14th Annual (International) Mechanical Engineering Conference – May 2006
Isfahan University of Technology, Isfahan, Iran

On Geometry Dependence of Weibull Parameters; BEREMIN Approach Revisited

Pooria Akbarzadeh[1], Saeid Hadidi-Moud[2]
Department of Mechanical Engineering, Ferdowsi University of Mashhad
po_akbarzadeh@yahoo.com

Abstract

Local stress based approaches to fracture prediction use Weibull distribution model based on weakest link theory. Accuracy of predictions as part of the integrity assessment of structural steels containing defects and often subjected to various conditions of temperature and loading relies on the accuracy in estimation of distribution parameters. In the model proposed by BEREMIN [2] it is argued that distribution parameters only depend on the material and are considered geometry independent. In this work their approach has been re-examined and based on independent series of experimental fracture tests using round notched bar specimens. The geometry dependence of parameters is highlighted.

Keywords: Weibull parameter, failure probability, fracture stress, linear regression, calibrated curve.

Introduction

"Local approaches" and "micromechanical modeling" of fracture have found increasing interest since several years. The general advantage, compared with classical fracture mechanics, is that, in principle, the parameters of the respective models are only material and not geometry dependent. Thus, these concepts guarantee transferability from specimens to structures over wide range of sizes and geometries and can still be applied when only small pieces of material are available which do not allow for standard fracture specimens.

Defect assessment of structural components require accurate estimates of fracture toughness for engineering materials under various temperature and loading and also geometry conditions. As we know, when the exact prediction about a subject is impossible, the statistical procedures should be used.

The Weibull stress model originally proposed by the Beremin group(Beremin 1983), based on the weakest link statistics, provides a framework to quantify the relationship between macro and microscale driving forces for cleavage fracture.

The Beremin model adopts a two-parameter description for the cumulative failure probability in the form :

$$\boldsymbol{P}f(\sigma w) = 1- \exp[-(\sigma w/\sigma u)^m] \qquad (1)$$

$\boldsymbol{m}$ and $\boldsymbol{\sigma}_u$ are Weibull parameters.

Beremin group said m (the Weibull exponent) depends on material and it is independent of geometry.

In this work, we want to show that Weibull parameters depend on both material and geometry.

[1] MSc Student
[2] Assistant Professor

Statistical studies

Local stress-based approaches to cleavage fracture use the Weibull distribution model as proposed by Beremin(1983). This method uses the Weibull parameters fitted to fracture test results from round-notched bar(*RNB*) specimens. In their round robin studies on micro-mechanical models, the European Structural Integrity Society(ESIS) suggests using the parameters fitted to the RNB test results to predict cleavage fracture in pre-cracked compact tension, C(T), specimens using:

$$\boldsymbol{P}_f(\sigma_f) = 1- \exp[-(\sigma_w/\sigma_u)\text{^}\boldsymbol{m}]$$

In this equation, σ_f represents the net section stress at a load level that is also used to evaluate the Weibull stress, σ_w.

The model based on weakest link theory in the form of Weibull probability distribution, where the parameter σ_f is a reference characteristic stress corresponding to a failure probability of 63.2%, and is referred to as the mean reference stress. The Weibull exponent, m, characterizes the spread of fracture data. Further modifications to the above distribution can be made by introducing a threshold stress, σ_{min}, below which fracture is impossible and failure does not occur unless $\sigma_w > \sigma_{min}$. Equation (1) is modified so that :

$$P_f(\sigma_f)=1-\exp[-((\sigma_w- \sigma_{min})/(\sigma_u-\sigma_{min}))\text{^}\boldsymbol{m}] \qquad (2)$$

The Weibull parameters are determined from experimental results. It should be noted that in both the Beremin and the ESIS studies, the analyses were conducted assuming $\sigma_{min}= 0$.

Method of linear regression

There are different approaches in estimation of the two Weibull parameters.

