Position Error Reduction of Kinematic Mechanisms Using Tolerance Analysis and Cost Function

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Abstract
It is practically impossible to manufacture a component exactly with the required dimensions. Therefore for each part dimension, a tolerance limit is prescribed. Also for all assemblies, a limit of variation is prescribed for a specified parameter of the assembly which is referred to as the assembly specification, and it could be the position of a point. If the assembly specification has limits of variation in two or more directions, the correlation between these variations also impresses the limit of variation. To determine the bivariate distribution of the assembly specification, in terms of part tolerances, the Direct Linearization Method (DLM) is used. In this paper, the Coupler Point (C.P.) position of a crank slider mechanism during one cycle of motion is considered as the assembly specification. The DLM results are validated with Monte Carlo simulation method and the percent contribution of each manufacturing variable in assembly specification is determined by multiple regression method. This paper proposes that by tightening the tolerances of those manufacturing variables that have the highest contribution in the maximun error of mechanism, the amount of maximum error of mechanism could be decreased significantly.

Keywords: Tolerance Analysis, Error, Multiple Regression

1 Introduction
An important aim in designing kinematic linkages is creating an accurate path by means of a point on the coupler. This point and the corresponding path are called Coupler Point (C.P.) and Coupler Point Path, respectively. In each cycle of motion, manufacturing tolerances of the parts cause a deviation in the C.P. path from its designed or ideal state. These deviations can lead to undesirable performance of the mechanism. There are several methods which were proposed to determine the effect of part tolerances on C.P. path deviations or the performance of the mechanism. An efficient method in assembly tolerance analysis, called the Direct Linearization Method (DLM). The Direct Linearization Method (DLM) is firstly presented by Marler [1]. This method has been extended by Parkinson and Chase for static structures and kinematic mechanisms [2]. Wittwer investigated path mechanism error by DLM and comprised with other statistics analysis methods [3]. They assumed that all components are rigid.
In Section 2 of this paper, the kinematic model of a four-bar mechanism including tolerances of manufacturing variables is expressed. In Section 3, the Direct Linearization Method is demonstrated and the equations of vector loops, sensitivity

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matrix and position error are obtained. In the following section, the DLM method is applied to find the bivariate distribution of the C.P. position error. The valid domains of the DLM are determined by means of Monte Carlo simulation in Section 5. So in next sections, the percent contribution of manufacturing variables in the assembly specification by multiple linear regression is then determined. Finally maximum error of mechanism, by changing tolerance of the variables have major affect in error function, is reduced. Therefore, a mechanism rejects is decreased.

2 Crank slider mechanism model

In the current work, The C.P position error of a crank slider mechanism is analyzed, (see Figure 1). The reference path of C.P. is generated by assuming nominal dimensions for all components.
For each component of the mechanism, the manufacturing tolerances are specified on the basis of corresponding manufacturing processes and length of dimension [4]. Hence, tolerances of each nominal dimensions based on Figure 2 are selected, and in table 1 is presented.

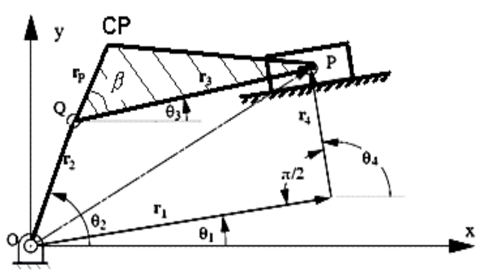

Figure 1: Crank slider mechanism with driving crank.

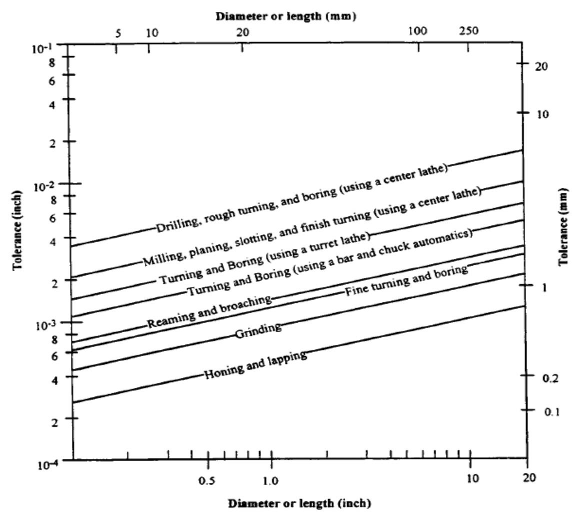

Figure 2: Tolerance range of machining processes [5].

