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INVESTIGATION OF STIRRUPS ROLE ON INCREASING OF LOAD BEARING AND DUCTILITY OF REINFORCED CONCRETE COLUMNS

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Abstract

Designing of widely used concrete structures requires certain information on the load bearing capacity, ductility, failure mode, etc., which should be obtained by expensive and time consuming experiments. Hence, analytical investigation of nonlinear behavior of concrete structures need to be studied.

Concrete is a relatively unknown material, although it is increasingly used, because of its brittle nature, which leads to sudden failure under compressive stresses. The amount of flexibility and energy absorption capacity in structures, especially under lateral load, are so important that new methods are needed to improve them. Strength and flexibility of concrete can be highly increased by controlling the lateral expansion by confining and enclosing. Using rectangular stirrups is one of the most common methods of confining the concrete members.

In this paper, in order to investigate the function of stirrups in increasing the strength and ductility of reinforced concrete columns, the behavior of such structures is studied using an analysis conducted by nonlinear layered finite element method, considering the characteristics of concrete within the theory of hypo-elasticity.

In considering the enclosing role of stirrups, a stress-strain curve is used to express the concrete’s behavior. In the curve, the role of enclosing in increasing the compressive stress and strain of the summit is taken into account. Furthermore, the descending branch of the compressive mode of its stress-strain curve relative to other curves has been modified. In addition, the effect of the section size on increasing/decreasing the stress and strain of the summit has been considered.

In the end, the results of numerical analysis and experimental studies are compared.

Key words: reinforced concrete; nonlinear finite element; columns; confining; ductility; load bearing capacity
1. Introduction
As concrete has suitable pressure strength, it is economical to use this substance in pressure structural elements such as columns. The sudden failure and low ductility of such members at failure, however, is considered as a major weak point of it. A good way of increasing the load bearing capacity and ductility of reinforced concrete columns is to enclose them.

In this paper, the effects of different parameters of enclosing on the behavior of reinforced concrete columns are first described, mentioning the results of experimental investigations. Then, the finite element model and stress-strain diagram used are described in brief. The efficiency of the model will be shown by a comparison of the analytical results (numerical analysis) with the experimental results. In the end, the role of certain effective parameters in confining is investigated by use of the named model.

2. Behavior of Confined Concrete
According to microscopic studies conducted at laboratories, concrete may be viewed as a combination of the three main phases of mortar, coarse aggregate and mortar-coarse aggregate interface (transfer area). The axial behavior of concrete under pressure due to the existence and spread of micro cracks in the contract area is usually nonlinear.

As long as the load exerted to the concrete is less than 30% of its load bearing capacity, the existing microcracks are relatively unchanged and do not spread. Therefore, the behavior of the concrete in this scope is linear. As the load is increased up to the maximum point the microcracks grow, the concrete stiffness decreases and gives rise to nonlinear behavior of concrete. The increased microcracks will cause expansion and increase in the volume of concrete. Beyond the maximum point, the microcracks spread in the mortar until, at failure point, the cracks existing in the mortar and the cracks of the transfer area joint together and cause failure of the sample. This is the time when a significant increase in volume has occurred [1].

The compressive stress-strain curve of the concrete descends steeply after the summit, which is indicative of the low flexibility and stiffness and rapid vanishing of stability in this area. Considering the importance of ductility in prevention of sudden failure and increasing the energy absorption in structures, this defect should be cured [2].

According to the experimental observations, the strength and ductility of concrete may be increased by exerting lateral pressure due to internal friction and aggregate interlock by confining. The main reason for increased stiffness under plane stress is the effect of the confined micro cracks [3]. By confining the concrete and controlling its lateral expansion, the strength and ductility of concrete may be increased. For this purpose, various methods may be used, such as putting the concrete in steel and polymer pipes, using welded wire fabrics, spiral reinforcements, or rectangular stirrups [2].

One of the most common methods used in concrete columns is the use of rectangular stirrups, on which this paper is focused. Using this method causes a lateral pressure around the core, leading to controlling and reducing the lateral expansion and increased ductility and load bearing capacity of the section. In concrete columns enclosed with stirrups, when the exerted loads are low and therefore the longitudinal strain of the members is insignificant, the core and the cover have the same function. As the load and longitudinal strain increase, the lateral expansion of the concrete increases; after this, the behaviors of the core and cover become different: the cover increases its volume and after a while, loses its bearing capacity due to big cracks. The core, however, cannot largely expand due to the lateral enclosing compression of the stirrups. As a result, by controlling the lateral expansion, the load bearing capacity and ductility continues to exist. Figure 1 shows the mechanism of the cover collapsing.