Linear regression is a special case of the least-squares method. Taking the logarithm of Equation (1) twice gives a linear equation:

$$\ln [\ln(1/(1-P_{f,n}))] =\boldsymbol{m} \ln \sigma_n -\boldsymbol{m} \ln \sigma_u \qquad (3)$$

with the slope b = $\boldsymbol{m}$ and a y-intercept of a = -$\boldsymbol{m}$ ln σ_u . The σ_n values are the experimentally determined fracture stresses ordered as fallows:

$$\sigma_1 < \sigma_2 < \ldots < \sigma_n < \ldots < \sigma_{N-1} < \sigma_N$$

A probability of fracture will be assigned to each σ_n such that:

$$\boldsymbol{P}_{f,1} < \boldsymbol{P}_{f,2} < \ldots < \boldsymbol{P}_{f,n} < \ldots < \boldsymbol{P}_{f,N-1} < \boldsymbol{P}_{f,N}$$

where $0 < P_f < 1$. Since the true value of $P_{f,n}$ for each σ_n is not known, it has to be estimated. This estimator is to be chosen such that on average, the errors arising each time due this estimation compensate each other

Specimen Geometry

Mechanical tests were performed using two sets of parts. The first set has the net section with 10mm of diameter and second has the net section with 15mm of diameter. Each set has different specimen geometries as listed below :

(i) Axisymmetric notched tensile bars with different radii, 5, 4 and 2 mm.

(ii) More sharply axisymmetric notched specimens, having 0.2mm notch, where employed in order to reach conditions corresponding to steeper stress-strain gradients.

The complete description of these geometries shown in figure (1).

Experimental procedure & Test results

Mechanical tensile tests were performed using different specimen geometries as shown before. These specimens enabled us to have different conditions of stress concentration and compare their fracture. These types of specimens are referred as ***D-R***, where ***D*** is a number, expressed in mm, giving the diameter of net section and ***R*** is a number too, expressed in mm and giving the notch radius.

Except the type 10-0.2, six specimens for each type were tested. Five specimens for type 10-0.2 were tested. All of forty seven specimens were tested at room temperature.

All of these specimens failed. The average stress at rupture is simply defined as

$\sigma_R= 4\boldsymbol{P}_R/\pi d_f^2$, where PR is the load at rupture and d_f is the diameter of net section at fracture.

Calibration of Weibull Parameters

The calibration process for the determination of the Weibull parameters using the fracture stress test data and linear regression method, was performed.

In all tests, the failure probability was determined from:

$$\boldsymbol{P}_f = (\boldsymbol{i}-0.5)/\boldsymbol{N} \qquad (4)$$

Where $\boldsymbol{N}$ is the total number of specimens in each type and $\boldsymbol{i}$ is the order number, respectively.

Fracture stress results (i.e., average net section stress at rupture) obtained from test of ***RNB*** specimens were summarized before.

For determining the Weibull parameters (m,σu), in first, fracture stresses of each type should be ranked from low to high. The results were interpreted in terms of failure probability against fracture stress, with the failure probability Pf determined using Eq (4).

For each type, except the type 10-0.2, N is six and for type 10-0.2, N is equal to five. Also, we did this work for the results of each set(10mm diameter and 15mm diameter). For the first set N is twenty three and for second N is twenty four. Finally, we did this work for all specimens as a set of data. In this case N is equal to forty seven. Then, values using equation (3), were calculated for each case. The results of some types are summarized in table (1). In this table $R_f = 1 - P_f$.

Table 1- The ranked stresses, respective Pf and values for fitting Linear curve

Type Name: 10-0.2			
σ(Mpa)	Pf (%)	Ln(σ)	LnLn(1/Rf)
673.8416	10	6.512995	-2.25037
689.0882	30	6.535369	-1.03093
725.258	50	6.586527	-0.36651
727.3831	70	6.589453	0.185627
733.6062	90	6.597972	0.834032

Type Name: 10-5			
σ(Mpa)	Pf(%)	Ln(σ)	LnLn(1/Rf)
1039.095	8.33	6.946105	-2.44213
1057.913	25	6.964053	-1.2459
1078.689	41.67	6.983502	-0.61794
1085.031	58.33	6.989364	-0.13309
1086.002	75	6.990258	0.326634
1124.339	91.67	7.024951	0.910396