Table 1. Nominal dimensions and tolerances of manufacturing variables (mm).

Manufacturing Variables	r_2	r_3	r_4	r_p	β	θ_1
Nominal Dimension	250	400	25	104	80°	0°
Tolerance	± 0.3	± 0.2	± 0.02	± 0.15	± 0.5°	± 0.5°

Angular position of link 2 (θ_2) is considered as an input to the mechanism. Therefore, it is not a manufacturing dimension and zero tolerance is assigned. All manufacturing dimensions are assumed to be normally distributed with a mean equal to the nominal link length. Also, the acceptable limit of distribution is taken according to common standard of 3σ.

3 Direct Linearization Method(DLM)

The Direct Linearization Method (DLM) can be used to determine the position error of a kinematic linkage. In this paper , point C.P. is designed to follow a specific path as the input crank (link 2) is rotated. The nominal position of point C.P. for a given input crank angle, θ_2, is found by solving one closed vector loop equations and one open vector loop equation (see Figure 3).

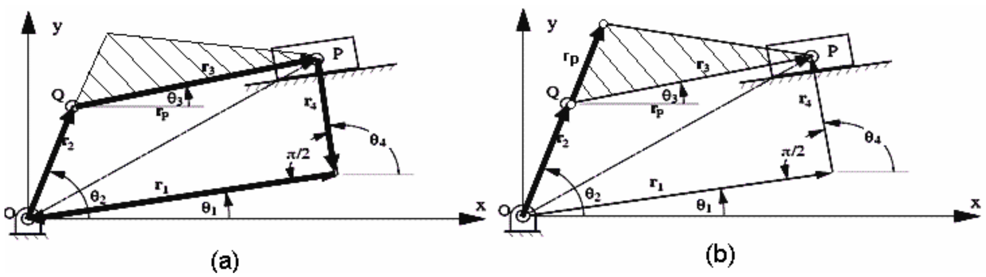

Figure 3: (a) Closed vector; (b) loop Open vector loop

Since position of point C.P. is defined by two direction x and y, so each vector loops is separated in to two equation. Closed and open loop equations are shown as follow, respectively:

$$h_x = r_2 \cos\theta_2 + r_3 \cos\theta_3 - r_4 \cos\theta_4 - r_1 \cos\theta_1 \tag{1}$$

$$h_y = r_2 \sin\theta_2 + r_3 \sin\theta_3 - r_4 \sin\theta_4 - r_1 \sin\theta_1 \tag{2}$$

$$(C.P.)_x = r_2 \cos\theta_2 + r_p \cos(\theta_3 + \beta) \tag{3}$$

$$(C.P.)_y = r_2 \sin\theta_2 + r_p \sin(\theta_3 + \beta) \tag{4}$$

In this method, the sensitivity matrix is derived using open and closed vector loops. The position error can be predicted by applying statistical approaches. Therefore ,It will be assumed that the actual dimensions are normally distributed with a mean equal to the nominal link length with a standard deviation 3σ.
Partial derivatives of equations 1 and 2 with respect to the manufacturing variables, give us limit of assembly variables. These equations are then linearized using a first-order Taylor's series expansion [6]. This is written as:

$$[A]\{dX\} + [B]\{dU\} = 0 \tag{5}$$

Where $\{X\}=\{r_2,r_3,r_4,r_p,\beta,\theta_1,\theta_2\}$is the vector of manufacturing variables and $\{U\}=\{\theta_3, r_1\}$ is the vector of assembly variables. [A] and [B] are matrices which represent first-order derivatives of equations (1) and (2) with respect to the manufacturing and assembly variables, respectively, i.e.

$$[A] = \partial h_i \Big/ \partial X_j \tag{6}$$

$$[B] = \partial h_i \Big/ \partial U_j \tag{7}$$

Equation (5) can be rewritten as:

$$\{dU\} = -[B]^{-1}[A]\{dX\} \tag{8}$$

A similar process is applied for open loop equations. Equation (9) expresses the variations of the assembly specification, i.e. C.P., in terms of the manufacturing and assembly variables.

$$\{d(C.P.)\} = [C]\{dX\} + [D]\{dU\} \quad (9)$$

Where [C] and [D] are first-order derivatives of equations (3) and (4) with respect to the manufacturing and assembly variables, respectively, i.e.