![Figure 1. Mechanism of the cover collapsing](image)
As one can see in Figure 2, the curve of loading capacity of the column has two summits. The first one, shows the moment the cover collapses, when the load bearing capacity is reduced abruptly. The second one relates to the load bearing capacity of the core due to being enclosed; beyond this point, its load bearing capacity decreases because of cracks in concrete and fails at the failure load[2]. Of course, in case the concrete confinement is performed properly, the curve will show only one summit.

![Figure 2. The idealized curve of load-deflection](image)

As the behavior and function of columns confined with rectangular stirrups depend to a considerable extent on the transverse reinforcements, the paper continues to review the results of investigations conducted by some researchers on various contributing parameters, such as the volume percentage of the confining reinforcement, the spacing and layout of stirrups, the yield stress of transverse reinforcements, etc.

In order to investigate the role of different parameters of enclosing in concrete, the two researchers named above tested 24 columns with square sections. Various variables such as longitudinal steel distribution, spacing and layout of stirrups, the amount of transverse steel, etc. were studied in these tests. The results in brief are as follows: as the longitudinal distribution of steel members and the layout of stirrups can affect the confined volume of the concrete, they are among the factors affecting the column's behavior. On the other side, however, the amount of longitudinal steel will not have much effect on the behavior of column. The amount of transverse steel has little effect on the strength of samples, but it directly affects the ductility. The spacing of stirrups hinders the buckling of longitudinal steels and plays an important role in the volume of the confined concrete, and therefore, it is inversely related to the ductility and strength of the samples. The yield stress of the stirrups helps control the lateral pressure on the concrete although it does not affect significantly the column’s behavior [4].

As the stirrups are designed so that the increase in strength obtained by enclosing the core compensates for the reduction in strength due to collapsing cover, the spacing of stirrups may be reduced, or the volume of steel may be added, to increase the strength and ductility. Sometimes, this leads to executive problems. Therefore, the yield stress of stirrups may be increased to raise the lateral pressure. In order to examine this parameter, the said researchers tested a number of columns with circular sections, with various diameters to examine the effect of the yield stress of the transverse steel. They observed that increasing the yield stress will increase the strength and ductility; in cases, however, where there is a high yield stress of steel, the confining reinforcement may not yield at the failure point and therefore, a significant error occurs in the calculations [5].

These researchers made studies on a number of samples with square and circular sections to investigate the effects of enclosing on concrete columns. The results are as follows: Due to the smaller relative lateral expansion, the ductility and normal strain of concrete columns of high strength is relatively less than other samples. The distribution of longitudinal reinforcement and the layout of stirrups play a significant part in the columns behavior due to its role in confining and enclosing of concrete, while the volume percentage of longitudinal reinforcement is of little, negligible effect. The tests well showed the contribution of reduced spacing of stirrups and increased volume of transverse
reinforcement in increasing the ductility and strength of the column. It was also observed that if the volume percentage of the confining reinforcements was reduced to less than 1%, its enclosing role would become ineffective [6].

These researchers conducted tests on a number of circular samples with the aim to determine the final strength and flexibility of circular columns with steel ties and spiral reinforcements under the effect of static loads and to study the effect of concrete cover on the strength of columns. The variables considered in the laboratory plan were: the strength of concrete, diameter and spacing of ties or spiral reinforcements, and the thickness of concrete cover. Not only are the test results consistent with other experimental test results mentioned in connection with the effect of different parameters on the behavior of columns, but point to the fact that after the stirrups yield a sudden failure occurs at the section. Also, the strength after the cover’s collapse and more final capacity are emphasized [7].

3. Finite Element Model
The fact that experimental tests on the structures behaviors are costly and time consuming, and as the study of a column’s behavior will entail its destruction, numerical methods of structural analysis prove necessary. In the present study, the nonlinear layered finite element model has been used to study the behavior of reinforced concrete columns. This method is applicable to cases where there are plane stresses. In this method, the model is divided into several elements, with each element divided into several layers. The stiffness matrix of each layer will be calculated in consideration of the stress and strain levels in that layer and the nonlinear behavior of the materials. Adding up the stiffness matrix of the layers gives the stiffness matrix of elements followed by that the whole structure [1].

The concrete’s behavior has been considered in the two cracked and non-cracked mode. In the non-cracked mode, the hypo-elasticity model proposed by Shayyanar [1] together with the rotation of the main axes has been used, considering the concept of equivalent uniaxial strains. According to this model, the constitutive relation of the materials is presented by Equation 1:

\[
\begin{pmatrix}
\frac{d\sigma_1}{d\tau_{12}} \\
\frac{d\sigma_2}{d\tau_{12}} \\
\end{pmatrix} = \frac{1}{(1-v^2)E_0} \times \begin{pmatrix}
E_1 & \frac{\nu E_1 E_2}{E_0} & 0 \\
\frac{\nu E_1 E_2}{E_0} & E_2 & 0 \\
0 & 0 & 4(E_1 + E_2 - 2\nu \frac{E_1 E_2}{E_0})
\end{pmatrix} \begin{pmatrix}
d\varepsilon_1 \\
d\varepsilon_2 \\
d\gamma_{12}
\end{pmatrix}
\]

(1)

Where, \(E_0\) stands for the initial stiffness module, \(\nu\) for the equivalent Poisson ratio, and \(E_1\) and \(E_2\) for the stiffness modules in the main directions 1 and 2, respectively, which are calculated according to the stress and strain levels at each layer. The other parameters are described in reference 1.

