Extended Abstracts of the 38^{th} Iranian International Conference on Mathematics
3-6 September 2007, University of Zanjan, Zanjan, Iran.

DIFFERENTIAL-ALGEBRAIC APPROACH FOR SOLVING NONLINEAR CONVEX PROGRAMMING PROBLEMS

SOHRAB EFFATI *, M.ABBASI, A.GHOMASHI

Abstract. In this paper we consider a differential-algebraic approach for solving nonlinear convex programming problems. The paper shows that the differential-algebraic approach is guaranteed to generate optimal solutions to nonlinear convex programming problems. The numerical results in this paper demonstrate that the proposed approach provides a promising alternative for solving nonlinear convex programming problems.

1. Introduction

In the 1980s, methods based on ordinary differential equation (ODE) for solving unconstrained optimization problems regained attention in parallel to the inception and development of interior-point methods [1-3]. Previously,the computational cost of ODE-based methods was thought to be higher than that of conventional methods. However, Brown and Bartholomew-Biggs [2] conducted numerical experiments and found that ODE-based methods for constrained optimization can perform better than some conventional methods. The aim of this paper is to propose a differential-algebraic approach, based on a barrier method, for solving nonlinear convex programming problems. After differentiating a set of algebraic equations, we obtain a second system of differential equations. In addition, the proposed differential-algebraic approach is very simple to use.

Key words and phrases. Nonlinear convex programming, dynamic systems, differential algebraic equations.

* Speaker.

2. Differential-Algebraic Equations problem formulation

The non-linear convex programming problem can be stated as follows:

$$\begin{aligned} &\text{minimize} && f(x) \\ &\text{subject to} && \mathbf{g}(x) \leq 0 \\ &&& x \geq 0 \end{aligned} \tag{2.1}$$

where $x \in R^n$, $f : R^n \to R$, $\mathbf{g} = [g_1, \cdots, g_m]^T : R^n \to R^m$ is an m-dimensional vector-valued continuous function of n-variables, and f and g_i's are convex functions on R^n.

After adding slack variable $s \in R^m$ we have:

$$\begin{aligned} &\text{minimize} && f(x) \\ &\text{subject to} && \mathbf{g}(x) + s = 0 \\ &&& x \geq 0, s \geq 0. \end{aligned}$$

The differential-algebraic approach considered in this is motivated by the application of the logarithmic barrier function technique, where the bound constraints are replaced by a logarithmic barrier term which is added to the objective function have [5]:

$$\begin{aligned} &\text{minimize} && \phi(x,s) = f(x) - \mu \left(\sum_{j=1}^{n} \log x_j + \sum_{i=1}^{m} \log s_i \right) \\ &\text{subject to} && \mathbf{g}(x) + s = 0 \end{aligned} \tag{2.2}$$

where $\mu > 0$ is the barrier penalty parameter. For a fixed μ, a Lagrangian function can be written for (2.2) as:

$$L(x,y,s) = f(x) - \mu \left(\sum_{j=1}^{n} \log x_j + \sum_{i=1}^{m} \log s_i \right) + y^T(-\mathbf{g}(x) - s) \tag{2.3}$$

where $y \in R^m$ is the Lagrangian multiplier. Defining vector $z \in R^n$ such that

$$z = (\frac{\mu}{x_1}, \frac{\mu}{x_2}, \cdots, \frac{\mu}{x_n})^T$$

taking the partial derivatives of $L(x,y,s)$ with respect to y, x, s, and setting them to zero, we obtain the following four sets of equations:

$$\begin{aligned} &\mathbf{g}(x) + s = \mathbf{0} \\ &\nabla \mathbf{g}(x)^T y + z = \nabla f(x)^T \\ &XZe_1 = \mu e_1 \\ &YSe_2 = -\mu e_2 \end{aligned} \tag{2.4}$$

where

$$\begin{aligned} &X = \operatorname{diag}(x_1, \cdots, x_n),\ Z = \operatorname{diag}(z_1, \cdots, z_n), Y = \operatorname{diag}(y_1, \cdots, y_m), \\ &S = \operatorname{diag}(s_1, \cdots, s_m),\ e_1 = [1, \cdots, 1]^T_{n\times 1},\ e_2 = [1, \cdots, 1]^T_{m\times 1}. \end{aligned}$$

Define

$$\theta(\mu) = \inf\{f(x) - \alpha\mu \left(\sum_{j=1}^{n} \log x_j + \sum_{i=1}^{m} \log s_i \right) \ \mid \ \mathbf{g}(x) + s = \mathbf{0}, x > \mathbf{0}, s > \mathbf{0}\} \tag{2.5}$$

where $\alpha > 1$, $\theta(\mu)$ is a convex program because both the objective function and the constraints are convex. For any fixed μ, $\theta(\mu)$ has a unique solution and hence is differentiable with respect to μ. The derivative of the function $\theta(\mu)$ is as follows:

$$\frac{d\theta(\mu)}{d\mu} = -\alpha \left(\sum_{j=1}^{n} \log x_j + \sum_{i=1}^{m} \log s_i \right) \tag{2.6}$$

From classical optimization theory [4] we have:

$$\inf_{\mu > 0} \theta(\mu) = \inf\{f(x) \ \mid \ \mathbf{g}(x) + s = \mathbf{0}, x \geq \mathbf{0}, s \geq \mathbf{0}\}. \tag{2.7}$$

By minimizing $\theta(\mu)$ we can be obtained the optimal solution to the problem (2.1). Using the steepest-descent method, we obtain the following differential equation for minimizing $\theta(\mu)$:

$$\frac{d\mu}{dt} = -\frac{d\theta}{d\mu} = \alpha \left(\sum_{j=1}^{n} \log x_j + \sum_{i=1}^{m} \log s_i \right) \tag{2.8}$$

where $x = [x_1, \cdots, x_n]$ and $s = [s_1, \cdots, s_m]$ satisfies (2.4).

References

[1] C. B. GARCIA, W. I. ZANGWILL , P*athways to Solutions, Fixed Points, Equilibria*, Prentice-Hall, Englewood Cliffs, New Jersey, (1981).

[2] A. A. BROWN, M. C. BARTHOLOMEW-BIGGS , *Some Effectie Methods for Unconstrained Optimization Based on the Solution of Systems of Ordinary Differential Equations*, Journal of Optimization Theory and Applications, **67**, (1989), pp. 211-224,.

[3] P. PAN, *New ODE Methods for Equality Constrained Optimization, Part 1:Equations*, Journal of Computational Mathematics, **10**, (1992), pp. 77-92.

[4] M. S. BAZARAA, C. M. SHETTY , *Nonlinear Programming, Theory and Algorithms*, John Wiley and Sons, New York, NY, (1979).

[5] R. D. C. Monteiro, I. Adler, *Interior Path-Following Primal-Dual Algorithms,Part 1: Linear Programming*, Mathematical Programming, **44**, (1989), pp. 27-41,.

Department of Mathematics, Teacher Training University of Sabzevar, Sabzevar, Iran.

E-mail address: effati@sttu.ac.ir