2007 International Symposium on Nonlinear Dynamics (2007 ISND) IOP Publishing
Journal of Physics: Conference Series **96** (2008) 012145 doi:10.1088/1742-6596/96/1/012145

Solitons of the Kadomstev-Petviashvili (KP) and the modified KP (mKP) equations for dust acoustic solitary waves in dusty plasmas with variable dust charge and nonthermal ions

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Abstract: By using the reductive perturbation theory, the Kadomstev-Petviashvili (*KP*) equation in unmagnetized dusty plasmas with variable dust charge, electrons with Boltzmann distribution and nonthermal ions, is derived and the propagation of nonlinear waves is analyzed. It is found that compressive and rarefactive solitons can be appeared. Amplitude of solitonic solutions of *KP* equation becomes diverges at the critical densities.

1. Introduction

The study of dusty plasmas represents one of the most rapidly growing branches of plasma physics. The dust grains are usually of micrometer or sub-micrometer size and their masses are very large. Experimental observations have confirmed the existence of linear and nonlinear feature of both dust acoustic waves (DAW) and dust ion acoustic waves (DIAW) [1]. DAWs with nonthermal ions and constant dust charge have been studied by Lin and Duan [2]. Duan have also investigated DAW with hot dusty plasmas [3]. Also Duan et al. have studied the nonlinear Schrödinger equation and stability of solitons in unmagnetized warm dusty plasmas and magnetized dusty plasma [4]. Also, Mamun and Shukla have already studied spherical and cylindrical dust acoustic waves. Gao and Tian have continued their works on DAW and DIAW. El.labany et al. have studied warm dusty plasmas with vortex like distributed electrons and have obtained modified *KdV* equation for different orders of dispersion terms. DAWs in the presence of hot and cold dust have been worked [5]. Wang et al. have studied the effects of negative ions on solitary waves in dusty plasmas by using the Sagdeev potential. Cylindrical *KP* equation in warm dusty plasmas with two ions has been studied by Wang and Zhang. Zhang and Xue have investigated dusty plasma systems containing dust charge fluctuation in which densities of electrons and ions vary with *x* coordinate. They did it successfully and derived shock waves and solitary waves in these mediums [6]. In above cases charge of dust particles is constant. Solitary waves of the *KdV* equation have been studied in dusty plasma with variable dust charge in [7,8]. Gill et al. have also analyzed solitons of *KP* equation for these plasmas with two temperature

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ions [9]. In present paper, we study the nonlinear waves in dusty plasmas with variable dust charge; Boltzmann distributed electrons and the nonthermal ions. In section 2 the basic set of equations is introduced. We derive the *KP* equation by using the reductive perturbation method in section 3. In section 4 the modified *KP* equation is derived at the critical density. Finally, conclusions and remarks are given in section 5.

2. Basic equations

We consider the propagation of dust acoustic waves in collisionless, unmagnetized dusty plasma consisting of high negatively charged dust grains, variable dust charges, nonthermal ions and Boltzmann distributed electrons. Total charge neutrality at equilibrium requires that $Z_{0d} n_{0d} + n_{0e} = n_{0i}$, where n_{0i}, n_{0e} and n_{0d} are the equilibrium values of ions, electrons and dust number densities respectively. Z_{0d} is the unperturbed number of charges on the dust particles. The following set of normalized two dimensional equations of motion describes the dynamics of dust acoustic wave in the variable dust charge plasmas:

$$\frac{\partial n_d}{\partial t} + \frac{\partial}{\partial x}(n_d u_d) + \frac{\partial}{\partial y}(n_d v_d) = 0 \quad , \quad \frac{\partial u_d}{\partial t} + u_d \frac{\partial u_d}{\partial x} + v_d \frac{\partial u_d}{\partial y} = Z_d \frac{\partial \phi}{\partial x} \tag{1}$$

