Records

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Abstract

In this paper, based on k-record values from the two parameters of the exponential distribution, the maximum likelihood and Bayes estimators for the unknown parameters are obtained. Bayes estimates are obtained based on the squared error loss functions. The occurrence of future k-record values based on observed k-record values from the two parameters of the exponential distribution is considered. Prediction, either point or interval, for future k-record values are derived from Bayesian view point. Some non-Bayesian results can be obtained as limiting cases from our results.

Keywords: Admissibility; Bayes prediction; Bayesian estimation; Beta distribution; Chi-square distribution; Jefferys prior

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1 Introduction and Preliminaries

Let $\{X_i,\ i \geq 1\}$ be a sequence of identically independent distributed (iid) continuous random variables each distributed according to cumulative distribution function (cdf) $F(t)$ and probability density function (pdf) $f(t)$. An observation X_j will be called an upper record value if its value exceeds that of all previous observations. Thus, X_j is an upper record if $X_j > X_i$ for every $i < j$. An analogous definition can be given for lower record values.

Record data arise in a wide variety of practical situations. Examples include industrial stress testing, meteorological analysis, hydrology, seismology, sporting and athletic events, and oil and mining surveys. Properties of record data have been extensively studied in the literature. First Chandler (1952) studied the stochastic behavior of random record values arising from the "classical record model", that is, the record model where the underling sample from which records are observed is considered to consist of iid observations from continuous probability distribution. Today there are over 500 papers and several books published on record-breaking data. Interested readers are refer to Arnold *et al.* (1998), Nevzorov (2001), where the second book focuses on the stochastic behavior of records. An important problem is that of predicting future record values. This problem has been studied by a number of statisticians [Ahsanullah (1980), Berred (1998) and Dunsmore (1983)]. Recently, some work has been done from Bayesian view point based on record values, see, Ahmadi and Doostparast(2005), Malinowska and Szynal (2004), Jaheen (2003).

In the record value theory, while inverse sampling considerations have given valuable insights, their practical implementation is greatly hindered by the sparsity of records. In fact, the expected waiting time is infinite for every record after the first; but, this problem will be fixed by considering k-records instead (see Theorem 2.1 of Hofmann and Balakrishnan, 2004). Upper k-record process is defined in terms of the k-th largest X yet seen. For a formal definition, we consider the definition in Arnold *et al.* (1998), p. 43, in the continuous case, let $T_{1,k} = k, R_{1,k} = X_{1:k}$ and for $n \geq 2$, let

$$T_{n,k} = \min\{j : j > T_{n-1,k}, X_j > X_{T_{n-1,k}-k+1:T_{n-1,k}}\},$$

where $X_{i:m}$ denotes the i-th order statistic in a sample of size m. The sequence of *upper k-records* are then defined by $R_{n,k} = X_{T_{n,k}-k+1:T_{n,k}}$ for $n \geq 1$, Arnold *et al.* (1998) call them *Type 2 k-record* sequence. For $k = 1$, note that the usual records are recovered. An analogous definition can be given for *lower k-records* as well. These sequence of k-records were introduced by Dziubdziela and Kopocinski (1976) and it have found acceptance in the literature. The theory of k-th records is still developing. In reliability analysis, the life length of the r-out-of-n system is the (n-r+1)-th order statistic in a sample of size n. Therefore, the n-th k-record value can be regarded as just the life length of a k-out-of-$T_{n,k}$ system.

We assume that this type of k-record data is available and the aim of this paper is to develop the Bayesian estimation as well as prediction of future k-records based on past observed k-records. Using the point density of records, we readily have the joint density of the first m, k-records $R_{1(k)}, R_{2(k)}, ..., R_{m(k)}$ as

$$f(x_1, ..., x_m) = k^m \prod_{i=1}^{m} \frac{f(x_i)}{1 - F(x_i)} (1 - F(x_m))^k, \tag{1}$$

see, Arnold *et al.* (1998) Nevzorov, (2001).

