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Isotropy design and optimization of a planar parallel manipulator with Combination of Fuzzy Logic and Genetic Algorithm

Hamidreza Kordjazi [1], Alireza Akbarzadeh-T [2], Ali Shokouhi Rostami [3]

[1] M.Sc.student, Mechanical Engineering Department, Ferdowsi University of Mashhad, Iran, Kordjazi@gmail.com
[2] Assistant professor, Mechanical Engineering Department, Ferdowsi University of Mashhad, Iran, Ali_Akbarzadeh_T@yahoo.com
[3] M.Sc. Electrical Engineering, Mapna group of iran, Alishokohirostami@gmail.com

1. Abstract
In an isotropic configuration, the sensitivity of a manipulator in both velocity and torque errors is at a minimum and the manipulator can be controlled equally well in all directions. If the Jacobian matrix is isotropic throughout the entire workspace, manipulator is fully-isotropic and therefore the condition number of the Jacobian matrix is one. Manipulator design can be expressed as a function of workspace requirements. This work presents a hybrid fuzzy logic - genetic algorithm (FL-GA) method for optimization and dimensional synthesis of a 3PRR (prismatic-revolute- revolute) planar parallel manipulator for a prescribed workspace. The algorithm is made of classical genetic algorithm coupled with fuzzy logic. The fuzzy logic controller monitors the variation of genetic algorithm variables during the first run of GA and modifies the initial bounding intervals to restart the next run of the algorithm. Links of robot are minimized while desired workspace is achieved.
2. Keywords: Genetic Algorithm, Fuzzy Logic, Isotropy, Workspace, Parallel Robot

3. Introduction
A parallel robot is defined as a mechanism having at least two kinematics chains connecting the base to the end effector. The performance of a machine tool with parallel kinematics can be evaluated by its kinematic, static and dynamic properties. Parallel manipulators have received great attention due to their properties of increased accuracy, high stiffness, high Rigidity, high payload capability, high speed, good dynamic characteristics and precise positioning capability. But one of their drawbacks is that their performances depend heavily on their geometry [1]. Due to their parallel topology, including limited workspace, difficulties in their analysis, synthesis, control and trajectory planning [2]. Serial manipulators have disadvantages of low precision, low stiffness and low power. Also, they are generally operated at low velocity to avoid excessive vibration and deflection. Parallel robots have many applications, such as industrial automation, telescopes, fine positioning devices, fast packaging, machine tool, medical, flight simulators, micromanipulators, and parallel machine tools [3]. However the optimization of parallel robot can be challenging since it can involve many parameters, like workspace, physical size of robot, stiffness, accuracy, singularity, isotropy [4]. Optimization of parallel manipulators is an important and challenging problem. There are two primary important issues in the field of parallel manipulator: one is mechanical architecture design and the other is optimum design of the manipulator.

An isotropic manipulator has optimum dexterity when it reaches an isotropic position, so it is desirable to develop manipulators that can reach more isotropic positions. With many isotropic positions to choose from, we can choose a preferable position for some specific applications. For example, an isotropic position with smaller singular values provides high resolution for fine position control of a manipulator. The isotropic manipulators are obtained by solving a system of nonlinear equations developed by the isotropy conditions.

In this paper we first introduce the isotropy problem for a planar parallel manipulator and then given specific area as a desired workspace, find the minimum dimension for parallel robot where the workspace contains the desired workspace [5, 6].

The old techniques for optimization are all gradient based search methods and hence require the calculation of derivatives. This characteristic makes these techniques very demanding in computation time and in some cases they may even fail to converge. New methods based on artificial intelligence or probabilistic approach has emerged. The most famous is genetic algorithm. Genetic algorithm is a robust method for searching the optimum solution to a complex problem, although it may not necessarily lead to the best possible solution. GA generally represents a solution using strings (also referred to as chromosomes) of variables that represent the problem.

Choosing a large bounding interval could lead to the same problem because the limited number of individuals of the population are scattered randomly over a large interval. Choosing large number of individual could cover more regions of the interval, but will slow the optimization without guaranteeing the global optimum, thus we improve the performance of GA search ability through the adaptive search range mechanism through fuzzy logic.

A combined fuzzy logic - genetic algorithm method (FL-GA) is proposed. This algorithm has the capability to adjust its starting population to avoid local minimum and to obtain accuracies that classical genetic algorithms fail to obtain. The process is started with an initial population chosen within the initial bounding intervals. The fuzzy logic controller monitors the evolution of the different variables during the optimization and adjusts the bounding intervals for each design variable. These new intervals are then used to start a second round of optimization in order to improve the final result [7].