For the cracked mode of concrete, the smeared cracking model has been used. In this model, the amount of stiffness module in Equation 2 in the direction perpendicular to the crack (\(E_1\) or \(E_2\)) is considered as zero. In this equation, the coefficient \(\beta\) (0< \(\beta\)<1) stands for the dowel action and the aggregate interlock after cracking occurs.

\[
\begin{pmatrix}
\frac{d\sigma_1}{d\tau_{12}} \\
\frac{d\sigma_2}{d\tau_{12}} \\
\end{pmatrix} = \begin{pmatrix}
E_1 & 0 & 0 \\
0 & E_2 & 0 \\
0 & 0 & \beta G
\end{pmatrix} \begin{pmatrix}
d\varepsilon_1 \\
d\varepsilon_2 \\
d\gamma_{12}
\end{pmatrix}
\]

(2)

The reinforcements may be modeled in bar element or distributed layer. Solving the nonlinear finite element is conducted by dividing the load into several small steps, and presuming linear behavior of the materials in each load step. Therefore, the load is exerted on the structure in a gradual and incremental manner [8]. In solving it, the combined incremental-iterative method has been used to reach a convergence criterion. The structure’s failure occurs when the convergence criterion is not satisfied [1].

4. Compressive Stress-strain Curve for Confined Concrete
As the compressive behavior of ordinary concrete and confined concrete are different in ways that were discussed in previous sections, in this study the stress-strain relation proposed by Hatanaka et al. (1999) [9] for confined concrete has been used. The relation has been given by the results of tests in which the effect of the sample size, section and spacing of stirrups, cover size, etc. have been taken into account. The curve presented in the ascending and descending branches is presented by Equations 3 and 4, respectively:

$$\frac{\sigma_i}{\sigma_{ic}} = \frac{N_a \left( \frac{\varepsilon_{iu}}{\varepsilon_{ic}} \right)}{N_a - 1 + \left( \frac{\varepsilon_{iu}}{\varepsilon_{ic}} \right)^N}$$

(3)

$$\frac{\sigma_i}{\sigma_{ic}} = \frac{1}{N_d} + \frac{(N_d - 1) \left( \frac{\varepsilon_{iu}}{\varepsilon_{ic}} \right)^2}{(N_d - 1) + \left( \frac{\varepsilon_{iu}}{\varepsilon_{ic}} \right)^{2N_d}}$$

(4)

In the above equations, $\sigma_i$ and $\varepsilon_i$ stand for the main stress and strain in the direction $i$, and $\sigma_{ic}$ and $\varepsilon_{ic}$ stand for compressive plane strength of concrete and the corresponding strain. $N_a$ and $N_d$ are the coefficients of consistency of the ascending and descending branches, respectively, described in Reference 9.

5. Numerical Analysis

In order to study the efficiency of the numerical model to be considered, a number of columns which were tested by Big, Park and Tanaka [6] were analyzed by this model. The results are given in Table 1 [10]. Figure 3 and Table 2 present the physical specifications of the models [6].

![Figure 3. Geometrical specifications of samples [6]](image-url)
Table 1. A comparison of analytical and experimental results of samples

<table>
<thead>
<tr>
<th>Sample</th>
<th>1A</th>
<th>10A</th>
<th>4B</th>
<th>7B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analytical load (KN)</td>
<td>4420</td>
<td>4037.5</td>
<td>4697.5</td>
<td>4605</td>
</tr>
<tr>
<td>Experimental load (KN)</td>
<td>4155</td>
<td>3945</td>
<td>4608</td>
<td>4510</td>
</tr>
<tr>
<td>Analytical load/ Experimental load</td>
<td>1.06</td>
<td>1.02</td>
<td>1.02</td>
<td>1.02</td>
</tr>
<tr>
<td>Analytical deflection (mm)</td>
<td>1.5</td>
<td>1.33</td>
<td>1.52</td>
<td>1.47</td>
</tr>
<tr>
<td>Experimental deflection (mm)</td>
<td>1.68</td>
<td>1.45</td>
<td>1.6</td>
<td>1.56</td>
</tr>
<tr>
<td>Analytical deflection/ Experimental deflection</td>
<td>0.89</td>
<td>0.92</td>
<td>0.95</td>
<td>0.94</td>
</tr>
</tbody>
</table>