$$[C] = \partial (C.P.)_i \,/\, \partial X_j \quad (10)$$

$$[D] = \partial (C.P.)_i \,/\, \partial U_j \quad (11)$$

By substituting equation (8) into (9), the following equation is obtained.

$$\{d(C.P.)\} = ([C] - [D][B]^{-1}[A])\{dX\} = [S_{ij}]\{dX\} \quad (12)$$

where $[S_{ij}]$ is the sensitivity matrix of the assembly variables and can be written as:

$$[S_{ij}] = [C] - [D][B]^{-1}[A] \quad (13)$$

Based on the sensitivity matrix, the influence of each manufacturing variable on the assembly specification can be evaluated using Root Sum Square (RSS) statistical approach. The variance of the univariate normal distribution, which expresses the spread of the distribution, is determined using DLM method and computed by equation (14) [7].

$$\sigma_{(C.P.)_i}{}^2 = Var\,(C.P.)_i = \sum_{j=1}^{n} (S_{ij})^2 \sigma_j{}^2 \quad (14)$$

In the above equation, σ_j^2 is the variance of j-th manufacturing. In the case of multivariate distribution, the variance of manufacturing variables is expressed as the variance matrix V, which presents the variance of each manufacturing variable along with the correlation between the variables [7]. It is assumed that there is no correlation between the manufacturing variables. Therefore, the variance matrix is diagonal and written as follows:

$$[V] = \begin{bmatrix} \sigma_1{}^2 & 0 & \cdots & 0 \\ 0 & \sigma_2{}^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma_n{}^2 \end{bmatrix} \quad (15)$$

4 Bivariate Normal Distribution

Brown estimated the concurrent variation limits of two assembly specifications $d(C.P.)_X$ and $d(C.P.)_Y$ by the following equation [7]:

$$[\Sigma] = [S_{ij}][V][S_{ij}]^T \quad (16)$$

The covariance matrix Σ for bivariate distribution of assembly specification is presented by:

$$[\Sigma] = \begin{bmatrix} V_X & V_{XY} \\ V_{XY} & V_Y \end{bmatrix} \quad (17)$$

The diagonal elements indicate the deviations of each individual variable while the off-diagonal elements describe correlation between variables. The eigenvalues of the covariance matrix indicate the magnitude and direction of greatest variations. These eigenvalues are principle variances that represent the major and minor diameters of the elliptic contour of distribution [3]. The eigenvalues of 2-order variance matrix are determined as follows:

$$V_1 = \frac{V_X + V_Y}{2} + \sqrt{V_{XY}^2 + \left(\frac{V_Y - V_X}{2}\right)^2} \quad (18)$$

$$V_2 = \frac{V_X + V_Y}{2} - \sqrt{V_{XY}^2 + \left(\frac{V_Y - V_X}{2}\right)^2} \quad (19)$$

Also, the rotation angle of principle axes to the y axis is given by:

$$\theta_R = \frac{1}{2}\tan^{-1}\left(\frac{2V_{XY}}{V_Y - V_X}\right) \quad (20)$$

The contour of equal probability can be presented by the following equation in polar coordinates (r,θ) [3].

$$\frac{\cos^2(\theta-\theta_R)}{V_2^2} + \frac{\sin^2(\theta-\theta_R)}{V_1^2} = \frac{n^2}{r^2} \quad (21)$$

Where n is the sigma-level of the process. The maximum normal-to-path error, which is defined as the maximum perpendicular distance between distribution contour and the nominal C.P. path, is estimated with standard deviation of ±3σ. For example, the distribution contour of C.P. at θ_2=3° is computed and demonstrated in Figure 4. The maximum normal error is also illustrated in the Figure.

5 Comparison to Monte Carlo Simulation

In this section, the results obtained by DLM are compared to the Monte Carlo simulation. For this purpose, the Monte Carlo simulation, based on reliability of 95%, is performed with 400,000 samples of mechanism at each value of θ_2 [7]. Figure 5 demonstrates the comparison of DLM with Monte Carlo at θ_2=3°.

