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NEW NEURAL NETWORK MODEL FOR SOLVING THE MAXIMUM FLOW PROBLEM

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Abstract. In this paper, a new neural network model for solving the maximum flow problem is presented. We solve neural network model with one of numerical methods.

1. Introduction

The problem is to find a flow of maximum value on a network from a source to a sink. Maximum flow problem is a linear programming problem. We can solve it by simplex method and modern numerical algorithms. However they do not lend themselves to problems which require solution in real time. One promising approach to solve optimization problems in real time is to use the neural network approach.

2. Problem Formulation

Consider a network with m nodes and n arcs. We associate with each arc (i,j), a lower bound on flow of l_{ij}=0 and an upper bound on flow u_{ij}. We shall assume throughout the development that u_{ij}'s are finite integers.
In such a network, we wish to find the maximum amount of flow from node 1 to node m. Let f represent the amount of flow in the network from node 1 to node m. Then the maximum flow problem may be stated as follow:

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$$
\begin{aligned}
&\text{Maximize} \quad f \\
&\text{subject to} \\
&\sum_{j=1}^{m} x_{ij} - \sum_{k=1}^{m} x_{ki} = \begin{cases} f & i=1 \\ 0 & i \neq 1 \text{ } or \text{ } m \\ -f & i=m \end{cases} \\
&x_{ij} \leq \ u_{ij} \ i = \ 1,2,\ldots,m \\
&x_{ij} \geq \ 0 \ j = \ 1,2,\ldots,m
\end{aligned} \tag{2.1}
$$

Now we can convert the above problem as follow:

$$
\begin{aligned}
&\text{Maximize} \quad cx \\
&\text{subject to} \\
&A_1 x = 0 \\
&A_2 x \leq \ b \\
&x \geq \ 0
\end{aligned} \tag{2.2}
$$

3. The neural network model

We transform the linear programming problem (2.2), to a neural network model. in general if the penalty method is applied to solve (2.2), then we can with assume $g(x) = A_2 x - b$ and $h(x) = A_1 x$ obtain unconstrained optimization problem as follow:

$$
\text{Maximize} \quad \mathrm{p(x) = cx} - \tfrac{\mu}{2}[\sum_{\mathrm{i=1}}^{\mathrm{n}} (\mathrm{g_i^+(x)})^2 + \sum_{\mathrm{j=1}}^{\mathrm{m}} (\mathrm{h_j(x)})^2] \tag{3.1}
$$

Thus, the necessary condition for optimality of (3.1) is $\frac{\partial p(x)}{\partial x} = 0$, i.e:

$$
c^t - \mu[\sum_{i=1}^{n} (g_i^+(x)) \frac{\partial g_i^+(x)}{\partial x} + \sum_{j=1}^{m} (h_j(x)) \frac{\partial h_j(x)}{\partial x}] = 0.
$$

The neural network model for the maximum flow problem can be described by the following nonlinear dynamical system :

$$
\tfrac{dx}{dt} = c^t - \mu[\sum_{i=1}^{n} (a^i)^t (a^i x - b_i)^+ + \sum_{j=1}^{m} (a^j)^t (a^j x)] \tag{3.2}
$$

4. Stability analysis of model neural network

First, consider that $v(x) = -p(x)$, then with respect to theorem(4.1) show that $v(x)$ is a Lyapunov function, then with respect to Lyapunov theorem show that dynamical system is asymptotically stable at equilibrium state. Then with respect to theorem(4.2) , obtain where optimal solution of maximum flow problem is the equilibrium state of (3.2).

Theorem 4.1. *Under the penalty method, $v(x)$ of (3.1) is a Lyapunov function of system (3.2).*

Theorem 4.2. *if for any μ (3.1) has an optimal solution, and if for system (3.2) we can find a state variable $x(t)$ such that the neural network is asymptotically stable at x^*, then the optimal solution to (3.1) will be the equilibrium state of(3.2).*

5. Conclusions

In this paper we present a new neural network models for maximum flow problem. In the neural network model we used the penalty method for obtained it. with start of any point system convergent to optimal solution and has a much faster convergence.

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