A random variable X is said to have a two-parameter Exponential distribution, denoted by $X \sim Exp(\mu, \sigma)$, if its cumulative distribution function (cdf) is

$$F(x; \mu, \sigma) = 1 - e^{-\frac{1}{\sigma}(x-\mu)} \quad x \geq \mu, \quad \sigma > 0, \tag{2}$$

and hence the probability density function (pdf) is given by

$$f(x;\mu,\sigma)=\frac{1}{\sigma}e^{-\frac{1}{\sigma}(x-\mu)} \quad x\geq\mu, \quad \sigma>0. \tag{3}$$

The rest of the paper is organized as follows, based on k-record values from the two parameters of the exponential distribution, the maximum likelihood and Bayes estimators for the unknown parameters are obtained in Section 2. The Bayes estimates are obtained based on the squared error loss (SEL) function. Bayesian prediction of the future k-record values, either point or interval, are obtained in Section 3. Some non-Bayesian results can be obtained as limiting cases from our results.

2 Estimation

In this section, we shall be concerned with estimation of the two unknown parameters μ and σ of the exponential model based on k-record values. Suppose, we observed the first m upper k-record values $R_{1(k)}=x_1,\ R_{2(k)}=x_2,...,R_{m(k)}=x_m$ from an Exponential distribution, with location parameter μ and mean $\mu+\sigma$, with cdf and pdf given, respectively, by (2) and (3). Notice that, from (1), (2) and (3) it is easy to verify that the likelihood function is given by

$$L(\mu,\sigma|\underline{\mathbf{x}})=(\frac{k}{\sigma})^m e^{-\frac{k}{\sigma}(x_m-\mu)}, \quad \mu\leq x_1<x_2<...<x_m,\ \sigma>0 \tag{4}$$

where $\underline{\mathbf{x}}=(x_1,x_2,...,x_m)$.

2.1 Maximum Likelihood Estimation (MLE)

In the case $k=1$, the MLE of the two-parameters of the exponential distribution can be found in Arnold *et al.* (1998), p. 123, or Ahsanullah (1995). Now, we present MLE based on k-record values, by (4) the natural logarithm of (4) is given by

$$l = m \ln k - m \ln \sigma - \frac{k}{\sigma}(x_m - \mu), \quad \mu \leq x_1 < x_2 < ... < x_m. \tag{5}$$

Assuming that the parameters μ and σ are unknown, from (5) we readily obtain the MLE of μ and σ as follows

$$\hat{\mu}_M = R_{1(k)}, \tag{6}$$

and

$$\hat{\sigma}_M = \frac{k}{m}(R_{m(k)} - R_{1(k)}). \tag{7}$$

It is easy to verify that

- $R_{1(k)} \sim Exp(\mu, \sigma/k)$,
- $R_{m(k)} - R_{1(k)}$ and $R_{1(k)}$ are independent random variables,
- $R_{m(k)} - R_{1(k)}$ has gamma distribution with parameters $m-1$ and k/σ.

Then by (6) and (7) we have

- $E(\hat{\mu}_M) = \mu + \frac{\sigma}{k}$,
- $MSE(\hat{\mu}_M) = 2\frac{\sigma^2}{k^2}$.

Also,

- $E(\hat{\sigma}_M) = \frac{m-1}{m}\sigma$,
- $MSE(\hat{\sigma}_M) = \frac{\sigma^2}{m}$, do not depend on k.
- $Cov(\hat{\mu}_M, \hat{\sigma}_M) = 0$.

Notice that $\hat{\mu}_M$ is a biased estimator μ, an unbiased estimator for μ given by

$$\tilde{\mu} = \frac{m+k-1}{m-1}R_{1(k)} - \frac{k}{m-1}R_{m(k)}.$$

2.2 Bayes Estimation

Our aim is to obtain Bayes estimation of the unknown parameters based on $x_1, ..., x_m$ under SEL function. We consider the following three cases for our Bayesian estimation problem.

a) *Scale parameter σ is known and μ is unknown.*

Without loss of generality, we may assume $\sigma = 1$ then by (4), we have

$$f(\underline{\mathbf{x}}|\mu) = k^m e^{-k(x_m-\mu)}, \quad \mu < x_1 < x_2 < ... < x_m. \tag{9}$$

Assuming a prior distribution of the parameter μ in the form

$$\pi(\mu) \propto 1. \tag{10}$$

Hence the posterior distribution of μ is

$$\pi(\mu|\underline{\mathbf{x}}) \propto f(\underline{\mathbf{x}}|\mu)\pi(\mu),$$

where $f(\underline{\mathbf{x}}|\mu)$ is the joint distribution function given by (9) and $\pi(\mu)$ is the prior density given by (10). So, we have

$$\pi(\mu|\underline{\mathbf{x}}) = ke^{k(\mu-x_1)}, \quad \mu < x_1. \tag{11}$$

Assuming a SEL function, the Bayes estimate of a parameter is its posterior mean. Therefore, by (11) the Bayes estimate of the parameter μ is given by

$$\hat{\mu}_{1B} = x_1 - \frac{1}{k}. \tag{12}$$

Also, $R_{1(k)} \sim Exp(\mu, \frac{1}{k})$, by (12) we have

- $E(\hat{\mu}_{1B}) = \mu$,
- $MSE(\hat{\mu}_{1B}) = \frac{1}{k^2}$.