$$\frac{\partial v_d}{\partial t} + u_d \frac{\partial v_d}{\partial x} + v_d \frac{\partial v_d}{\partial y} = Z_d \frac{\partial \phi}{\partial y} \quad , \quad \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = Z_d n_d + n_e - n_i$$

in which u_d and v_d are velocity components of the dust particles in x and y-directions and normalized by the dust acoustic speed $c_d = \sqrt{Z_{0d} T_i / m_d}$ where T_i is the temperature of ions, m_d is the mass of dust particles. n_d and ϕ are the dust number density and electrostatic potential that have been normalized by $n_{\circ d}$ and T_i / e and e is the magnitude of the electron charge, respectively. n_e and n_i are the electron and ion number densities which are normalized by n_{0e} and n_{0i}, respectively. The Space and Time variables are normalized by the Debye length $\lambda_D = \sqrt{T_i / 4\pi n_{0d} Z_d e^2}$ and the inverse of dust plasma frequency $\omega_{pd}^{-1} = \sqrt{m_d / 4\pi n_{0d} Z_{od}^2 e^2}$, respectively. Normalized number densities for Boltzmann distributed electrons and nonthermal distributed ions are [2]

$$n_e = (\mu/1-\mu) e^{\sigma_i \phi} \qquad , \qquad n_i = (1/1-\mu)\left[1 + \beta(\phi + \phi^2)\right] e^{-\phi} \tag{2}$$

where $\mu = n_{0e} / n_{0i}$, $\sigma_i = T_i / T_e$ and $\beta = 4\alpha/(1+3\alpha)$ in which α is a parameter that determines the population of nonthermal ions. The dust charge variable $Q_d = m_d Z_d$ is obtained from the charge-current balance equation $(\partial/\partial t + \vec{\nabla}.\vec{V}) Q_d = I_e + I_i$, where $\vec{V} = (u_d, v_d)$ and I_e, I_i are the electron and ion currents. We further suppose that the streaming velocities of electrons and ions are much smaller than the thermal velocities. Notice that the characteristic time for dust motion is around $10^{-3} s$ while the dust charging time is typically about $10^{-9} s$ [9]. So the dust charge reaches its equilibrium position quickly. Thus $dQ_d / dt << I_e, I_i$ and charge-current balance equation reads [8]

$$I_e + I_i \approx 0 \tag{3}$$

The electron and ion currents for spherical dust grains with radius r are

$$I_e = -e\pi r^2 \sqrt{8T_e / \pi m_e}\, n_e \exp(e\Phi / T_e)$$
$$I_i = e\pi r^2 \sqrt{8T_i / \pi m_i}\, n_i \exp(e\Phi / T_i) \tag{4}$$

in which Φ denotes the dust grain surface potential relative to the plasma potential ϕ. If the thermal velocities of electrons and ions are larger than their streaming velocities then from (3) we have

$$\sqrt{\sigma_i/\mu_i}\left[1+\beta(\phi+\phi^2)\right]\exp(-\phi)(1-\psi)-\mu\exp(\sigma_i\phi)\exp(\sigma_i\psi)=0 \tag{5}$$

where $\psi = e\Phi/T_i$ and $\mu_i = m_i/m_e \cong 1840$. The dust charge $Q_d = C\Phi$ is calculated by using (5) in which C is capacitance of dust grains(C=r). Z_d is defined as $Z_d = \dfrac{\psi}{\psi_0}$, where $\psi_0 = \psi(\phi=0)$ is the dust surface floating potential with respect to the unperturbed plasma potential at an infinite region. By substituting $\phi = 0$ into (5) we have

