,j}^\text{max} \quad \forall t \in T, \forall i \in G, \forall (b+1) \in B$$

(2)

Real power output limits:

$$u_{t,j} \cdot p_{t,j}^i \leq p_{t,j}^i \leq u_{t,j} \cdot p_{t,j}^\text{max} \quad \forall t \in T, \forall i \in G$$

(3)

Which $p_{t,j}^i$ is the produced real power of unit $i$ at period
$t$ and it can be derived by:

$$p_{t,j}^i = u_{t,j} \cdot p_{t,j}^\text{min} + \sum_{b \in B} \sum_{b>1} p_{t,j}^i \quad \forall t \in T, \forall i \in G$$

(4)

Real power balance:

$$\sum_{i \in G} p_{t,j}^i \geq D_t \quad \forall t \in T$$

(5)

Ramp rate limits:

$$- \Delta R_i \leq p_{t,j}^i - p_{t-j-1,j}^i \leq \Delta R_i \quad \forall t \in T, \forall i \in G$$

(6)

Minimum starting up times:

$$(x_{t-1,i} - UT_i) (u_{t-1,i} - u_{t,i}) \geq 0 \quad \forall t \in T, \forall i \in G$$

(7)

Minimum starting down times:

$$(x_{t-1,i} + DT_i) (u_{t-1,i} - u_{t,i}) \leq 0 \quad \forall t \in T \forall i \in G$$

(8)

The addressed unit commitment problem is a mixed-
integer non-linear optimization problem with linear
objective function, binary decision variables, continuous
variables for operation processes, time couplings and non-
linear constraints, such as minimum up and down time
constraints. The difficulties related to resolution of non-
linear optimization problems with binary variables force to
make use of an alternative linear formulation of the
problem.

IV. TRANSLATE TO LINEAR MODEL

Mathematic model of UC is stated in the previous
section. This model contains nonlinear constraints such as
minimum up time and minimum down time of each unit.
This section presents an alternative linear formulation of
problem.

A. Linearization of Minimum Up Time Constraints

Nonlinear constraints "(7)," are replaced by the
equivalent linear constraints "(9)-(11)," below:

To satisfy the minimum up time constraint consider three
distinct situations.

- Suppose that the unit was initially in operation less than
its minimum up time, following equation will guaranty
the minimum up time constraint.

$$L_i = \text{Min}[T,(UT_i - U_i) \cdot \omega_{t,j}]$$

(9)

$$L_i = \text{Min}[T,(VT_i - V_i) \cdot \omega_{t,j}]$$

(10)

$$L_i = \text{Min}[T,(AT_i - A_i) \cdot \omega_{t,j}]$$

(11)

where $T_i$, $UT_i$, $VT_i$, $AT_i$ are the total,
minimum up time and minimum down time of each
unit.
Clearly, if the unit is initially de-committed or \( UT_j \leq U_j^0 \), constraints "(9)," are not included in the formulation.

- Unit must remain committed at least for \( UT_j \) hours at any consecutive time during the day; if it is necessary. Clearly if \( L_j + 1 \geq T - UT_j + 1 \) these constraints are not included.

\[
\sum_{j=k}^{k+UT_j-1} u_{j,i} \geq UT_j [u_{k,i} - u_{k-1,i}] \quad (10)
\]

\[ k = L_j + 1, \ldots, T - UT_j + 1 \]

- The unit that its minimum up time is greater than 2 must satisfy the minimum up constraint time in the last \( UT_j - 1 \) hours. It can be modeled by equation "(11)."

\[
\sum_{j=k}^{T} u_{j,i} - (u_{k,i} - u_{k-1,i}) \geq 0 \\
F_i \quad (12)
\]

\[ k = T - UT_j + 2, \ldots, T \]

**B. Linearization of Minimum Down Time Constraints**

To replace nonlinear constraints "(8)," by the equivalent linear constraints three distinct situations are considered.

- Suppose that the unit was initially off less than its minimum up time, following equation will guarantees the minimum down time constraint.

\[
F_i \quad (13)
\]

Clearly if the unit is initially committed or \( DF_j \leq U_j^0 \), constraints "(12)," are not included in the formulation.

- Unit must remain committed at least for \( DT_j \) hours at any consecutive time during the day. Clearly if \( F_j + 1 \geq T - DT_j + 1 \), these constraints are not included.

\[
\sum_{j=k}^{k+DT_j-1} (1 - u_{j,i}) \geq DT_j [u_{k-1,j} - u_{k,j}] \quad (13)
\]

\[ k = F_j + 1, \ldots, T - DT_j + 1 \]

- The unit that its minimum down time is greater than 2 must satisfy the minimum down time constraint in the last \( DT_j - 1 \) hours. It can be modeled by equation "(14)."

\[
\sum_{j=k}^{T} [1 - u_{j,i} - (u_{k,i} - u_{k-1,i})] \geq 0 \\
F_i \quad (14)
\]

\[ k = T - DT_j + 2, \ldots, T \]

**V. TEST SYSTEMS AND RESULTS**

The test carried out for the system that its structure is the same as Iranian electricity market. It is programmed in GAMS mathematical modeling language.

In appendix A, table A.1 addresses power demand for each time of the day, for simplicity just five hours reported. Table A.2 presents some characteristics of the unit generations available and the energy offer blocks and their bid prices are shown in table A.3.

Proposed linearization method and 0/1 mixed integer programming is employed to solve unit commitment problem and optimal schedule of units find. In [14] another linearization method presents to convert the nonlinear minimum up time and minimum down time constraints in to linear one. So this method employed and results are reported and compared with the result of our paper.

The test system includes 7 units: two gas plants, two steam units and three combined cycle plants. The total generation capacity amounts to 626 MW. The transmission network constraints are not included in this study. Results and optimal schedule reported only for five hours.

**Fig. 1. Optimal scheduling according to present method**

**Fig. 2. Optimal scheduling according to [14]**

In this section "fig. 1," shows described technique results and "fig. 2," shows results of employing [14] technique. Both "fig. 1," and "fig. 2," suggest the most economic way taking into account power balance equations and technical limitations and the objective function has the same value.

An approach in [14] suggests three sets of binary variable to linearize nonlinear constraints and it becomes problem more complex than one set of binary variable used in this paper. In addition, comparing results shown in "fig. 1," and "fig. 2," clarify that turn on or switch off generation units strategy in figure1 is more fair. Generating units 4 and 5 (G4, G5) are suitable to be used to compare the solution obtained in both papers.