b) *Scale parameter σ is unknown and μ is known.*

Without loss of generality, we may assume $\mu = 0$, then by (4), we have

$$f(\underline{\mathbf{x}}|\sigma) = (\frac{k}{\sigma})^m e^{-\frac{k}{\sigma}x_m}, \quad 0 < x_1 < x_2 < ... < x_m. \tag{13}$$

Assuming σ^{-1} is distributed as $\Gamma(\alpha, \beta)$, i.e.,

$$\pi(\sigma) = \frac{\beta^\alpha}{\Gamma(\alpha)} \frac{1}{\sigma^{\alpha+1}} e^{-\frac{\beta}{\sigma}}, \quad \sigma > 0, \tag{14}$$

where $\alpha > 0$, $\beta > 0$ are known. Then the posterior distribution of σ is

$$\pi(\sigma|\underline{\mathbf{x}}) \propto f(\underline{\mathbf{x}}|\sigma)\pi(\sigma),$$

where $f(\underline{\mathbf{x}}|\sigma)$ is the joint distribution function given by (13), $\pi(\sigma)$ is the prior density given by (14). So, we have

$$\pi(\sigma|\underline{\mathbf{x}}) = \frac{(\beta + kx_m)^{m+\alpha}}{\Gamma(m+\alpha)} \frac{1}{\sigma^{m+\alpha+1}} e^{-\frac{1}{\sigma}(\beta+kx_m)}, \quad \sigma > 0. \tag{15}$$

Therefore, by (15) the Bayes estimate of the parameter σ is given by

$$\hat{\sigma}_{1B} = \frac{\beta + kx_m}{m+\alpha-1}. \tag{16}$$

Obviously, $\hat{\sigma}_{1B}$ is the unique Bayes estimate of σ and hence is admissible. Also, $R_{m(k)} \sim \Gamma(m, \frac{k}{\sigma})$ by (16) we have

- $E(\hat{\sigma}_{1B}) = \frac{\beta+m\sigma}{m+\alpha-1}$,
- $MSE(\hat{\sigma}_{1B}) = \frac{m^2+(\alpha-1)(m-\sigma)+\beta^2}{(m+\alpha-1)^2}$, do not depend of k.

c) *μ and σ are both unknown.*

Under the assumption that both of the parameters μ and σ are unknown, we may consider the joint density as a product of the conditional density of μ for given σ and a two parameter inverted gamma density for σ. So the joint prior density of μ and σ can be written as

$$\pi(\mu, \sigma) = \pi_1(\mu|\sigma)\pi_2(\sigma), \tag{17}$$

where

$$\pi_1(\mu|\sigma) \propto \sigma^{-1}, \tag{18}$$

which is the Jeffreys non-informative prior distribution [see Berger, (1985)] of the parameter μ for fixed value of the parameter σ, and

$$\pi_2(\sigma) = \frac{\beta^\alpha}{\Gamma(\alpha)} \frac{1}{\sigma^{\alpha+1}} e^{-\frac{\beta}{\sigma}}, \quad \sigma > 0, \alpha > 0, \beta > 0, \tag{19}$$

i.e., $\sigma^{-1} \sim \Gamma(\alpha, \beta)$, which is the conjugate prior distribution of the parameter σ for fixed value of μ. Substituting (18) and (19) in (17), we get

$$\pi(\mu, \sigma) \propto \frac{\beta^\alpha}{\Gamma(\alpha)} \frac{1}{\sigma^{\alpha+2}} e^{-\frac{\beta}{\sigma}}. \tag{20}$$

Hence, the joint posterior distribution of μ and σ is

$$\pi(\mu, \sigma | \underline{\mathbf{x}}) \propto f(\underline{\mathbf{x}} | \mu, \sigma) \pi(\mu, \sigma), \tag{21}$$

where $f(\underline{x}|\mu, \sigma)$ is the joint distribution function given by (4) and $\pi(\mu, \sigma)$ is the joint prior density given by (20).

