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Name: Masoud Tahani

Affiliation: Ferdowsi University of Mashhad

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TIME HISTORY ANALYSIS OF STRESSES IN FUNCTIONALLY GRADED THICK HOLLOW CYLINDER SUBJECTED TO THERMAL SHOCK LOADING USING ANALYTICAL METHOD

M. Tahani[1,2], S.M. Hosseini[3], A. Safari[1], A. Todoroki[2]
[1]Department of Mechanical Engineering, Ferdowsi University of Mashhad, Mashhad, Iran
[2]Tokyo Institute of Technology, 2-12-1 O-okayama, Meguro, Tokyo 152-8552, Japan
[3]Industrial Engineering Department, Ferdowsi University of Mashhad, Mashhad, Iran
mtahani@ferdowsi.um.ac.ir

1. Summary

In this paper, an analytical method is presented to investigate time history analysis of stresses in a functionally graded thick hollow cylinder under thermal shock loading. Mechanical properties of FG thick hollow cylinder are assumed to be temperature independent and vary continuously in the radial direction. Constitutive equation of motion is derived for FG cylinder with infinite length and axisymmetry condition. The governing equations and boundary conditions are transferred to Laplace domain and analytically solved using series solution technique. To study the dynamic behavior of stresses, the obtained results in Laplace domain are transferred to time domain employing inverse Laplace transform technique. The time histories of thermo-mechanical stresses are studied and discussed for various grading patterns of mechanical properties across thickness of FG cylinder. Also, the thermo-mechanical stresses are presented in closed form for a functionally graded thick hollow cylinder subjected to thermal shock loading. The presented results show good agreement with published data in previous literatures.

2. Introduction

In the recent years, to determine the stress distribution in homogeneous and FG cylindrical shells, a number of analytical solutions have been obtained by many researchers. Wang et al. [1, 2] presented an effective analytical method to analyze the histories and distribution of the dynamic thermo-elastic stresses in an homogeneous thick hollow cylinder subjected to thermal and mechanical shock loadings. The transient thermo-mechanical stresses in a FGM hollow circular cylinder with infinite length have been determined using Laplace transform and series solution method by Shao et al. [3]. The inertia term in equation of motion was not considered by them and they considered pseudo dynamic conditions (considering elastic governing equation without inertia term in equation of motion) for their problem. Also, an analytical method based on composition of Bessel's functions has been presented by Hosseini et al. [4] for pseudo dynamic problem in FG thick hollow cylinder subjected to transient thermal loading.

In this article, the dynamic thermo-elastic stresses have been studied using an analytical method in functionally graded thick hollow cylinder subjected to thermal shock loading. The distributions of stresses in FG cylinder are presented and time histories of stress field are evaluated for various grading patterns of mechanical properties distribution.

3. Constitutive Equation and Solution Method

Consider a FG thick hollow cylinder with infinite length, inner radius a and outter radius b. The equation of motion is

$$\frac{\partial\sigma_{rr}(r,t)}{\partial r}+\frac{\sigma_{rr}(r,t)-\sigma_{\theta\theta}(r,t)}{r}=\rho(r)\frac{\partial^2 u(r,t)}{\partial t^2} \quad (1)$$

where σ_{rr} and $\sigma_{\theta\theta}$ are radial and hoop stresses, respectively. Also, r, t, $E(r)$ and $\rho(r)$ are radius of FG cylinder, time, modulus of elasticity and density, respectively. The stress-strain relations can be written as [3]

$$\sigma_{rr}(r,t)=\frac{E(r)}{(1+\nu)(1-2\nu)}\left[(1-\nu)\varepsilon_{rr}(r,t)+\nu\,\varepsilon_{\theta\theta}(r,t)\right]-\frac{\alpha(r)E(r)}{(1-2\nu)}T(r,t) \quad (2)$$

$$\sigma_{\theta\theta}(r,t)=\frac{E(r)}{(1+\nu)(1-2\nu)}\left[(1-\nu)\varepsilon_{\theta\theta}(r,t)+\nu\,\varepsilon_{rr}(r,t)\right]-\frac{\alpha(r)E(r)}{(1-2\nu)}T(r,t) \quad (3)$$