Type Name: 15-0.2			
σ(Mpa)	Pf(%)	Ln(σ)	LnLn(1/Rf)
673.2271	8.33	6.512083	-2.44213
673.813	25	6.512953	-1.2459
683.2578	41.67	6.526872	-0.61794
722.1074	58.33	6.582174	-0.13309
724.6683	75	6.585714	0.326634
743.8529	91.67	6.611843	0.910396

.Type Name: 15-4			
σ(Mpa)	Pf(%)	Ln(σ)	LnLn(1/Rf)
1064.869	8.33	6.970607	-2.44213
1065.134	25	6.970856	-1.2459
1065.737	41.67	6.971422	-0.61794
1072.877	58.33	6.978099	-0.13309
1091.57	75	6.995372	0.326634
1094.413	91.67	6.997973	0.910396

Set Name: 15mm			
σ(Mpa)	Pf(%)	Ln(σ)	LnLn(1/Rf)
673.2271	2.08	6.512083	-3.86231
673.813	6.25	6.512953	-2.74049
683.2578	10.42	6.526872	-2.20693
722.1074	14.58	6.582174	-1.84776
724.6683	18.75	6.585714	-1.57195
743.8529	22.92	6.611843	-1.34582
807.8499	27.08	6.694376	-1.15262
809.9782	31.25	6.697007	-0.98165
814.7615	35.42	6.702895	-0.82721
834.717	39.58	6.727093	-0.68548
842.6751	43.75	6.736581	-0.55275
856.3201	47.92	6.752644	-0.42711
1064.869	52.08	6.970607	-0.30702
1065.134	56.25	6.970856	-0.19034
1065.455	60.42	6.971158	-0.07597
1065.737	64.58	6.971422	0.037193
1072.877	68.75	6.978099	0.151133
1084.783	72.92	6.989135	0.267256
1091.57	77.08	6.995372	0.38741
1092.806	81.25	6.996504	0.515202
1094.413	85.42	6.997973	0.655196
1096.02	89.58	6.999441	0.816003
1097.847	93.75	7.001106	1.019781
1102.093	97.92	7.004966	1.353978

A linear curve was fitted to these values of each case and for this reason, the MATLAB code was used (Fig. 2). The Weibull exponent(***m***) is the slope of this line (fitted line). The y-intercept of this line is equal to $-m \ln\sigma_u$, so we can calculate the σ_u. The values of Weibull parameters (m,σ_u) for each case, are summarized in table (2).

Table 2. Weibull parameters of various cases

Type Name	*m*	σ_u
10-0.2	32	717
10-2	39	1110
10-4	117	1081
10-5	42	1091
15-0.2	25	718
15-2	45	837
15-4	78	1083
15-5	94	1095
10(all parts)	6	1085
15(all parts)	6	998
all parts	6	1040

After determination of Weibull parameters for each case, calibrated curves using equation (1) were plotted. Figure (3) shows experimental points and calibrated curves.

Discussion

As it was said before, we want to investigate whether the Weibull parameters are depended on both material and geometry or not?.

Beremin in his paper in 1983 said that for his tested material, the Weibull exponent, ***m***, is constant for every geometry. In his work the axisymmetric notched specimens with different minimum diameter-5, 10, 15, 20mm- were tested. The notch radius of tested specimens were 20, 10, 4 and 2mm and for more sharply notched specimens, the notch radius was 0.2mm.

He put the all results of experiments in one set of data and then for this set of data he determined the Weibull parameters. For his tested material he found ***m***=22, of course, he did his experiments in two temperature (77k and over 223k) and for different temperature, he said that ***m***=22 too.

In our work, we tested specimens in two sets with 10 and 15mm diameter of net section and for each set with 5, 4, 2 and 0.2mm notch radius. We tested theses eight types of specimens and then for each type we determined the Weibull parameters (Table 2). We found the different values for Weibull parameters and they had very difference with together, so we can't choose one value for all specimens.

With the obtained values, we plot the calibrated curves and we observed the curves covered the experimental points very well (Fig. 3). Of course, it should be said that for three cases(set 10, set 15 and all specimens), the Weibull exponent, ***m***, is approximately constant. It means that for these cases the value of ***m*** with a little difference is equal to six. The point should be noted is that the calibrated curves with this value can not cover the experimental points very well and so we can't have a good prediction of failure.