Applying (4) and (20) in (21) the joint posterior density is given by

$$\pi(\mu, \sigma | \underline{\mathbf{x}}) = \frac{k[\beta + k(x_m - x_1)]^{m+\alpha}}{\Gamma(m+\alpha)} \frac{1}{\sigma^{m+\alpha+2}} e^{-\frac{1}{\sigma}[\beta + k(x_m - \mu)]}. \tag{22}$$

Therefore, by (22) under SEL the Bayes estimate of the parameter σ is given by

$$\hat{\sigma}_{2B} = \frac{\beta + k(x_m - x_1)}{m + \alpha - 1}. \tag{23}$$

Notice that, as $\beta \to 0$ and $\alpha \to 1$, $\hat{\sigma}_{2B} \to \hat{\sigma}_{ML}$. By (23) we have

- $E(\hat{\sigma}_{2B}) = \frac{\beta + \sigma(m-1)}{m+\alpha-1}$,
- $MSE(\hat{\sigma}_{2B}) = \frac{m-1}{(m+\alpha-1)^2}\sigma^2 + \frac{(\beta - \sigma\alpha)^2}{(m+\alpha-1)^2}$.

Also, the Bayes estimate of the parameter μ is given by

$$\hat{\mu}_{2B} = x_m + \beta - \frac{m+\alpha}{k(m+\alpha-1)}[\beta + k(x_m - x_1)]. \tag{24}$$

By (24), we have

- $E(\hat{\mu}_{2B}) = \mu + \alpha\frac{\sigma}{k} + [1 - \frac{(m+\alpha)}{k(m+\alpha-1)}]\beta$,
- $Var(\hat{\mu}_{2B}) = \frac{\sigma^2}{k^2}[1 + \frac{m-1}{(m+\alpha-1)^2}]$,
- $Cov(\hat{\mu}_{2B}, \hat{\sigma}_{2B}) = -\frac{(m-1)\sigma^2}{(m+\alpha-1)k}$.

3 Bayes Prediction

In this section, we consider the problem of prediction, either point or interval, for future k-record values from Bayesian approach. Assume that we have the first m upper k-records $R_{1(k)} = x_1, R_{2(k)} = x_2, ..., R_{m(k)} = x_m$ from the $Exp(\mu, \sigma)-$distribution. Based on such a sample, prediction, either point or interval, is needed for s-th upper k-record, $1 \leq m < s$. Now, let $Y = R_{s(k)}$ be the s-th upper k-record value, $1 \leq m < s$. The conditional pdf of Y for the given vector parameter θ and that the m-th 1-record had been observed is given by

$$f^*(y|\underline{\mathbf{x}}, \theta) = \frac{[H(y) - H(x_m)]^{s-m-1}}{\Gamma(s-m)} \frac{f(y|\theta)}{1 - F(x_m|\theta)}, \quad x_m < y < \infty, \tag{25}$$

where $f(.), F(.)$ are, respectively, the pdf and cdf of X, $H(.) = -\ln[1 - F(.)]$ [see Arnold *et al.* (1998)]. For the k-record, it is enough substituted F in (25) by $1 - (1 - F(.))^k$. Also, the Bayes predictive density function of Y given $\underline{\mathbf{x}}$ is given by

$$h^*(y|\underline{\mathbf{x}}) = \int_\Theta f(y|\underline{\mathbf{x}}, \theta)\pi(\theta|\underline{\mathbf{x}})d\theta. \tag{26}$$

We consider the following three cases.

a) σ *is known and* μ *is unknown*

Without loss of generality, we may assume $\sigma = 1$, then By (2), (3) and (25), we have

$$f^*(y|x_m, \mu) = \frac{k^{s-m}}{\Gamma(s-m)}(y - x_m)^{s-m-1} e^{-k(y-x_m)}, \quad y > x_m, \tag{27}$$

which is independent of μ. So by (26) and (27) we have

$$\begin{aligned} h^*(y|\underline{\mathbf{x}}) &= \int_{-\infty}^{x_1} f^*(y|\underline{\mathbf{x}}, \mu)\pi(\mu|\underline{\mathbf{x}})d\mu \\ &= \frac{k^{s-m}}{\Gamma(s-m)}(y - x_m)^{s-m-1} e^{-k(y-x_m)}, \quad y > x_m, \end{aligned} \tag{28}$$

for any posterior distribution (therefore, for any prior distribution) $\pi(\mu|\underline{\mathbf{x}})$. By (28), we have

$$Y - x_m|\underline{\mathbf{x}} \sim \Gamma(s-m, k).$$

So,

$$\hat{Y}_1 = x_m + \frac{s-m}{k}. \tag{29}$$

By (29) we have

- $E(\hat{Y}_1) = \mu + \frac{s}{k}$,
- $MSE(\hat{Y}_1) = \frac{s-m}{k^2}$.