$$\sqrt{\sigma_i/\mu_i}(1-\psi_0)-\mu\exp(\sigma_i\psi_0)=0 \tag{6}$$

Z_d can be expanded respect to ϕ as

$$Z_d = 1+\gamma_1\phi+\gamma_2\phi^2+\gamma_3\phi^3+.... \tag{7}$$

where $\gamma_1 \equiv \dfrac{\psi_0'}{\psi_0}$, $\gamma_2 \equiv \dfrac{\psi_0''}{2\psi_0}$ [8] and $\gamma_3 = \dfrac{1}{6}\dfrac{\psi_0'''}{\psi_0}$ come from expanding ψ near ψ_0 so we can write

$$\psi_0' = \frac{(\beta+\sigma_i-1)(1-\psi_0)}{1+\sigma_i(1-\psi_0)}, \qquad \psi_0'' = \frac{(1+\sigma_i^2)(1-\psi_0)+2[1-\beta-\sigma_i^2(1-\psi_0)]\psi_0'-\sigma_i^2(1-\psi_0)\psi_0'^2}{1+\sigma_i(1-\psi_0)}$$

$$\psi_0''' = \frac{3[1-\beta-3\sigma_i^2(1-\psi_0)(1+\psi_0')]\psi_0''-[(3+4\beta)\psi_0'+\sigma_i^3(1-\psi_0)(1+\psi_0')^3]-(3\beta+1)(1-\psi_0)}{1+\sigma_i(1-\psi_0)} \tag{8}$$

3. Derivation of the *KP* equation

According to the reductive perturbation method, we choose the independent variables as $\xi = \varepsilon(x-\lambda t)$, $\eta = \varepsilon^2 y$ and $\tau = \varepsilon^3 t$ where λ is the phase velocity of waves and ε is a small parameter which is characterized the strength of the nonlinearity. Dependent variables are expanded as follows

$$\begin{aligned} n_d &= 1+\varepsilon^2 n_{1d}+\varepsilon^4 n_{2d}+... , \qquad u_d = \varepsilon^2 u_{1d}+\varepsilon^4 u_{2d}+... \\ v_d &= \varepsilon^3 v_{1d}+\varepsilon^5 v_{2d}+... , \qquad \phi = \varepsilon^2\phi_1+\varepsilon^4\phi_2+... \\ Z_d &= 1+\varepsilon^2 Z_{1d}+\varepsilon^4 Z_{2d}+\varepsilon^6 Z_{3d} \end{aligned} \tag{9}$$

By substituting (9) into (2) and collecting the terms in the different powers of ε we have

$$\begin{aligned} & n_{1d} = -\frac{\phi_1}{\lambda^2}, \quad u_{1d} = -\frac{\phi_1}{\lambda}, \quad \frac{1}{\lambda^2} = \gamma_1+\frac{\mu\sigma_i+1-\beta}{1-\mu}, \quad \lambda\frac{\partial v_{1d}}{\partial\xi} = \frac{-\partial\phi_1}{\partial\eta} \\ & \frac{\partial^2\phi_1}{\partial\xi^2} = Z_2+n_{2d}+Z_{1d}n_{1d}+\frac{1}{1-\mu}\left[\sigma_i\mu+(1-\beta)\right]\phi_2+\frac{1}{2(1-\mu)}(\mu\sigma_i^2-1)\phi_1^2 \\ & \frac{\partial n_{1d}}{\partial\tau}-\lambda\frac{\partial n_{2d}}{\partial\xi}+\frac{\partial}{\partial\xi}(n_{1d}u_{1d}+u_{2d})+\frac{\partial v_{1d}}{\partial\eta} = 0 \\ & \frac{\partial u_{1d}}{\partial\tau}-\lambda\frac{\partial u_{2d}}{\partial\xi}+u_{1d}\frac{\partial u_{1d}}{\partial\xi} = \frac{\partial\phi_2}{\partial\xi}+Z_{1d}\frac{\partial\phi_1}{\partial\xi} \end{aligned} \tag{10}$$

