SVD-Based Receiver for Downlink MIMO MC-CDMA Systems

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Abstract — In this paper, a new receiver for downlink multicarrier code-division multiple-access (MC-CDMA) system is proposed when the channel between the transmitter and each user is multiple-input multiple-output (MIMO). The structure of the proposed receiver is based on the singular value decomposition (SVD) method. The conventional SVD method is able to eliminate the co-channel interference (CSI) in single user scenario, however, due to multi-channel environment, it is not able to eliminate the co-space interferences caused by the transmitted signals to the other users that is called multiuser interference (MUI). By combining the SVD and space frequency spreading code, a receiver called total interference cancellation (TIC) is developed that completely eliminates both the CSI and the MUI. The performance of the TIC suffers from small singular values of the subchannel matrices. To improve the performance, a symbol-chip level MMSE receiver is developed in which joint equalization and beamforming is utilized at chip level when the MMSE criterion is employed at symbol level (after despreading). The performance of the proposed receiver is evaluated and compared under different scenarios by computer simulations. The results show that there is no error floor in the bit error rate (BER) performances of the SVD based receiver under both criteria in the multiuser environment. Compared with the TIC criterion, the receiver designed based on the symbol-chip level MMSE criterion significantly improves the BER performance.

Index Terms — Downlink MC-CDMA, OFDM-CDMA, MIMO, SVD, Space-frequency spreading, MMSE.

I. INTRODUCTION

Multicarrier code division multiple access (MC-CDMA) system is regarded as a promising candidate for next generation broadband wireless communications. In the frequency selective fading channels, the MC-CDMA mitigates the intersymbol interference (ISI) due to using orthogonal frequency division multiplexing (OFDM) method and achieves frequency diversity due to employing code division multiple access (CDMA) technology.

To increase bandwidth efficiency and link reliability in the MC-CDMA system, multiple-input multiple-output (MIMO) channel is used by utilizing multiple antennas in both transmitting and receiving sides. The MIMO system potentially has more capacity in scattering environment compared with a single input single output system in which only one transmitting and receiving antenna is used. However, co-space interference is one of the main obstacles to exploit the capacity of the MIMO MC-CDMA system. In addition to co-space interference caused by other transmitted signals to the desired user due to using multiple antennas at the transmitter, in multiuser downlink MIMO MC-CDMA, the transmitted signals to the other users interfere to the desired user's signal as well that this type of interference is called multiuser interference (MUI). The MUI is another challenging issue that should be mitigated in multiuser environment.

Different techniques have been used in literature for receiver design in the MIMO MC-CDMA systems. One group of the employed techniques is based on the layered space-time architecture [1,2] in which the proposed algorithms mitigate the CSI and the MUI in an iterative manner. In some proposed designs, outer coder such as convolutional code or turbo code is also utilized to improve the system performance [2,3]. Chip level and symbol level are two strategies that have been considered in implementation of receiver design criteria. [2,4].

In this paper, we develop a new approach to design the downlink MIMO MC-CDMA receiver based on the singular value decomposition (SVD). The SVD method has been used to evaluate the capacity of the downlink MIMO MC-CDMA system [5,6], however to our best knowledge, it is the first time that the SVD approach is proposed for receiver design in downlink MIMO MC-CDMA system. A combination of the SVD and space frequency spreading code is employed to design a joint chip equalization and beamforming method that eliminates CSI and MUI totally; we call it total interference cancellation (TIC). The performance of the TIC receiver is degraded by small singular values of the channel matrix. To mitigate the effect of small singular values, another joint equalization and beamforming method is developed based on symbol level MMSE criterion when equalization and beamforming are utilized at the chip level. We call it symbol-chip level MMSE (SC-MMSE) receiver. The SC-MMSE receiver significantly improves the BER performance of the MIMO MC-CDMA system in comparison with the TIC receiver.