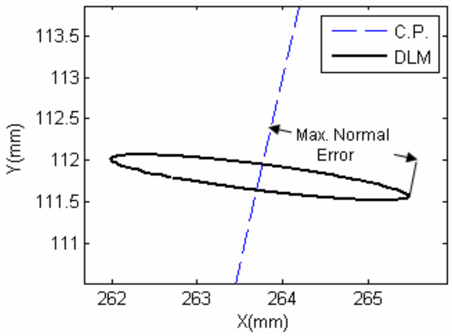

Figure 4: Bivariate distribution of C.P. at. θ_2=3°

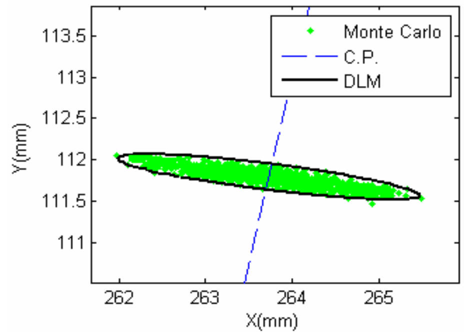

Figure 5: Comparison of DLM with Monte Carlo at θ_2=3°.

After evaluation of the maximum normal-to-path error at each value of θ_2, the variations of the error for one cycle of motion is determined. In Figure 6, the maximum normal-to-path error using DLM method is compared to the Monte Carlo simulation.

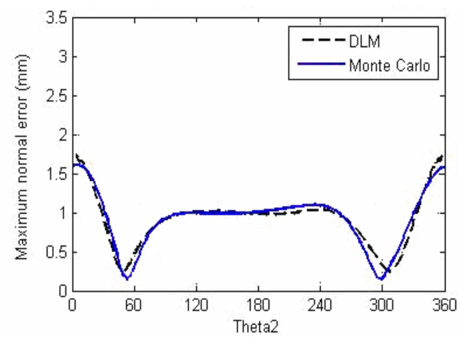

Figure 6: Comparison of maximum normal-to-path error using DLM with Monte Carlo.

6 Multiple linear regression model

A regression model that contains more than one variable is called a multiple regression model. In this paper, it is supposed that the maximum normal error mechanism is produced at θ_2=3° which depends on the manufacturing variables. A multiple regression model which describes this relationship is [9]:

$$y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_K X_K + \in \qquad (22)$$

where y represents the maximum normal error, X_i represents the manufacturing variables, and $\in$ is a random error term. In this method, β_i coefficients are determined by changing manufacturing variables and calculating the maximum normal error at θ_2=3°. These coefficients are obtained by 3072 combinations of the manufacturing variables and given in the following relationship:

$$\mathrm{Error} = 0.05 + 0.35\mathrm{r}_2 + 3.34\mathrm{r}_\mathrm{p} + .69\beta + 1.5068\theta_1 \qquad (23)$$

The valid-ness of the above relationship is verified by the normality test. The difference between initial errors and the errors obtained by equation 23 are computed. If the variation of the computed difference is normally distributed, the error estimated by equation 23 is valid.

In order to determined the percent contribution of each manufacturing variable on the error, β_i are divided by the summation of β_i and the sensitivity of each manufacturing variables is determined (see Figure 8). It can be observed that r_p and θ_1 have the major percent contribution. Thus, decreasing their tolerance limits can substantially decrease the error. Because of improving tolerance limits of angular dimension (i.e. θ_1, β_i) imposes manufacturing cost with higher rate, r_2 and r_p are chosen for decreasing the error and the modified tolerance limits are reported in Table 2.

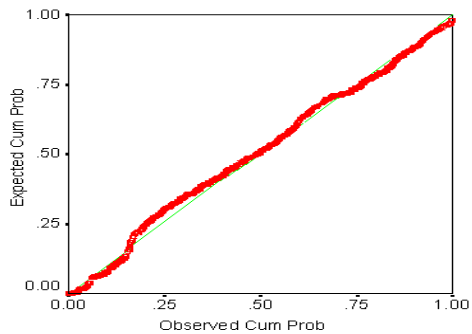

Figure 7: Chart of Test accurate equation23

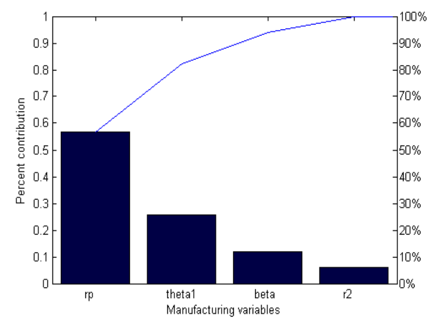

Figure 8: Present contribution of manufacturing variables at θ_2=3°

Table 2. New nominal dimensions and tolerances of manufacturing variables (mm).