Table 2. Physical specifications of samples

<table>
<thead>
<tr>
<th>sample</th>
<th>f’c (MPa)</th>
<th>Longitudinal reinforcement</th>
<th>Transverse reinforcement</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Diameter (mm)</td>
<td>fy (MPa)</td>
<td>Diameter (mm)</td>
</tr>
<tr>
<td>1A</td>
<td>60</td>
<td>12</td>
<td>443</td>
</tr>
<tr>
<td>10A</td>
<td>60</td>
<td>12</td>
<td>443</td>
</tr>
<tr>
<td>4B</td>
<td>72.3</td>
<td>12</td>
<td>443</td>
</tr>
<tr>
<td>7B</td>
<td>72.3</td>
<td>12</td>
<td>443</td>
</tr>
</tbody>
</table>

As the results given by the presented model were satisfactory, it was used to study analytically the role of stirrups in the behavior of reinforced concrete columns. For this purpose, the effects resulting from change in the volume percentage, spacing and yield stress of stirrups on the strength and ductility corresponding to the maximum load bearing capacity has been evaluated.

For this purpose, a concrete column of a compressive strength of 60 MPa, 520 mm high, with a square section 240 mm wide, with four longitudinal bars 14 mm thick, and with yield stress of 445 MPa was chosen. Stirrups 6 mm diameter, with a yield stress of 445 MPa and spacing of 60 mm, with 12.5 mm of concrete cover, were chosen (Figure 3).

To study the role of each of the said parameters, the variable of interest has been changed. Such changes and the results of analysis are given in Tables 3, 4 and 5 [10].

Table 3. Final load and the corresponding deflection for different volume ratios of confining reinforcement

<table>
<thead>
<tr>
<th>Diameter (mm)</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Final load (KN)</td>
<td>3932.5</td>
<td>39387.5</td>
<td>4050</td>
<td>4235</td>
<td>4477.5</td>
<td>4785</td>
</tr>
<tr>
<td>Final deflection (mm)</td>
<td>1.333</td>
<td>1.336</td>
<td>1.359</td>
<td>1.428</td>
<td>1.536</td>
<td>1.66</td>
</tr>
</tbody>
</table>

Table 4. Final load and the corresponding deflection for different spacing of confining reinforcement

<table>
<thead>
<tr>
<th>spacing (mm)</th>
<th>20</th>
<th>40</th>
<th>60</th>
<th>80</th>
<th>100</th>
<th>120</th>
<th>240</th>
</tr>
</thead>
<tbody>
<tr>
<td>Final load (KN)</td>
<td>4452.5</td>
<td>4150</td>
<td>4050</td>
<td>3990</td>
<td>3960</td>
<td>3950</td>
<td>3895</td>
</tr>
<tr>
<td>Final deflection (mm)</td>
<td>1.49</td>
<td>1.373</td>
<td>1.359</td>
<td>1.345</td>
<td>1.345</td>
<td>1.345</td>
<td>1.347</td>
</tr>
</tbody>
</table>
Table 5. Final load and the corresponding deflection for different yield stresses of confining reinforcement

<table>
<thead>
<tr>
<th>Yield stress (MPa)</th>
<th>270</th>
<th>320</th>
<th>395</th>
<th>445</th>
<th>490</th>
<th>540</th>
</tr>
</thead>
<tbody>
<tr>
<td>Final load (KN)</td>
<td>3965</td>
<td>3987.5</td>
<td>4025</td>
<td>4050</td>
<td>4077.5</td>
<td>4092.5</td>
</tr>
<tr>
<td>Final deflection (mm)</td>
<td>1.335</td>
<td>1.34</td>
<td>1.352</td>
<td>1.359</td>
<td>1.371</td>
<td>1.379</td>
</tr>
</tbody>
</table>

6. Conclusion
1. A comparison of the experimental and analytical results show that the model presented may predict the final load and the corresponding deflection with a reasonable, considerable accuracy for reinforced concrete columns.
2. The section of the transverse reinforcements plays an important role in the creation of confining in concrete, and its increase will significantly increase the load bearing capacity and ductility of reinforced concrete columns.
3. The spacing of the confining reinforcement, if the spacing between the stirrups is less than one-third of the column’s size, will have a considerable effect on the load bearing capacity and ductility of reinforced concrete columns, so that they significantly improve the load bearing capacity (about 14%) and ductility (about 10%) of reinforced concrete columns.
4. The yield stress of the stirrups will not play a significant part in the columns behavior; increasing the yield stress of the confining reinforcement will improve the load bearing capacity and ductility of reinforced concrete columns, although such improvement is very small (about 3%) and is negligible compared to the other parameters.

References