The paper is organized as follows. After introduction, we consider multiuser detection via SVD in Section II. SVD-based total interference cancellation (TIC) method is proposed in Section III. In Section IV, a receiver is developed based on symbol-chip MMSE criterion. By computer simulations, the performance of the proposed receiver is evaluated in Section V and Section VI concludes the paper.
II. MULTIUSER DETECTION VIA SVD

We consider a downlink MIMO MC-CDMA system with $K$ users and $L$ subcarriers where $N$ transmit antennas and $M$ receive antennas are deployed. The transmitted signal vector for user $m$ $(m = 1, 2, \ldots, K)$ is given as $d_m = [d_{m,1}, d_{m,2}, \ldots, d_{m,N}]^T$. Each symbol of $d_m$ is spreaded by spreading code vector $c_m = [c_m(0), c_m(1), \ldots, c_m(N-1)]^T$ where $\gamma$ is the processing gain, $c_m^H c_m = 1$. Note that $(\cdot)^T$ and $(\cdot)^H$ represent the transpose and transposed complex conjugate operations, respectively. Without loss of generality, number of subcarriers is assumed to be equal with the processing gain $L = \gamma$. Thus, the data vector of $g$ th subcarrier becomes:

$$s_m^{(g)} = c_m^{(g)} d_m, \quad g = 1, \ldots, \gamma$$

(1)

After adding the signal of other users, it is converted to the OFDM symbol and transmitted by the base station.

By doing some processing at the receiver, the received signal of the $m$ th user at the $g$ th subcarrier becomes:

$$y_m^{(g)} = H_m^{(g)} s_m^{(g)} + H_m^{(g)} \sum_{k=1}^{K} s_k^{(g)} + \eta_m^{(g)}$$

(2)

where $H_m^{(g)}$ is the $M \times N$ flat fading MIMO subchannel in the $g$ th subcarrier between the transmitter and the $m$ th user, the size of the received vector $y_m^{(g)}$ is $M \times 1$ and $\eta_m^{(g)}$ is the $M \times 1$ vector of the additive white Gaussian noise with zero mean and covariance matrix $R_\eta = \sigma_\eta^2 \mathbf{I}_M$. Singular value decomposition (SVD) of the $g$ th subchannel matrix is:

$$H_m^{(g)} = U_m^{(g)} \Lambda_m^{(g)} V_m^{(g)H}$$

(3)

in which, $U_m^{(g)}$ and $V_m^{(g)}$ are unitary matrices of sizes $M \times P$ and $N \times P$. Also, $\Lambda_m^{(g)}$ is the diagonal $P \times P$ matrix that $P \leq \min(M,N)$ is the rank of the subchannel. To mitigate the effect of the channel in the rank of the user by using the SVD based beamforming, we can send $\tilde{s}_m^{(g)} = V_m^{(g)H} c_m d_m$, where the sizes of $\tilde{s}_m^{(g)}$ and $s_m^{(g)}$ are $P \times 1$ and $N \times 1$, respectively. Thus, the received signal of the $m$ th user at the $g$ th subcarrier is given as

$$\tilde{y}_m^{(g)} = H_m^{(g)} \sum_{k=1}^{K} V_k^{(g)H} c_k d_k + \eta_m^{(g)}$$

(4)

After using beamformer matrix $U_m^{(g)}$ at the $g$ th subcarrier and doing some manipulations, the receiver signal becomes

$$\tilde{y}_m^{(g)} = U_m^{(g)H} y_m^{(g)}$$

$$= c_m^{(g)} d_m + \sum_{k=1}^{K} V_k^{(g)H} c_k^{(g)} d_k + \Lambda_m^{(g)} U_m^{(g)H} \eta_m^{(g)}$$

(5)

As it can be seen, co-space interference (CSI) is fully mitigated. To suppress multiuser interference (MUI), despreading is performed:

$$\hat{d}_m = \sum_{g=1}^{\gamma} \gamma_m^{(g)} y_m^{(g)}$$

(6)

$$\hat{d}_m = d_m + \sum_{g=1}^{\gamma} c_m^{(g)} d_k + \Lambda_m^{(g)} U_m^{(g)H} \eta_m^{(g)}$$

(7)

As seen, the interfences of other subchannel beamformer matrices ruin the orthogonality of the spreading codes between the users, thus the MUI is not suppressed. In the following section, we combine the SVD with space and frequency spreading code to overcome the MUI challenging issue.