4. Description of the manipulator

In this section, we introduce 3PRR planar parallel manipulator. The manipulator consists of a base plate, a movable platform, and three links, each of which has a prismatic joint and two consecutive revolute joints. The detailed description about the manipulator can be found in Figure 1. Only the prismatic joints are actuated. Three degrees-of-freedom (DOF) of the PRR manipulator are the translations along the X and Y axis and the rotation about the Z axis. P_i is the length of ith prismatic joint.

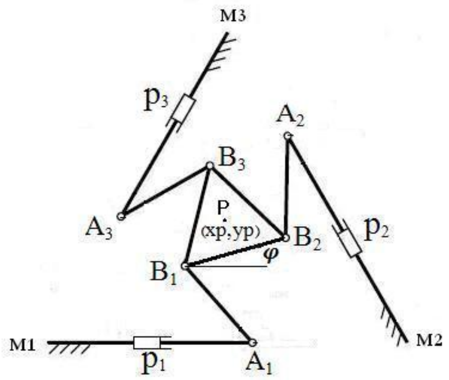

Figure 1. 3PRR Planar Parallel Manipulator

5. Isotropy

Generation of smooth and bounded joint velocities greatly depends on the rank preservation and conditioning of the Jacobian matrix [8]. Isotropy of a manipulator is related to condition number of its Jacobian matrix, which can be calculated as the ratio of the largest and the smallest singular values. Jacobian matrix of a robotic manipulator is the matrix mapping the actuated joint velocity space and the end effector velocity space, and also the static load on the end effector and the actuated joint forces or torques. For parallel manipulator we can write:

$$Jt + K\dot{p} = 0 \tag{1}$$

Where J and K are Jacobian matrices, $\dot{p}$ is the vector of actuated joint rates and t defines as twist vector such that:

$$t = [\dot{x} \quad \dot{y} \quad \dot{\varphi}]^T \tag{2}$$

$$\dot{p} = [\dot{p_1} \quad \dot{p_2} \quad \dot{p_3}]^T \tag{3}$$

The condition number of the Jacobian matrix is not only a measure of how accurate the force and motion transfer between the actuators and the end effector, but also the measure of the ease with which the manipulator can arbitrarily change its position and orientation and apply forces in arbitrary directions. Because of this fact, the condition number of the manipulator Jacobian matrix has been recognized as an optimization criterion in structural synthesis of manipulators even though it is not the only synthesis criterion [2]. The condition number is based on a concept common to all matrices, whether square or not, i.e., their singular values. For an m × n matrix A, with m<n, we can define its m singular values as the non-negative square roots of the non-negative eigenvalues of the matrix AA^T.

$$k = \frac{\sigma_m}{\sigma_s} \tag{4}$$

Here σ_m and σ_s are largest and smallest singular values. It is apparent that a singular matrix has a minimum singular value of zero, and hence, its condition number becomes infinite and if the singular values of a matrix are identical, then the condition number of the matrix attains a minimum value of unity. The reason why isotropic matrices are desirable is that they can be inverted at no cost because the inverse of an isotropic matrix or the generalized inverse of a rectangular isotropic matrix for that matter is proportional to its transpose, the proportionality factor being the reciprocal of its multiple singular values.

From the above discussion, and considering that the Jacobian matrices are configuration dependent, it is apparent that the condition number of the Jacobian matrices of a manipulator is configuration-dependent as well. We define a parallel manipulator as isotropic if both J and K are being the proportional to an identity matrix such that:

$$J\,J^T = \sigma^2\ I_{3\times3} \tag{5}$$
$$K\,K^T = \tau^2\ I_{3\times3} \tag{6}$$
For the velocity of point P (center of mobile triangle) we can write:
$$\vec{\dot{P}} = \vec{V}_{Ai} + \left(\vec{V}_{Bi} - \vec{V}_{Ai}\right) + \left(\vec{\dot{P}} - \vec{V}_{Bi}\right) \tag{7}$$
Also we have:
$$\vec{V}_{Bi} - \vec{V}_{Ai} = \dot{\theta}_i\,\boldsymbol{E}\,\vec{e} \tag{8}$$
Where **E** is rotation matrix and defined as:
$$E = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \tag{9}$$
And J and K obtained such that:
$$J = \begin{bmatrix} \overrightarrow{A_1B_1}^T & -\left(\overrightarrow{A_1B_1}^T\right)\boldsymbol{E}(\overrightarrow{B_1P}) \\ \overrightarrow{A_2B_2}^T & -\left(\overrightarrow{A_2B_2}^T\right)\boldsymbol{E}(\overrightarrow{B_2P}) \\ \overrightarrow{A_3B_3}^T & -\left(\overrightarrow{A_3B_3}^T\right)\boldsymbol{E}(\overrightarrow{B_3P}) \end{bmatrix} \tag{10}$$
$$K = \begin{bmatrix} \overline{A_1B_1} & 0 & 0 \\ 0 & \overline{A_2B_2} & 0 \\ 0 & 0 & \overline{A_3B_3} \end{bmatrix} \tag{11}$$
For normalizing matrix J, as needed to define its condition number, each term of the third column of J is divided by the characteristic length L.
$$J = \begin{bmatrix} \overrightarrow{A_1B_1}^T & -\left(\overrightarrow{A_1B_1}^T\right)\boldsymbol{E}(\overrightarrow{B_1P})/L \\ \overrightarrow{A_2B_2}^T & -\left(\overrightarrow{A_2B_2}^T\right)\boldsymbol{E}(\overrightarrow{B_2P})/L \\ \overrightarrow{A_3B_3}^T & -\left(\overrightarrow{A_3B_3}^T\right)\boldsymbol{E}(\overrightarrow{B_3P})/L \end{bmatrix} \tag{12}$$
By replacing (11) and (12) in Eq.(5) we have:
$$(A_iB_i)^2 + ((A_iB_i)\ \boldsymbol{E}\ (B_iP)/L)^2 = \sigma^2 \tag{13}$$
$$\overrightarrow{A_iB_i}^T\,\overrightarrow{A_jB_j} + ((\overrightarrow{A_iB_i}^T\,\boldsymbol{E}\,(\overrightarrow{B_iP})\,(\overrightarrow{A_jB_j}^T\,\boldsymbol{E}\,(\overrightarrow{B_jP}))/L)^2 = 0 \qquad i \neq j \tag{14}$$
Therefore from relation (13) and (14) we should have:
$$\overline{A_1B_1} = \overline{A_2B_2} = \overline{A_3B_3} \tag{15}$$
$$\overline{B_1P} = \overline{B_2P} = \overline{B_3P} \tag{16}$$
$$A_1B_1{}^T\,A_2B_2 = A_2B_2{}^T\,A_3B_3 = A_3B_3{}^T\,A_1B_1 \tag{17}$$
Also for isotropy of matrix K we have:
$$(A_1B_1)^2 = (A_2B_2)^2 = (A_3B_3)^2 \tag{18}$$
In summary, the constraints defined in the Eq.(15) to (18) are:

Bi should be placed at the vertices of an equilateral triangle, segments A_iB_i form an equilateral triangle; the axes of the prismatic pairs define an equilateral triangle and point P placed in the center of the base and movable triangles. Therefore A_1, A_2 and A_3 should be vertices of an equilateral triangle. Also B_i should made an equilateral triangle as the vertices places on A_iB_i. We can see that there are infinite numbers of equilateral triangles for $A_1A_2A_3$ and $B_1B_2B_3$ (Figure 2). This means that robot has many isotropic configurations.

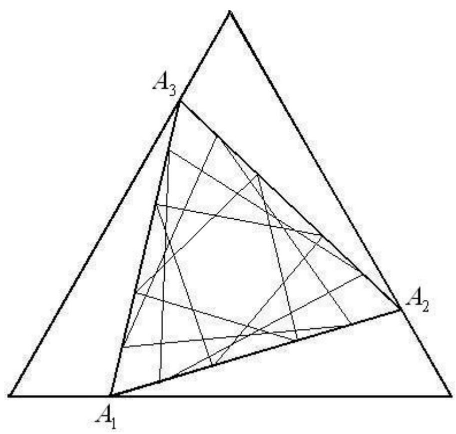

Figure 2. Equilateral triangle for $A_1A_2A_3$ and $B_1B_2B_3$

6. Workspace

The workspace of a robot is defined as the set of all end effector configurations which can be reached by some choice of joint coordinates. Compared with the serial manipulators, parallel manipulators have relatively small workspaces. Thus the workspace of a parallel manipulator is one of the most important aspects to reflect its working capacity, and it is necessary to analyze the shape and volume of the workspace for enhancing applications of parallel manipulators [9]. Therefore for design a manipulator it is of particular interest to determine how the workspace varies with different values of the architectural parameters.