Finally, it should be said that we can't ignore the effect of geometry on Weibull parameters. In fact,, we can say that the main reason of difference between Weibull parameters of each geometry, is the stress concentration that affects on failure.

Acknowledgments

We are very grateful to the department of Mechanical Engineering of Ferdowsi university of Mashhad and to the workers in machine shop. We would also like to thank Dr. Zabet and his material's lab contribution to this work.

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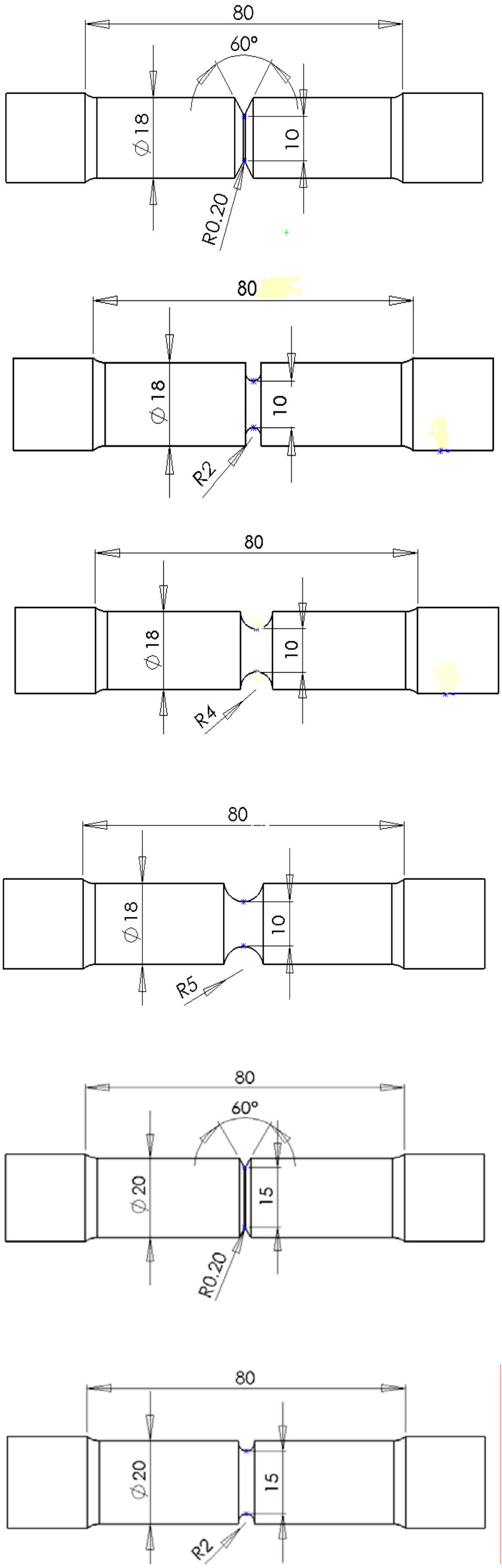