The parameters $\varepsilon_{rr}(r,t)$, $\varepsilon_{\theta\theta}(r,t)$, ν, $\alpha(r)$ and $T(r,t)$ are radial and hoop strains, Poisson's ratio, thermal expansion coefficient and temperature of FG cylinder, respectively. In a cylinder with infinite length and axisymmetry condition, the strains are related to displacements as follows [3]

$$\varepsilon_{rr}(r,t)=\frac{du(r,t)}{dr},\quad \varepsilon_{\theta\theta}(r,t)=\frac{u(r,t)}{r} \quad (4)$$

The nondimensional parameters used in the study are

$$\xi=\frac{r}{\bar r},\ \xi_1=\frac{a}{\bar r},\ \xi_2=\frac{b}{\bar r},\ \bar E(\xi)=\frac{E(r)}{E_0},\ \bar\alpha(\xi)=\frac{\alpha(r)}{\alpha_0},\ \bar\rho(\xi)=\frac{\rho(r)}{\rho_0},\ t^*=\left(\frac{C_V}{\bar r}\right)t$$
$$\bar r=\frac{a+b}{2},\ \bar T(\xi,t^*)=\frac{T(r,t)}{T_0},\ \bar u(\xi,t^*)=\frac{u(r,t)}{\alpha_0 T_0\bar r},\ \sigma_r(\xi,t^*)=\frac{\sigma_{rr}(r,t)}{\alpha_0 T_0 E_0},\ \sigma_\theta(\xi,t^*)=\frac{\sigma_{\theta\theta}(r,t)}{\alpha_0 T_0 E_0} \quad (5)$$

where the term C_V is a reference value for speed of stress wave propagation. Also, the parameters α_0, T_0, E_0 and ρ_0 are reference values. By substituting Eqs. (2)-(4) into Eq. (1) and using the nondimensional parameters the equation of motion in displacement and temperature terms can be derived as

$$\frac{\partial^2\bar u(\xi,t^*)}{\partial\xi^2}+\left(\frac{1}{\xi}+\frac{\bar E'(\xi)}{\bar E(\xi)}\right)\frac{\partial\bar u(\xi,t^*)}{\partial\xi}+\frac{\bar u(\xi,t^*)}{\xi}\left(\frac{\nu}{1-\nu}\frac{\bar E'(\xi)}{\bar E(\xi)}-\frac{1}{\xi}\right)-\bar T(\xi,t^*)\frac{\bar E'(\xi)}{\bar E(\xi)}\bar\alpha(\xi)\left(\frac{1+\nu}{1-\nu}\right)$$
$$-\frac{\partial\bar T(\xi,t^*)}{\partial\xi}\bar\alpha(\xi)\left(\frac{1+\nu}{1-\nu}\right)=\frac{(1+\nu)(1-2\nu)}{(1-\nu)}\frac{\bar\rho(\xi)}{\bar E(\xi)}\frac{\partial^2\bar u(\xi,t^*)}{\partial t^{*2}} \quad (6)$$

The boundary conditions in nondimensioanl form and for thermal shock loading can be considered as follow

$$\sigma_r(\xi_1,t^*)=0:\quad (1-\nu)\frac{\partial\bar u(\xi_1,t^*)}{\partial\xi}+\frac{\nu}{\xi_1}\bar u(\xi_1,t^*)-(1+\nu)\bar\alpha(\xi_1)\bar T(\xi_1,t^*)=0 \quad (7)$$

$$\sigma_r(\xi_2,t^*)=0:\quad (1-\nu)\frac{\partial\bar u(\xi_2,t^*)}{\partial\xi}+\frac{\nu}{\xi_2}\bar u(\xi_2,t^*)-(1+\nu)\bar\alpha(\xi_2)\bar T(\xi_2,t^*)=0 \quad (8)$$

$$\bar T(\xi,t^*)=H(t^*)=\begin{cases}0 & t^*<0\\ 1 & t^*\ge 0\end{cases} \quad (9)$$