There are different types of workspaces namely maximal workspace or reachable workspace, constant orientation workspace, inclusive orientation workspace, and total orientation workspace. The maximal workspace or reachable workspace is the set of locations of the end effector that may be reached with at least one orientation of the platform. The reachable workspace of a 3PRR manipulator presented in Figure 3.

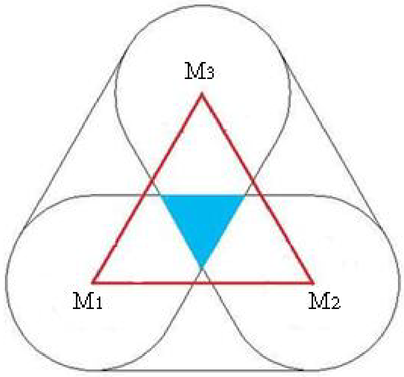

Figure 3. Workspace Of 3PRR Planar Parallel Manipulator

7. Genetic Algorithm

Genetic algorithm is a probabilistic technique that uses a population of designs rather than a single design at a time and utilizes the concept of biological structure to natural selection and survival of the fittest. This method requires no previous experience on the problem. An initial randomized population that consists of a group of chromosomes and represent the problem variables, produces new populations through successive iterations, using various genetic operators. The common operators are selection, mutation, crossover and elitism.GA use objective function information, not derivatives or other auxiliary knowledge, use probabilistic transition rules, not deterministic rules. Coding components of possible solutions into a chromosome is the first part of a GA formulation. Each chromosome is a potential solution and is comprised of a series of substrings or genes, representing components or variables that either form or can be used to evaluate the objective function of the problem. Chromosomes in the population with high fitness values have a high probability of being selected for combination with other chromosomes of high fitness. Combination is achieved through the crossover of pieces of genetic material between selected chromosomes. Mutation allows for the random mutations of bits of information in individual genes. Through successive generations, fitness should progressively improve.

8. Algorithm of the optimization

The aim of this section is to develop and to solve the optimization problem of selecting the geometric design variables for the 3PRR planar parallel manipulator having a prescribed workspace. Given a specified area defines by some points and we want to find the parameters of the 3PRR robot having a workspace that includes the specified area. To define the objective function, we note this theorem that the total distance of a point in the equilateral triangle from 3 lines of the equilateral triangle is equal to the height of equilateral triangle. The algorithm is as follow:

Select 2 points $A(x_1, y_1)$ and $B(x_2, y_2)$ and compose 2 equilateral triangles with these 2 points (Figure 4). Then we determine the distance of each point of prescribed workspace from 3 lines of 2 triangles. If the total distance in each part is equal to the height, this triangle can be the workspace of 3PRR planar parallel manipulator. We want to find the minimum area for this triangle.

A penalty function method is used to handle the constraints. The objective function is accordingly constructed as:

$$F = F_1 + F_2 \tag{19}$$

Where:

$$F_1 = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \tag{20}$$

And F_2 is a penalty function defined as follows:

$$F_2 = C.V \tag{21}$$

Here, C is a large positive constant and V is a number that relate to the minimum number of point of desired workspace where outside of 2 triangles.

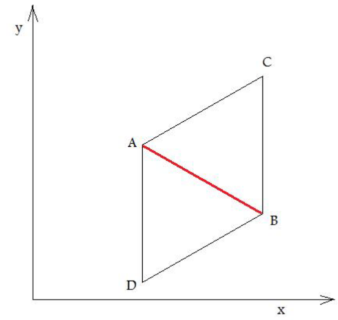

Figure 4. Objective Function definition

Figure 5 shows the combined FL-GA algorithm. The process, which is the classical genetic algorithm optimization, is started with initial population chosen randomly within the initial bounding intervals. The FL monitors the evolution of the different variables during the optimization process and adjusts the bounding intervals of each variable for the next round of optimization process. These new intervals are then used to start a second round of the genetic algorithm.