Finally the *KP* equation is obtained

$$\frac{\partial}{\partial\xi}\left[\frac{\partial\phi_1}{\partial\tau}+a\phi_1\frac{\partial\phi_1}{\partial\xi}+b\frac{\partial^3\phi_1}{\partial\xi^3}\right]+c\frac{\partial^2\phi_1}{\partial\eta^2} = 0 \tag{11}$$

Coefficients of nonlinear and dispersion terms are

$$a=-\left[\gamma_2+\frac{\mu\sigma_i^{\,2}-1}{2(1-\mu)}\right]\left[\gamma_1+\frac{\mu\sigma_i+1-\beta}{1-\mu}\right]^{\frac{-3}{2}}+3\gamma_1\left[\gamma_1+\frac{\mu\sigma_i+1-\beta}{1-\mu}\right]^{\frac{-1}{2}}-\frac{3}{2}\left[\gamma_1+\frac{\mu\sigma_i+1-\beta}{1-\mu}\right]^{\frac{1}{2}}$$

$$b=\frac{1}{2}\left[\gamma_1+\frac{\mu\sigma_i+1-\beta}{1-\mu}\right]^{\frac{-3}{2}},c=\frac{1}{2}\left[\gamma_1+\frac{\mu\sigma_i+1-\beta}{1-\mu}\right]^{\frac{-1}{2}} \tag{12}$$

Recently Zhang S. has derived generalized solutions of (3+1)-dimensional KP equation [10]. One-solitonic solution for (11) is given by [9]

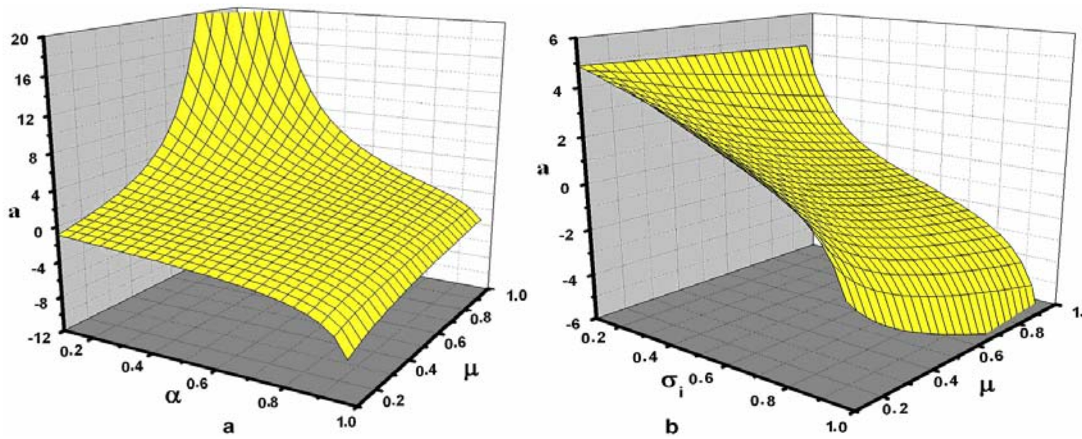
$$\phi_1=\phi_m\sec h^2(\chi/w) \tag{13}$$

where $\chi=\xi+\eta-u\tau$. The amplitude and width of the solitons are

$$\phi_m=3(u-c)/a \qquad w=2\sqrt{b/(u-c)} \tag{14}$$

Now we can compare the above results with the other authors' results. The above mentioned equations agree with [9] for dusty plasma containing one ion with Boltzmann distribution. Results of Zhang and Xue [8] for warm dusty plasma with the external static magnetic field agree with our results. For $\gamma_1=\gamma_2=0$, the above solitonic solutions agree with those of Duan for warm dusty plasma with Boltzmann distribution ion, and also are reduced to equation of wave propagation in one dimension which mentioned in [2]. Lin and Duan [11] have studied this medium with two ions and *N* different species of dust grains and our results are compatible with their results.