Let $t_1(Y) = 2k(Y - x_m)|\underline{\mathbf{x}}$, then it is easy to verify that has Chi-square distribution with $2(s-m)$ degrees of freedom. Let χ^2_γ denotes the γ-th percentage of Chi-square distribution with $2(s-m)$ degrees of freedom. Then one can obtain the $100(1-\gamma)\%$ Bayesian prediction interval for $Y_s = R_{s(k)}$, that is given by

$$(L_1,\ U_1),$$

where L_1 and U_1 are the lower and upper bounds, respectively and given by

$$L_1 = x_m + \frac{\chi^2_{\frac{\gamma}{2}}}{2k},$$

and

$$U_1 = x_m + \frac{\chi_{1-\frac{\gamma}{2}}{}^2}{2k}.$$

b) *σ is unknown and μ is known.*

Without loss of generality, we may assume $\mu = 0$, then by (2), (3) and (25), we have

$$f^*(y|\underline{\mathbf{x}}, \sigma) = (\frac{k}{\sigma})^{s-m} \frac{(y-x_m)^{s-m-1}}{\Gamma(s-m)} e^{-\frac{k}{\sigma}(y-x_m)}, \quad y > x_m. \tag{30}$$

By (15), (26) and (30) the Bayesian predictive density function of $Y = R_{s(k)}$, for the given past m, k-records, is given by

$$h(y|\underline{\mathbf{x}}) = \int_0^\infty f^*(y|x_m, \sigma)\pi(\sigma|\underline{x})d\sigma$$

$$= \frac{1}{B(s-m,m+\alpha)}(\frac{kx_m+\beta}{ky+\beta})^{m+\alpha}(1-\frac{kx_m+\beta}{ky+\beta})^{s-m}$$
$$\times\frac{1}{y-x_m}, \qquad y > x_m, \tag{31}$$

where $B(.,.)$ is the complete beta function. Hence by (30)

$$E(\beta+kY|\underline{\mathbf{x}}) = \frac{(\beta+kx_m)(s-1+\alpha)}{m-1+\alpha}. \tag{32}$$

So, the Bayes point predictor of the s-th upper k-record is given by

$$\hat{Y}_2 = E(Y|\underline{x}) = \frac{s+\alpha-1}{m+\alpha-1}x_m + \frac{s-m}{m+\alpha-1}\frac{\beta}{k}. \tag{33}$$

By (33) we have

- $E(\hat{Y}_2) = \frac{m\sigma(s+\alpha-1)+\beta(s-m)}{k(m+\alpha-1)}$,
- $MSE(\hat{Y}_2) = (s-m)\{\frac{\sigma^2}{k^2}(1+\frac{m(s-m)}{(m+\alpha-1)^2}) + \frac{(s-m)}{(m+\alpha-1)^2}(\frac{\beta}{k}+1-m)\}$.

Let $t_2(Y) = \frac{kx_m+\beta}{kY+\beta}$, then it is easy to verify that $t_2(Y)$ has a Beta distribution with parameters $m+\alpha$ and $s-m$ which is independent of Y, thus $t_2(Y)$ is a pivotal quantity and one can use $t_2(Y)$ for constructing a Bayesian prediction interval for Y. Let b_γ be the γ-th percentage of $Beta(m+\alpha, s-m)$-distribution. Hence, the $100(1-\gamma)\%$ Bayesian prediction interval for $Y = R_{s(k)}$ is given by

$$(L_2,\ U_2),$$

where

$$L_2 = \frac{x_m}{b_{1-\frac{\gamma}{2}}} + \frac{\beta}{k}(\frac{1}{b_{1-\frac{\gamma}{2}}} - 1),$$

and

$$U_2 = \frac{x_m}{b_{\frac{\gamma}{2}}} + \frac{\beta}{k}(\frac{1}{b_{\frac{\gamma}{2}}} - 1).$$

c) *μ and σ are both unknown*

Let $Y = R_{s(k)}$ be the s-th upper k-record value, $1 \le m < s$. So, by (2), (3) and (25) we have

$$f^*(y|\underline{\mathbf{x}},\mu,\sigma) = (\frac{k}{\sigma})^{s-m}\frac{(y-x_m)^{s-m-1}}{\Gamma(s-m)}e^{-\frac{k}{\sigma}(y-x_m)}. \tag{34}$$