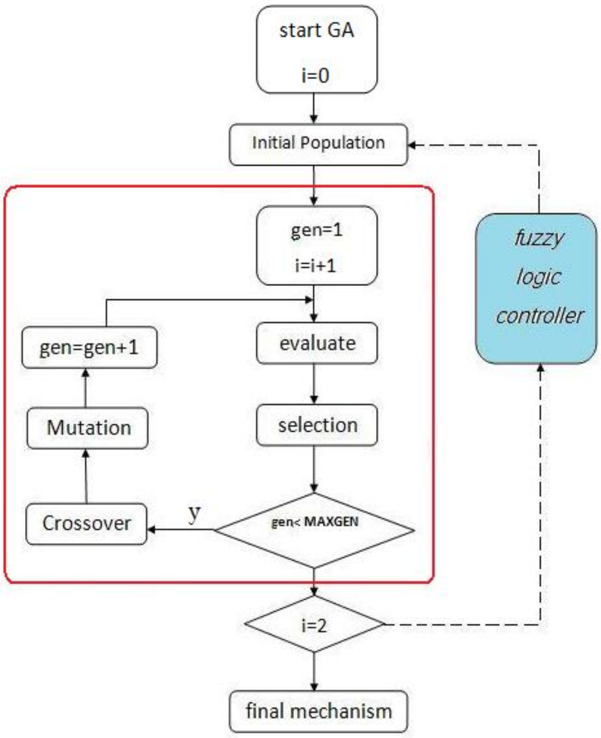

Figure 5. The modified FL- GA optimization scheme

The knowledge base is built during the first run of the genetic algorithm optimization. By analyzing the final result, a parameter D_x is determined and the bounding interval for the design parameter x is corrected using the following equation:

$$x_{min}^{*} = x_{ave} - \frac{D_x}{2}\,(x_{max} - x_{min}) \tag{22}$$

$$x_{max}^{*} = x_{ave} + \frac{D_x}{2}\,(x_{max} - x_{min}) \tag{23}$$

Where x_{ave} is the average value of the design variable x of all the individuals of the last generation and

$[x_{min}\ x_{max}]$ is the initial bounding interval. The coefficient D_x (the output variable) is obtained from the knowledge of the two inputs, variables E and K_x where E is the error found after the first run of the genetic algorithm optimization and K_x is a counter of the variation of each parameter during the last 30 generations. K_x is a counter for each one of the variables and it is ranging between 0 and 30. It starts at 0 and during the last 30 generations it is incremented by 1 each time the variable changes.

$$E = \frac{x_{last} - x_{min}}{x_{min}} \tag{24}$$

Fuzzification of the input variables is the first step in the design of a fuzzy logic controller. Fuzzification of the input variables involves quantizing the universes into a number of fuzzy sets. The output variables also need to be quantized in a similar manner. Quantization involves breaking up a fuzzy input (and also output) variable into several fuzzy subsets. Figure 6 shows the membership functions chosen for 2 fuzzy input variables and the output variable and Table 1 contains the definition of the linguistic parameters.

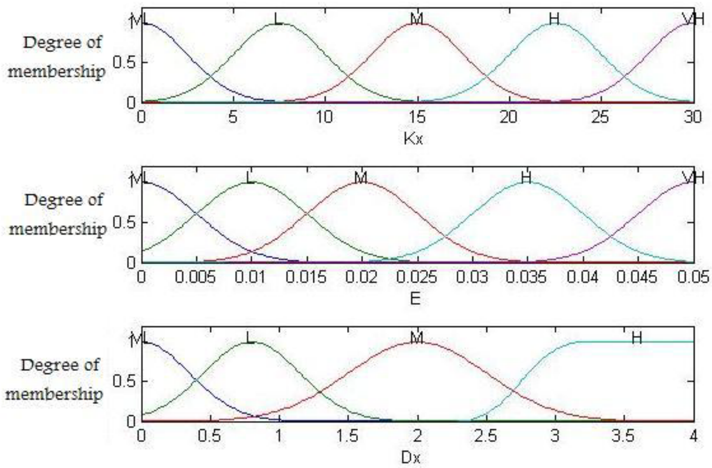

Figure 6. Membership function for two inputs and the output

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Table 1.The linguistic parameters