Figures1 and 2 show the strength of nonlinear term of the KP equation and the solitonic profiles with different values of the parameters, respectively. The γ_1 and γ_2 are zero in all the cases. From (12) one can find that "*a*" is always negative for $\alpha=0$ when dust charge is constant. Also Figure 1 clearly shows that "*a*" can be positive or negative for different values of α,μ and σ_i. Figures1 and 2 present that the parameter "*a*" is negative for $\alpha=0.5$ and $0<\mu<0.43$. Thus we have rarefactive soliton in this range of parameters. Also for $\mu>0.43$ "*a*" is positive so the compressive soliton can be appeared. Figure 2 shows this situation too. These results show that the parameter α has important role. Also, it is clear that the amplitude of compressive (rarefactive) solitons increases (decreases) as μ and σ_i increases and amplitude of compressive (rarefactive) solitons decreases (increases) as α increases. The width of soliton decreases when μ and σ_i increase and increases when α increases.



a b

Figure 1: Parameter "*a*" as a function of α, μ and σ_i .In figure-a $\sigma_i = 0.3$ and in figure-b $\alpha = 0.5$

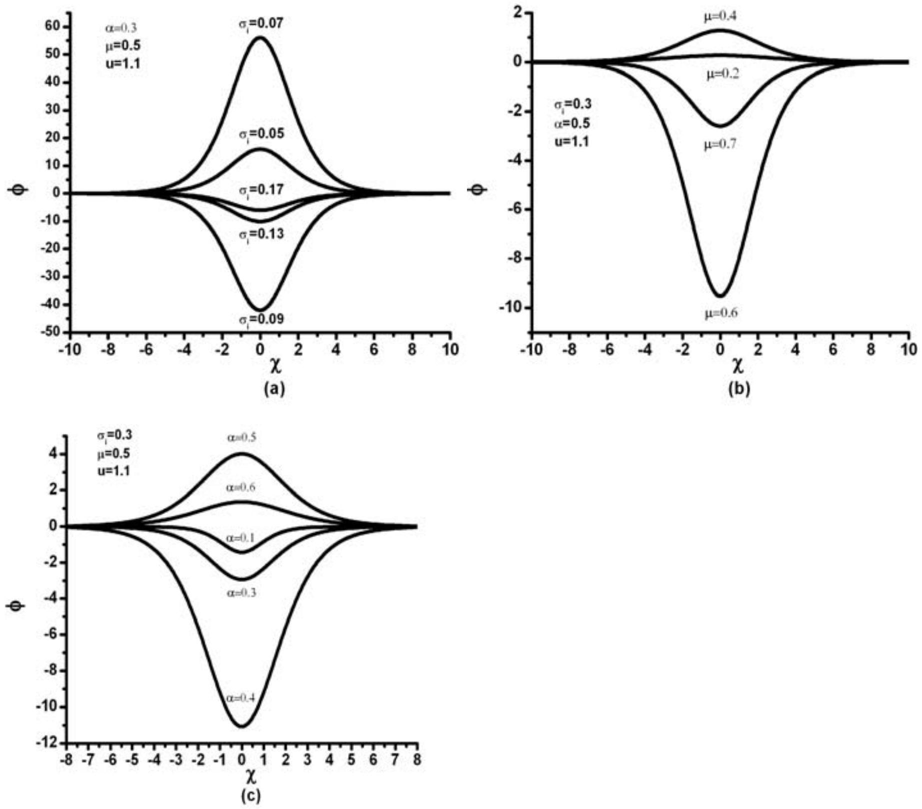


Figure2. The soliton profiles for different values of parameters. γ_1 and γ_2 are zero and *u*=1.1 in all plots.