By (3), (22) and (34) Bayesian predictive density function of $Y = R_{s(k)}$, for the given past m records, is given by

$$h(y|\underline{\mathbf{x}}) = \int_{-\infty}^{x_1}\int_0^{\infty} f^*(y|\underline{\mathbf{x}},\mu,\sigma)\pi(\mu,\sigma|\underline{\mathbf{x}})d\sigma d\mu$$
$$= \frac{1}{B(m+\alpha, s-m)}(\frac{k(x_m - x_1)+\beta}{k(y-x_1)+\beta})^{m+\alpha}$$
$$\times(1 - \frac{k(x_m - x_1)+\beta}{k(y-x_1)+\beta})^{s-m}\frac{1}{y-x_m},\ y > x_m. \tag{35}$$

Now, by (35) the Bayes point predictor of the s-th upper k-record is given by

$$\hat{Y}_3 = E(Y|\underline{\mathbf{x}}) = \frac{s+\alpha-1}{m+\alpha-1}x_m + \frac{s-m}{m+\alpha-1}(\frac{\beta}{k} - x_1). \tag{36}$$

By (36) we have

- $E(\hat{Y}_3) = \frac{k\mu(\alpha+m-1)+s\sigma(m-1)+\beta(s-m)}{k(m+\alpha-1)}$,
- $MSE(\hat{Y}_3) = (s-m)\{\frac{\sigma^2}{k^2}(1+\frac{(m-1)(s-m)}{(m+\alpha-1)^2}) + (s-m)(\frac{\alpha}{m+\alpha-1} - \frac{\beta}{k})^2\}$.

Let $t_3(Y) = \frac{k(x_m - x_1)+\beta}{k(Y-x_1)+\beta}$, by (35), it can be shown that $t_3(Y)$ has a Beta distribution with parameters $m+\alpha$ and $s-m$ which is independent of Y, thus $t_3(Y)$ is a pivotal quantity and one can use $t_3(Y)$ for constructing a Bayesian prediction interval for Y. Hence, the $100(1-\gamma)\%$ Bayesian prediction interval for $Y = R_{s(k)}$ is given by

$$(L_3,\ U_3),$$

where

$$L_3 = \frac{x_m - x_1}{b_{1-\frac{\gamma}{2}}} + \frac{\beta}{k}(\frac{1}{b_{1-\frac{\gamma}{2}}} - 1) + x_1,$$

and

$$U_3 = \frac{x_m - x_1}{b_{\frac{\gamma}{2}}} + \frac{\beta}{k}(\frac{1}{b_{\frac{\gamma}{2}}} - 1) + x_1.$$

4 Conclusion

In this paper, the maximum likelihood and Bayes methods of estimation are used for estimation of the parameters of the exponential distribution based on

k-record values. Bayesian prediction bounds are obtained for future k-record values.

• It is interesting to compare the results of this paper with non-Bayesian results. For instance, for $k = 1$, we consider the Bayes estimator in (23) with the usual estimator $(R_{m(1)} - R_{1(1)})/m$ [BLIE, See Arnold et al. (1998), p. 143]. Before any observations are taken, the estimator from the Bayesian approach is the expectation of the prior, by (19) is given by $\beta/(\alpha - 1)$. Once the first m record data have been observed, the non-Bayesian (for example, BLIE) estimator is $(R_{m(1)} - R_{1(1)})/m$. The Bayes estimator σ given by (23) lies between these two. In face,

$$\frac{R_{m(1)} - R_{1(1)} + \beta}{m + \alpha - 1} = (\frac{\alpha - 1}{m + \alpha - 1})\frac{\beta}{\alpha - 1} + (\frac{m}{m + \alpha - 1})\frac{R_{m(1)} - R_{1(1)}}{m},$$

is a weighted average of $\beta/(\alpha - 1)$, the estimator of σ before any observation are taken, and $(X_{T_m} - X_{T_1})/m$, the BLIE without consideration of a prior.

• In Bayes theory, the prior parameter is assumed to be known. If the prior parameter are unknown the empirical Bayes approach may be used to estimate such parameters, [see, for example, Martiz and Lwin (1989)].

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