VS	Very Small
S	Small
M	Medium
H	High
VH	Very High

Table 2. Rule matrix

D_x		K_x				
		VS	**S**	**M**	**H**	**VH**
E	**VS**	VS	S	S	M	M
	S	VS	S	M	H	M
	M	S	S	H	M	H
	H	M	M	M	H	H
	VH	H	H	M	M	M

Table 3. Coordinates of the points of desired workspace

Point number	1	2	3	4	5	6	7
x (cm)	5	12	26	30	20	10	2
y (cm)	5	2	10	17	23	25	14

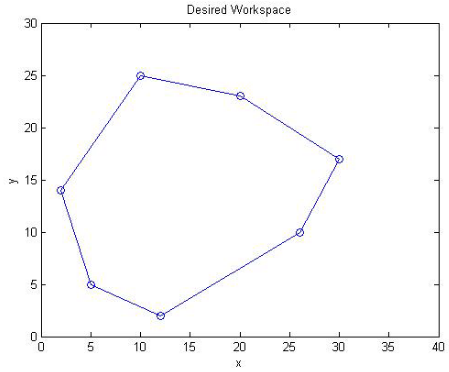

Figure 7. Desired workspace for 3PRR robot

Table 4. GA parameters

The Size Of The Population	30
Maximum Number Of Generations	80
Number Of Variables	4
Number Of Bits	8
Crossover Rate	0.7
Mutation	0.02

9. Results

Figure 8 shows the comparison of two methods for 80 generations and table 5 shows the initial and corrected boundary for each parameter of GA. Results shows the efficiency of combination of FL with GA.

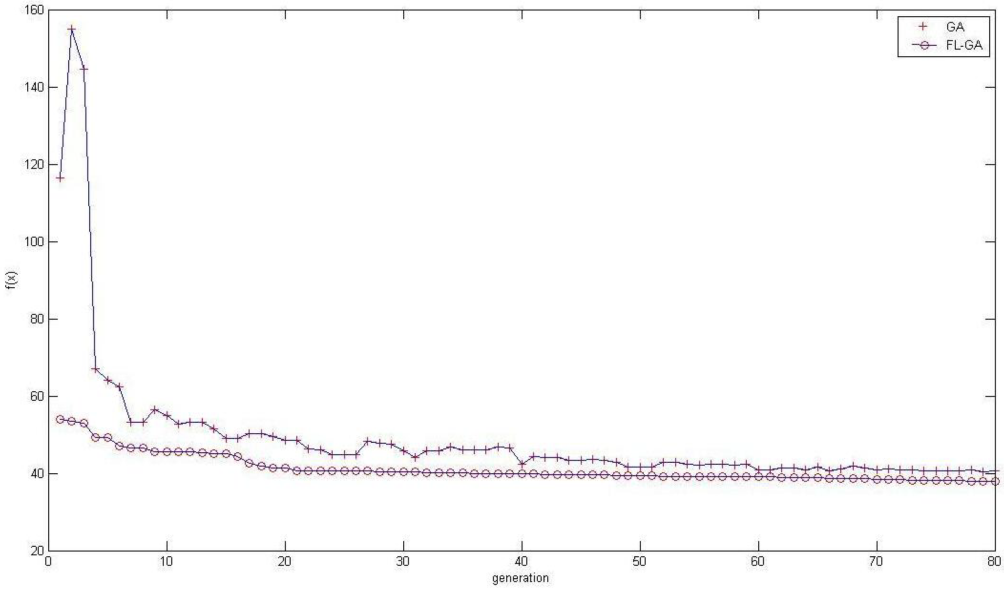

Figure 8. Comparison of GA and FL-GA

Table 5. Parameters of the FL-GA method

	X1	Y1	X2	Y2
Initial boundary	[0 80]	[0 80]	[0 80]	[0 80]
Kx	12	14	10	16
E	0.0029			
Dx	1.4940	1.3812	1.1921	1.4116
Corrected boundary	[18.6 54.6]	[-1.5 34.7]	[-12.3 18.5]	[12.7 47.8]
F (GA)	40.47			
F (FL-GA)	37.84			

Therefore:

$$\overline{M_1M_2} = \overline{M_2M_3} = \overline{M_1M_3} = 37.84\ cm \tag{25}$$

$$\overline{A_\iota B_\iota} + \overline{B_\iota P} = \frac{\sqrt{3}}{2}\ \overline{M_1M_2} = 32.77\ \ cm \tag{26}$$

10. Conclusion

In this paper we first discuss about isotropy of the Jacobian matrices of parallel manipulator and used this concept to design a 3PRR planar parallel manipulator. We obtained the relation between the parameters of manipulator. Then we presented a combined fuzzy logic - genetic algorithm method for workspace optimization of a 3PRR planar parallel manipulator. The proposed method is made of a classical genetic algorithm coupled with a fuzzy logic controller. This controller monitors the variation of the variables during the first run of the genetic algorithm and modifies the initial bounding intervals to restart a second round of the genetic algorithm. The desired workspace of the robot was obtained. The links for this robot are all minimized. Using this method showed that these results are always better than those obtained by genetic algorithm lonely.

11. References

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