III. SVD BASED TOTAL INTERFERENCE CANCELLATION METHOD

In this section, the desired user's transmitted signal vector $d_m = [d_{m,1}, d_{m,2}, \ldots, d_{m,J}]^T$ is spreaded both in the frequency and space domains where $J \leq N$. It is assumed that $M \geq N$ and $N$ is the rank of each subchannel matrix [1]. Note that general case will be considered latter. We define orthogonal spreading code matrix $C_m = [c_{m,1}, c_{m,2}, \ldots, c_{m,J}]$ with the size of $NL \times J$ that consists of $J$ different spreading orthogonal code vectors. Each code vector $c_{m,j} = [c_{m,j}^{(0)}, c_{m,j}^{(1)}, \ldots, c_{m,j}^{(NL-1)}]^T$ has $\gamma = NL$ processing gain where $C_m^{H} C_m = \mathbf{I}_J$ for $m = 1, 2, \ldots, K$.

The signal of the $m$ th user that is transmitted over $L$ subcarriers is given as

$$s_m = C_m d_m$$

(8)

where the size of $s_m$ is $NL \times 1$. The received signal of the $m$ th user at the output of the fast Fourier transform (FFT) block becomes

$$y_m = H_m \sum_{k=1}^{K} C_k d_k + \eta_m$$

(9)

where $H_m$ is the space-frequency channel matrix that has the block diagonal form.

$$H_m = \begin{bmatrix} H_m^{(1)} & 0 & \cdots & 0 \\ 0 & H_m^{(2)} & \cdots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & H_m^{(J)} \end{bmatrix}$$

(10)

The size of $H_m$ is $ML \times NL$ , $y_m$ is the $ML \times 1$ received signal vector and $\eta_m$ is the white additive white Gaussian noise vector with zero mean and covariance matrix $R_\eta = \sigma_\eta^2 \mathbf{I}_{ML}$. The block channel matrix, $H_m$, can be decomposed as

$$H_m = U_m \Lambda_m V_m^H$$

(11)
where \( U_m \) and \( V_m \) are block unitary matrices of sizes \( M L \times N L \) and \( N L \times N L \), respectively. Also, \( \Lambda_m \) is the diagonal \( N L \times N L \) matrix. After utilizing beamforming and equalization matrices, we have

\[
\hat{y}_m = \Lambda_m^H U_m^H y_m
\]

\[
= V_m^H \sum_{k=1}^{K} C_k d_k + \Lambda_m^H U_m^H \xi_m
\]

(12)

To suppress the MUI, instead of applying the conventional despreading matrix \( C_m^H \), we use \( G_m^H \) matrix that satisfies the following relation

\[
C_m^H = G_m^H V_m^H
\]

(13)

After utilizing the new despreading code matrix, \( G_m^H \), from (12) and (13) we have

\[
\hat{d}_m = G_m^H \hat{y}_m = G_m^H V_m^H \sum_{k=1}^{K} C_k d_k + G_m^H \Lambda_m^H U_m^H \xi_m
\]

\[
= C_m^H \sum_{k=1}^{K} C_k d_k + G_m^H \Lambda_m^H U_m^H \xi_m
\]

(14)

Due to orthogonality properties of spreading code \( C_m \), \( C_m^H C_i = I, \delta(m - k) \), after despreading the output becomes

\[
\hat{d}_m = C_m^H d_m + C_m^H \sum_{k=1}^{K} C_k d_k + n_m = d_m + n_m
\]

(15)

where \( n_m = G_m^H \Lambda_m^H U_m^H \xi_m \) is the \( J \times 1 \) noise vector at the output of the despreader (or at the input of slicer). As it can be seen in (15), both CSI and MUI are completely mitigated and total interference becomes zero; consequently, the proposed method is called total interference cancellation (TIC).

To evaluate the performance of the TIC method, we consider the SNR of the received signal after despreading. From (15) the \( j \) th element of \( \hat{d}_m \) becomes

\[
\hat{d}_{m,j} = d_{m,j} + n_{m,j}
\]

(16)

Since \( E[\hat{d}_m \hat{d}_m^H] = I_j \), from (15) and (16), the SNR of \( \hat{d}_{m,j} \) signal becomes

\[
SNR(\hat{d}_{m,j}) = \frac{1}{E[|n_{m,j}|^2]} = \frac{1}{\sum_{i=1}^{K} \sigma_i^2 \Lambda_{m,j,i}^2}
\]

(16)

where \( \Lambda_{m,j} \) is the \( j \)th diagonal element of \( \Lambda_m = \text{diag}(\lambda_{m,1}, \ldots, \lambda_{m,NL}) \) and \( g_{m,j} \) is also the \( j \)th element of \( \Lambda_m \) such that \( \Lambda_m = [g_{m,1}, g_{m,2}, \ldots, g_{m,NL}] \). As it can be seen the performance of the TIC method depends on the singular values of channel matrix. The small singular values degrade the performance of the TIC method.