Also, the following initial condisions are assumed for the probelm

$$u\left(r,0\right)=0,\quad \frac{\partial u\left(r,t\right)}{\partial t}=0 \tag{10}$$

The constitutive and boundary conditions are transferred to Laplace domain. The solution of Navier's equation in Laplace domain can be considered as

$$\tilde{u}\left(\xi,s\right)=\sum_{n=0}^{\infty}\left[f_n(s)\psi_0+g_n(s)\psi_1+h_n(s)\right]\left(\xi-1\right)^n \tag{11}$$

where the coefficients ψ_0 and ψ_1 should be determined using boundary conditions and also

$$\tilde{u}\left(\xi,s\right)=\Im\left\{\bar{u}\left(\xi,t^*\right)\right\} \tag{12}$$

The term $\Im$ defines the Laplace operator and terms $f_n(s)$, $g_n(s)$ and $h_n(s)$ can be calculated using a recurrence equation that are obtained as follows

$$\begin{aligned}
&-\left(n+1\right)\left(n+2\right)\varphi_{n+2}=\left(n+1\right)\left(2n-1\right)\varphi_{n+2}+\left(n^2-1\right)\varphi_n\\
&+\sum_{j=0}^{n}\left[\left(j-1\right)\varphi_{j-1}+2j\varphi_j+\left(j+1\right)\varphi_{j+1}\right]\Gamma_{1,n-j}+\left(\frac{\nu}{1-\nu}\right)\sum_{j=0}^{n}\left[\varphi_{j-1}+\varphi_j\right]\Gamma_{1,n-j}\\
&-\left(\frac{\left(1+\nu\right)\left(1-2\nu\right)}{1-\nu}\right)s^2\sum_{j=0}^{n}\left[\varphi_{j-2}+2\varphi_{j-1}+\varphi_j\right]\Gamma_{3,n-j}-\left(\frac{1+\nu}{1-\nu}\right)\frac{1}{s}\left[\Gamma_{2,n-2}+2\Gamma_{2,n-1}+\Gamma_{2n}\right]
\end{aligned} \tag{13}$$

where $\varphi_n=\left[f_n \quad g_n \quad h_n\right]^T$, $\varphi_{-1}=\varphi_{-2}=0$ and $n=0,1,2,...,\infty$. Also, the terms $\Gamma_{1,n}$, $\Gamma_{2,n}$ and $\Gamma_{3,n}$ are defined as

$$\begin{aligned}
&\frac{\bar{E}'(\xi)}{\bar{E}(\xi)}=\frac{1}{\bar{E}(\xi)}\frac{d\,\bar{E}(\xi)}{d\,\xi}=\sum_{n=0}^{\infty}\Gamma_{1,n}\left(\xi-1\right)^n,\quad \frac{1}{\bar{E}(\xi)}\frac{d\left[\bar{E}(\xi)\bar{\alpha}(\xi)\right]}{d\,\xi}=\sum_{n=0}^{\infty}\Gamma_{2,n}\left(\xi-1\right)^n,\\
&\frac{\bar{\rho}(\xi)}{\bar{E}(\xi)}=\sum_{n=0}^{\infty}\Gamma_{3,n}\left(\xi-1\right)^n
\end{aligned} \tag{14}$$

Using radial displacement function in (11), the stresses are analytically determined as functions of radius and time.

4. Numerical Example

Consider a FG thick hollow cylinder whose inner surface to be made of alumina and outer surface to be made of aluminum. The mechanical properties of these materials are shown in Table 1.

Table 1 Material properties.