Manufacturing Variables	r_2	r_3	r_4	r_p	β	θ_1
Nominal Dimension	250	400	25	104	80°	0°
Tolerance	± 0.3	± 0.2	± 0.02	± 0.15	± 0.5°	± 0.5°
Modified Tolerance	± 0.15	± 0.2	± 0.02	± 0.07	± 0.5°	± 0.5°

Based on the improved parameters, the distributed of normal-to-path error at θ_2=3° along with the variation of maximum error for whole cycle are computed and depicted in Figures 9 and 10, respectively.

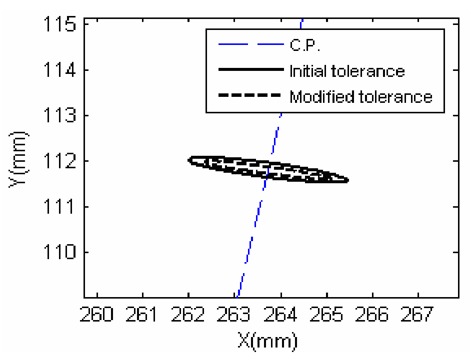

Figure 9: Comparison of bivariate distribution of C.P. position at θ_2=3°,after and before modify.

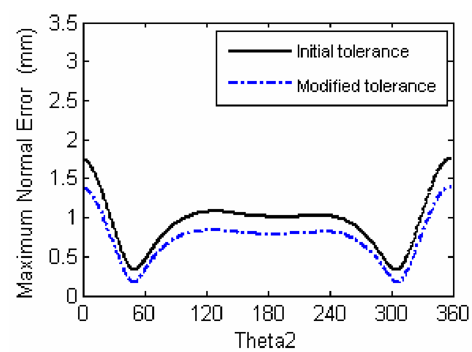

Figure 10:.Variation of maximum error with modified and initial tolerances.

7 Optimization of error and manufacturing costs

A promising method of selecting part tolerances is assigning tolerances such that the manufacturing expenses are minimized. This can be accomplished by the cost-tolerance function for each component. Chase et. al. [10] proposed the following general form for this purpose:

$$C = A + B/tol^k \qquad (24)$$

Where the constant coefficient A represents fixed costs. It may include setup cost, tooling, material, prior operations, etc [10]. The B term determines the cost of producing a single component dimension to a specified tolerance and includes the machine cost rate. Costs are calculated on a per part basis. In order to reach tighter tolerances, speeds and feeds should be reduced and the number of passes increased, requiring more time and higher costs. The exponent k describes how sensitive the process cost is to changes in tolerance specifications. In this study, optimized tolerances of manufacturing variables for minimized costs is obtained by considering equation 23 and 24 and finding the minimum cost for the given max error. For example, information of point A is presented in table 3.

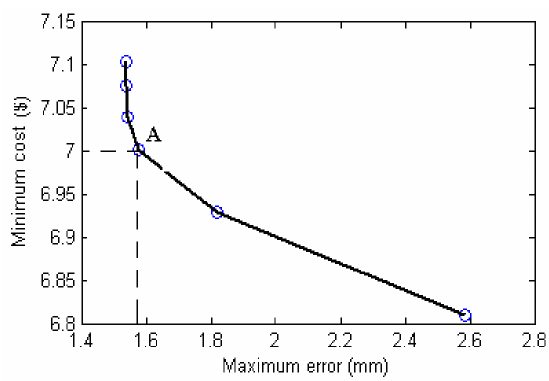

Figure11: Optimized max error versus minimum cost

Table 3 Information of point A

Manufacturing Variables	r_2	r_3	r_4	r_p	β	θ_1
Nominal Dimension	250	400	25	104	80°	0°
Tolerance	± 0.15	± 0.2	± 0.02	± 0.07	± 0.5°	± 0.5°

8 Conclusion

The Coupler Point (C.P.) position of a crank slider mechanism during one cycle of motion is considered as the assembly specification which has variations in two directions. The correlation between these variations also impresses the limit of variation. The bivariate distribution of the assembly specification is determined using the Direct Linearization Method (DLM). The valid range of DLM is firstly determined by means of the time-consuming Monte Carlo simulation and then the percent contribution of each input variable to assembly specification is computed by DLM. Thus, the most critical variables, which have the highest contribution to the variations of the assembly specification, can be recognized. By improving the tolerance limits of these critical variables, the maximum error of mechanism can be decreased significantly. According to Figure 10, tolerance improvements result in 17% reduction of the maximum normal error at θ_2=3°. In addition, the curve which represents optimize max error versus minimum cost is derived based on the regression equation and the cost function.

9 References

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