4. The modified *KP* equation

From (14) it is clear that the amplitude of solitons is highly depends on "*a*" which is a function of μ , β , σ_i , γ_1 and γ_2 . There exist values of density (which we called it the critical density) for which the coefficient "*a*" becomes zero and thus ϕ_m increases to infinity. Some researchers have studied *KP* and *KdV* equations at the critical density and have obtained modified *KP* and modified *KdV* equations [2,11,12]. Here with $\gamma_1 = \gamma_2 = 0$, the critical density is

$$\mu_c = \frac{[6\sigma_i(1-\beta)+1+\sigma_i^{\,2}] \pm \{[6\sigma_i(1-\beta)+1+\sigma_i^{\,2}]^2 - 8\sigma_i^{\,2}(2+3\beta^2-6\beta)\}^{\frac{1}{2}}}{4\sigma_i^{\,2}} \tag{15}$$

In this case we save the stretching coordinates transform in section 3, but we use the new perturbation expansions as

$$\begin{aligned} n_d &= 1+\varepsilon n_{1d}+\varepsilon^2 n_{2d}+\varepsilon^3 n_{3d}+\ldots \quad , \quad u_d = \varepsilon u_{1d}+\varepsilon^2 u_{2d}+\varepsilon^3 u_{3d}+\ldots \\ v_d &= \varepsilon^2 v_{1d}+\varepsilon^3 v_{2d}+\varepsilon^4 v_{3d}+\ldots \quad , \quad \phi = \varepsilon\phi_1+\varepsilon^2\phi_2+\varepsilon^3\phi_3+\ldots \\ Z_d &= 1+\varepsilon^2\gamma_1\phi_1+\varepsilon^4(\gamma_1\phi_2+\gamma_2\phi_1)+\varepsilon^6(\gamma_1\phi_3+2\gamma_2\phi_1\phi_2+\gamma_3\phi_1^{\,3}) \end{aligned} \tag{16}$$

Substituting the above expansions into (2) and collecting different orders of ε we can derive the following equation

$$\frac{\partial}{\partial \xi}\left[\frac{\partial \phi_1}{\partial \tau} + A\phi_1^2\frac{\partial \phi_1}{\partial \xi} + E\frac{\partial}{\partial \xi}(\phi_1\phi_2) + B\frac{\partial^3 \phi_1}{\partial \xi^3}\right] + C\frac{\partial^2 \phi_1}{\partial \eta^2} = 0 \tag{17}$$

where *A*, *E*, *B* and *C* are

$$A=\frac{1}{2}\left(\frac{4}{3}\gamma_2+\frac{\gamma_1^2}{2}\right)\left[\gamma_1+\frac{\mu\sigma_i+1-\beta}{1-\mu}\right]^{\frac{-1}{2}}-\frac{1}{2}\left[\gamma_3+\frac{\mu\sigma_i^3+1-3\beta}{2(1-\mu)}\right]\left[\gamma_1+\frac{\mu\sigma_i+1-\beta}{1-\mu}\right]^{\frac{-3}{2}}+$$

$$\frac{15}{4}\left[\gamma_1+\frac{\mu\sigma_i+1-\beta}{1-\mu}\right]^{\frac{3}{2}}-\gamma_1\left[\gamma_1+\frac{\mu\sigma_i+1-\beta}{1-\mu}\right]^{\frac{1}{2}}$$

$$B=\frac{1}{2}\left[\gamma_1+\frac{\mu\sigma_i+1-\beta}{1-\mu}\right]^{\frac{-3}{2}} \quad , \quad C=\frac{1}{2}\left[\gamma_1+\frac{\mu\sigma_i+1-\beta}{1-\mu}\right]^{\frac{-1}{2}} \tag{18}$$

$$E=-\left[\gamma_2+\frac{\mu\sigma_i^2-1}{2(1-\mu)}\right]\left[\gamma_1+\frac{\mu\sigma_i+1-\beta}{1-\mu}\right]^{\frac{-3}{2}}+3\gamma_1\left[\gamma_1+\frac{\mu\sigma_i+1-\beta}{1-\mu}\right]^{\frac{-1}{2}}-\frac{3}{2}\left[\gamma_1+\frac{\mu\sigma_i+1-\beta}{1-\mu}\right]^{\frac{1}{2}}$$