Note that in developing of the TIC method, it is assumed that \( M \geq N \) and the rank of the \( l \)th subchannel \( H_m^{(l)} \) matrix is \( \text{Rank}(H_m^{(l)}) = N \) for \( l = 1, \ldots, L \). However, when \( M \leq N \) or \( \text{Rank}(H_m^{(l)}) = P \leq N \), the SVD of \( H_m^{(l)} \) can be written as

\[
H_m^{(l)} = U_m^{(l)} \Lambda_m^{(l)} [V_m^{(l)} : V_m^{(l)2}]^H
\]

(17)

where \( U_m^{(l)} \) is the \( M \times N \) unitary matrix, \( \Lambda_m^{(l)} \) is the \( N \times N \) diagonal matrix, and \( V_m^{(l)} \) is the \( N \times P \) unitary matrix and \( V_m^{(l)2} \) is the \( N \times (N - P) \) unitary matrix. Note that based on the SVD of \( H_m^{(l)} \) in (17), \( (N - P) \) diagonal elements of \( \Lambda_m^{(l)} \) are zeros. Therefore, when \( \text{Rank}(H_m^{(l)}) = P < N \), due to having zero singular value, the performance of the TIC method becomes poor. To overcome this challenging issue, in the next section, we develop another SVD based receiver using MMSE criterion.

IV. SVD BASED SYMBOL-CHIP MMSE RECEIVER

Consider the received signal at the \( m \)th user, Eq. (9), by utilizing the beamforming matrix, \( U_m \), at the receiver, we have

\[
\hat{y}_m = U_m^H y_m
\]

\[
= \Lambda_m^H V_m^H C_m d_m + \Lambda_m^H V_m^H \sum_{k=1}^{K} C_k d_k + U_m^H \xi_m
\]

(18)

Instead of using \( \Lambda_m \) that causes the CSI and MUI to be eliminated, a diagonal matrix \( F_m \) is utilized before despreading.

\[
\hat{d}_m = G_m^H F_m^H \hat{y}_m
\]

(19)

Without loss of generality, let us consider the \( j \)th element of \( \hat{d}_m \), it can be written as

\[
\hat{d}_{m,j} = g_{m,j}^H F_m \hat{y}_m
\]

(20)

We can convert the diagonal weight matrix of \( F_m \) to the vector \( f_{m,j} \) and also convert the vector \( g_{m,j} \) to the diagonal form \( G_{m,j} = \text{diag}(g_{m,j}^{(1)}, g_{m,j}^{(2)}, \ldots, g_{m,j}^{(N)}) \), in result, (19) can be rewritten as

\[
\hat{d}_{m,j} = \hat{f}_{m,j}^H \hat{G}_{m,j}^H \hat{y}_m
\]

\[
= \hat{f}_{m,j}^H \hat{G}_{m,j}^H \Lambda_m^H V_m^H C_m d_m + \hat{f}_{m,j}^H \hat{G}_{m,j}^H U_m^H \xi_m
\]

(21)

To compute the joint equalization and beamforming vector, \( f_{m,j} \) that is utilized at the chip level, the MMSE criterion is employed after despreading process at the symbol level. The
mean square error is defined as follows

\[ \varepsilon = E\left[ (d_{m,j} - \hat{d}_{m,j})^2 \right] \]  

(22)

By minimizing \( \varepsilon \) with respect to \( f_{m,j} \) and after doing some manipulations, we have

\[ f_{m,j} = (A_{m,j} + B_{m,j})^{-1} a_{m,j} \]  

(23)

where \( A_{m,j} \), \( B_{m,j} \) and \( a_{m,j} \) are defined as

\[ A_{m,j} = \sum_{k=1}^{K} \lambda_{k} C_{k} C_{k}^H V_{n} V_{n}^H \]  

(24)

\[ B_{m,j} = \sigma_n^2 \tilde{G}_{m,j} \tilde{G}_{m,j}^H \]  

(25)

\[ a_{m,j} = \tilde{G}_{m,j} A_{n} V_{n}^H c_{m,j} \]  

(26)

By utilizing \( f_{m,j} \) obtained from (23) instead of \( A_n^{-1} \), the orthogonality property is not preserved and as a result the MUI is not completely eliminated, however the effect of interference plus noise on detection at the symbol level is minimized.