Material	Modulus / GPa	Poisson's ratio	Density / kg$\cdot$m^{-3}	Thermal expansion / ($1/^\circ C$)
Alumina	380	0.3	3800	$8*10^{-6}$
Aluninium	70	0.3	2760	$23.6*10^{-6}$

The mechanical properties are assumed to vary across the thickness of FG cylinder as nonlinear function as follows

$$p\left(r\right)=p_c\left[1-\left(\frac{r-a}{b-a}\right)^{\beta}\right]+p_m\left(\frac{r-a}{b-a}\right)^{\beta} \tag{12}$$

where p is mechanical property such as elasticity modulus, thermal expansion coefficient and density and subscripts c and m stand for ceramic and metal, respectively.

The time histories of stresses in several points across thickness of FG cylinder under thermal shock loading are obtained for various grading patterns. Figs.1- 4 show the effects of variations in values of β on dynamic behavior of stresses.

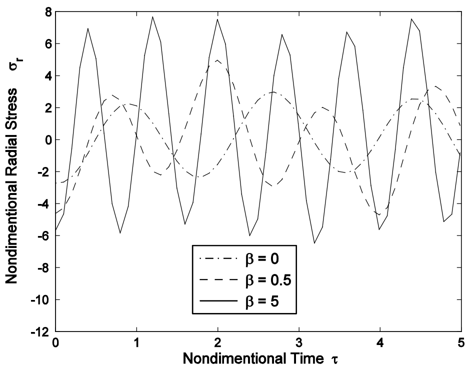

Figure 1 Time history of radial stress at middle point of thickness for $b/a = 2$ and various β.

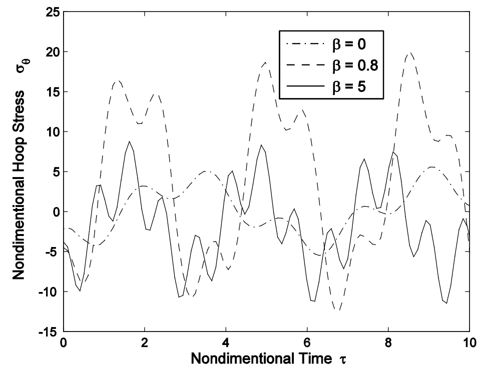

Figure 2 Time history of hoop stress at inner surface of FG cylinder for $b/a = 3$ and various β.

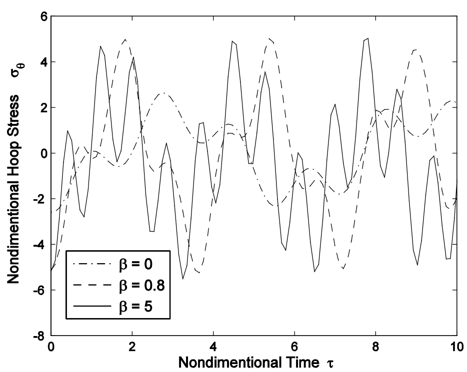

Figure 3 Time history of hoop stress at middle point of thickness for $b/a = 3$ and various β.

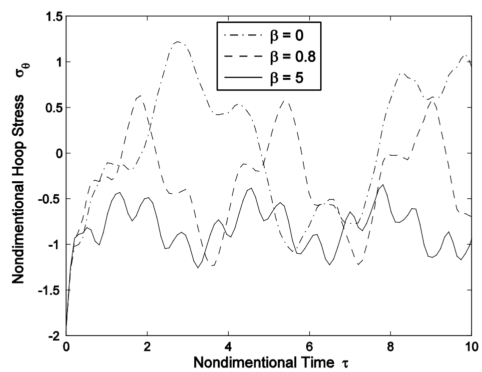

Figure 4 Time history of hoop stress at outer surface of FG cylinder for $b/a = 3$ and various β.

5. Conclusion

In this article, an analytical method based on series solution is presented to study dynamic behaviors of stresses in functionally graded thick hollow cylinder subjected to thermal shock loading. The radial displacement and stresses are obtained as functions of radius and time. The time histories of stresses are investigated for various grading patterns of FGM in several points across thickness of FG cylinder using presented analytical solution.

6. References

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