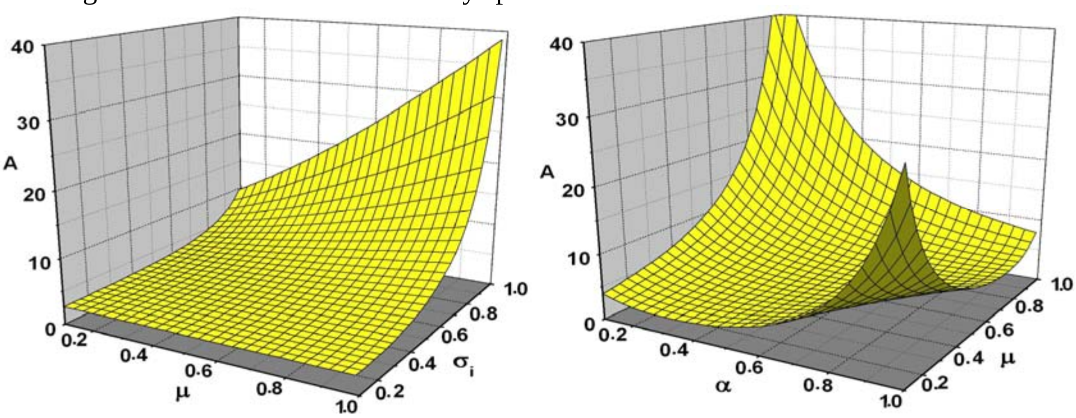
For critical density (μ_c) "*E*" becomes zero and in this situation (17) reduces into the modified *KP* equation

$$\frac{\partial}{\partial \xi}\left[\frac{\partial \phi_1}{\partial \tau} + A\phi_1^2\frac{\partial \phi_1}{\partial \xi} + B\frac{\partial^3 \phi_1}{\partial \xi^3}\right] + C\frac{\partial^2 \phi_1}{\partial \eta^2} = 0 \tag{19}$$

This equation has solitonic solutions. One solitonic solution for this equation is [2,9]

$$\phi_1 = \pm\phi_m \sec h\left[(\xi+\eta-u\tau)/W\right] \tag{20}$$

where *u*, $\phi_m=\sqrt{6(u-C)/A}$ and $W=\sqrt{B/(u-C)}$ are velocity, amplitude and width of solitary wave. The above results for one dimensional propagation with $\gamma_1=\gamma_2=\gamma_3=0$ can be compared with results of [2]. Figures 3 show the parameter "*A*" as functions of μ, α and σ_i. The γ_1,γ_3 and γ_3 are zero in these Figures. It is clear that "*A*" is always positive.



a b

Figure3: Parameter "*A*" as a function of μ, α and σ_i. In figure(a) $\sigma_i = 0.3$, in figure(b) $\alpha = 0.3$

5. Conclusion and remark

The *KP* equation was obtained in unmagnetized dusty plasma with variable dust charge; Boltzmann distributed electrons and nonthermal ions. In this equation γ_1 and γ_2 are appeared and the effects of non-thermal ions, relative density and relative temperature on the behavior of the solitons are discussed. Since the nonlinear coefficient of the *KP* equation "*a*" can be positive or negative, it can also be zero at a critical density (μ_c). In critical density the amplitude of solitons diverges. In order to find definite solutions, the new perturbation expansions are introduced so that the modified *KP* equation is derived. In the *mKP* equation, a new parameter (γ_3) is appeared. It can be concluded that when $\mu \to \mu_c$, solitary waves of *mKP* equation has finite amplitudes at the critical density. Some results which are presented in this paper can be compared with [2, 8, and 9]. The stability and energy of the solitons of *mKP* equation and variations of them with respect to charge of dust particles can be investigated in further works.

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