V. SIMULATIONS AND RESULTS

A MIMO MC-CDMA system with \( M = N = 2 \) and \( 4 \) is considered in simulations. A sequence of independent, identically distributed signal vector with different modulation schemes is sent from transmitter antenna arrays. The utilized spreading code is orthogonal Walsh-Hadamard. The number of subcarriers is the same as the processing gain and is equal to \( L = 64 \). Each frequency selective channel between Tx-Rx antennas has been realized based on an exponentially decaying power delay profile with \( L_c=16 \) resolvable paths. The power decay factor of each path is \( \beta = 0.1 \). Also cyclic prefix with the same length of the channel is added to avoid ISI.

For comparison, conventional SVD based beamforming algorithm is considered in the MIMO MC-CDMA system. To have a fair comparison in the sense of processing gain, the transmitted signal vector is spreaded both in the frequency and space domains. The transmit weight matrix, \( V_{n} \), is deployed at the transmitter and the beamforming and equalization matrices are utilized at the desired user’s receiver in order to mitigate the CSI. Finally, the despreading is performed by the use of the same orthogonal spreading code matrix used in the transmitter. We call this algorithm as CSVD. The CSVD fully mitigates the co-space interference but the interferences of other subchannel beamformer matrices ruin the orthogonality of the spreading codes between the users, hence the MUI is not suppressed.

In Fig.1 the BER performance of the TIC algorithm is compared with that of the CSVD algorithm for \( J = M = N = 2 \) using BPSK modulation. In this figure, \( K \) is the number of active users. For the single-user case, it can be seen that both approaches have the same BER performance. But when \( K = 64 \), as expected, the performance of the CSVD is very poor but the TIC completely removes the MUI in the desired user receiver. In the other words, TIC performance in the multiuser environment is the same as its performance in the single-user case.

BER performance of the TIC in multiuser environments is compared with single-user performance of the CSVD utilizing different modulations in Fig.2 and Fig.3 for \( J=M=N=2 \) and \( J=M=N=4 \), respectively. It can be seen that in the single-user scenario, the BER performance of TIC and CSVD are the same. Moreover, by utilizing the TIC method, the MUI is completely suppressed even under full-load condition; thus, the BER performance of the TIC is independent of the number of users.

Fig.1. BER versus SNR for the proposed TIC and CSVD method for BPSK modulation when \( J=M=N=2 \).

Fig.2. BER versus SNR for the proposed TIC and CSVD methods for different modulation schemes when \( J=M=N=2 \).
The BER performance of the SC-MMSE approach is compared with that of the TIC method in Fig. 4 for 16QAM modulation and $M = N = 2$ in the full-load case when $J = 1$ and 2. It can be seen that the single-user BER performance of the SC-MMSE is much better than the single-user performance of the TIC; because the small singular values of the channel matrix enhance the noise power in the TIC but the SC-MMSE avoids this problem. On the other hand, since the SC-MMSE cannot preserve the spreading code orthogonality, its performance in the presence of the MUI is slightly degraded. However its full-load performance is still much better than the single-user TIC performance. Fig.5 demonstrates the BER performance comparison of the SC-MMSE and TIC receivers for 16QAM modulation and $M = N = 4$ when $J = 1$ and 4. As it can be seen the achieved results are similar to the results of the $M = N = 2$ case.

VI. CONCLUSIONS

In MIMO MC-CDMA systems, the presence of the desired user’s co-space interference (CSI) and multiuser interference (MUI) can degrade the system performance dramatically. To combat these interferences, a receiver structure based on the singular value decomposition (SVD) method has been proposed in this paper. The proposed receiver, in which spreading code is used in both frequency and space domains, completely eliminates the CSI and MUI that is called total interference cancellation (TIC) scheme. However, the TIC is vulnerable to noise power amplification when the singular values of subchannel matrices are small. By employing symbol-chip level MMSE criterion, both the interference and noise powers are taken into account in receiver design. The MMSE receiver shows better ability to suppress the interference plus noise power. In multiuser environment, simulation results indicate that the proposed SVD based receiver achieves a good performance even in fully loaded scenario.

REFERENCES