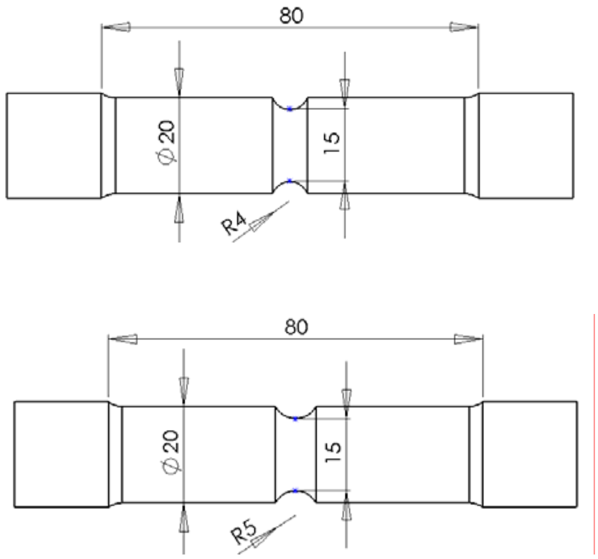


Figure 1. The complete description of specimen geometries

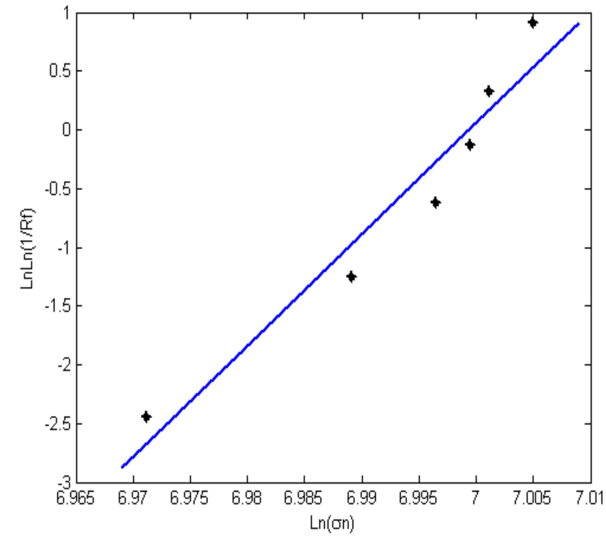
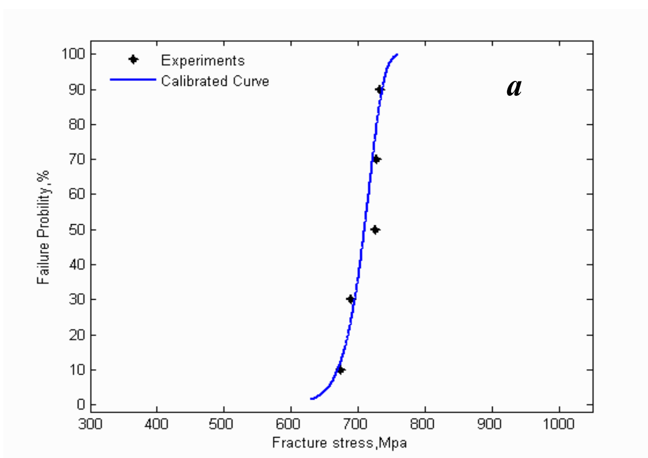
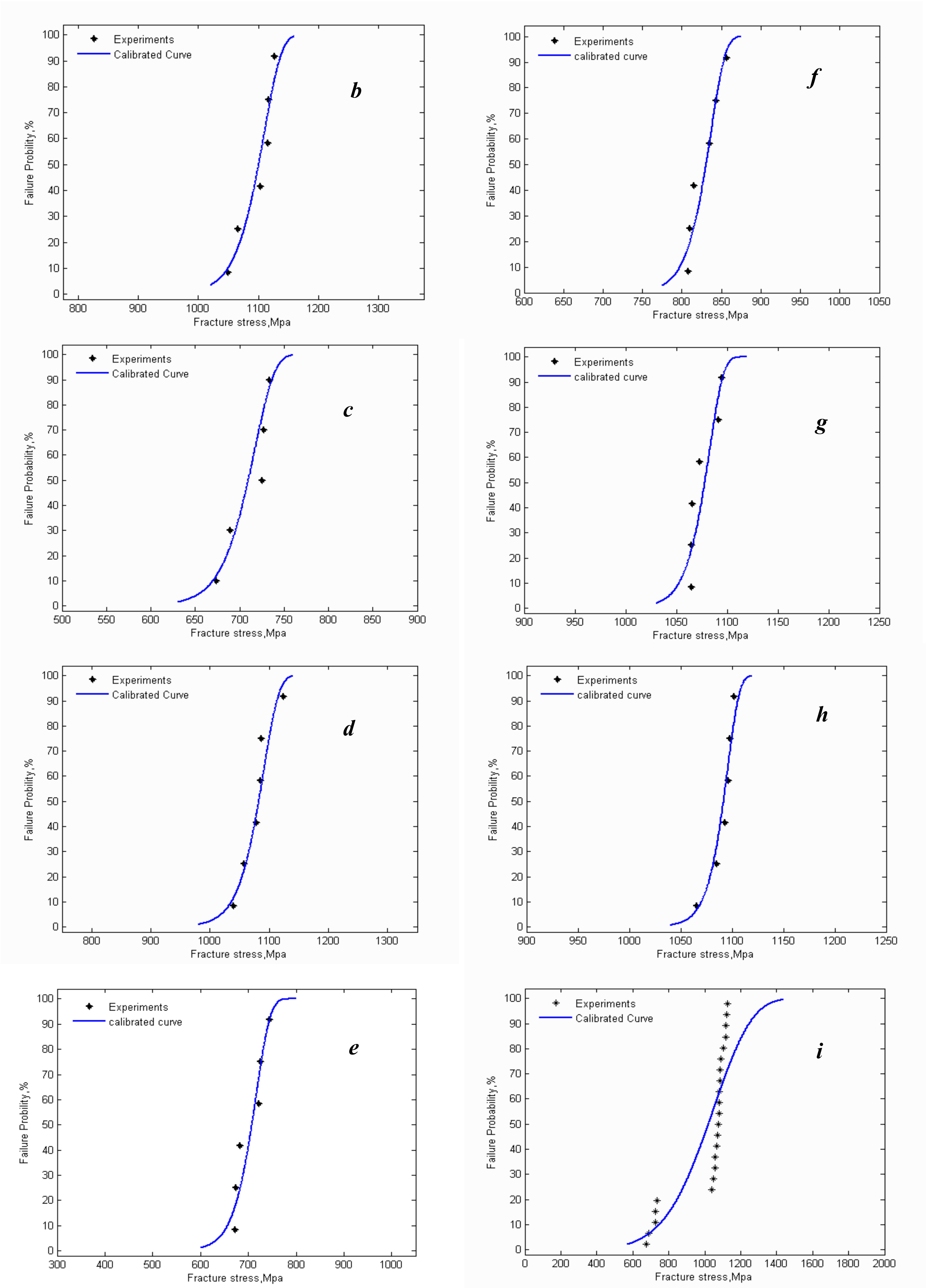


Figure 2. Fitted linear curve to logarithmic values




Experiments
Calibrated Curve
b
Failure Probility,%
Fracture stress,Mpa
Experiments
calibrated curve
f
Failure Probability,%
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c
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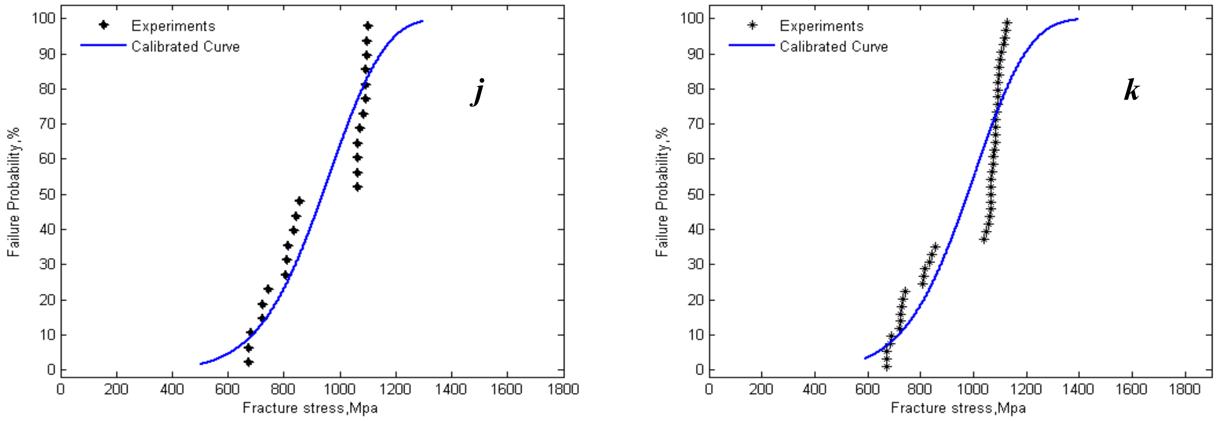

Figure 3. Experimental points and calibrated curves for each case. (a)Type 10-0.2 (b)Type 10-2 (c)Type 10-4 (d)Type 10-5 (e)Type 15-0.2 (f)Type 15-2 (g)Type 15-4 (h)Type 15-5 (i)Set 10 (j)set 15 (k)All specimens