REDUCTION OF POSITION ERROR OF KINEMATIC MECHANISMS BY TOLERANCE ANALYSIS METHOD, PART I: THEORY

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Abstract. It is practically impossible to manufacture a component exactly with the required dimensions. Therefore for each part dimension, a tolerance limit is prescribed. Also for all assemblies, a limit of variation is prescribed for a specified parameter of the assembly which is referred to as the assembly specification. In this research the Direct Linearization Method (DLM) is used to determine the distribution limit of the assembly specification in terms of part tolerances. It has been assumed that the assembly is a mechanism with flexible parts; therefore, in addition to manufacturing tolerances, external loading will impose external variations on part dimensions which result in extra errors on assembly specification. The effect of flexible components will cause change in mean, variance and correlation of the assembly specification. FEM is used to model the mechanism in order to compute part dimension variations under external loading. The percent contribution of each input variable on the assembly specification is obtained by the proposed multiple linear regression model. It has been proposed an optimization algorithm to assign part tolerances which minimizes manufacturing expenses while the maximum error of the assembly specification is kept within the desired limit.

Keywords: **Tolerance Analysis, FEM, Multiple Regression**

1. INTRODUCTION

An important aim in designing kinematic linkages is creating an accurate path by means of a point on the coupler. This point and the corresponding path are called Coupler Point (C.P.) and Coupler Point Path, respectively. In each cycle of motion, manufacturing tolerances of the parts and also extra variations due to flexibilities and loading cause a deviation in the C.P. path from its designed or ideal state. These deviations can lead to undesirable performance of the mechanism. There are several methods which were proposed to determine the effect of part tolerances on C.P. path deviations or the performance of the mechanism. The Direct Linearization Method (DLM) is firstly presented by Marler [1]. This method has been extended by Parkinson and Chase for static structures and kinematic mechanisms [2]. However, they assumed that all components are rigid. Markley presented a method to analyze assemblies with flexible parts [3]. He used the linear elastic assumption for contact of two parts which was proposed by Francavilla and Zienkiewicz [4]. He also presented a method to determine the variance and mean value of a dimension under loading. The current work implemented FEM model of kinematic mechanisms and investigated variations of assembly specification during one cycle of motion under external loading.

In this paper we are going to applying simultaneity the DLM and FEM, the effected of external loading on path error variation of a Crank Slider mechanism is obtained. Therefore, in this section of paper will be description theory of DLM, bivariate distribution and FEM and then in next section, the above discussion are using on the Crank Slider mechanism.

In Section 2 of this paper, the kinematic model of a crank slider mechanism including tolerances of input variables is expressed. In Section 3, the Direct Linearization Method is demonstrated and the equations of vector loops, sensitivity matrix and position error are obtained. In the following section, the DLM method is applied to find the bivariate distribution of the C.P. position error. In section 4, the FEM model of mechanism and a method for obtaining variation of component are described. Finally, the multiple linear regression method and present contribution are defined.

2. CRANK SLIDER MECHANISM MODEL

In the current work, The C.P. position error of a crank slider mechanism is analyzed, (see Figure 1). The reference path of C.P. is generated by assuming nominal dimensions for all components.
For each component of the mechanism, the manufacturing tolerances are specified on the basis of corresponding manufacturing processes and length of dimension[5]. Hence, tolerances of each nominal dimensions are selected based on Figure 2 [5], and reported in Table 1 is presented.

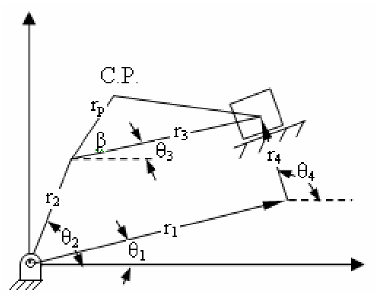

FIGURE 1: Crank slider mechanism with driving crank.

Angular position of link 2 (θ_2) is considered as an input to the mechanism. Therefore, it is not a manufacturing dimension and zero tolerance is assigned. All manufacturing dimensions are assumed to be normally distributed with a mean equal to the nominal link length. Also, the acceptable limit of distribution is taken according to common standard of 3σ.

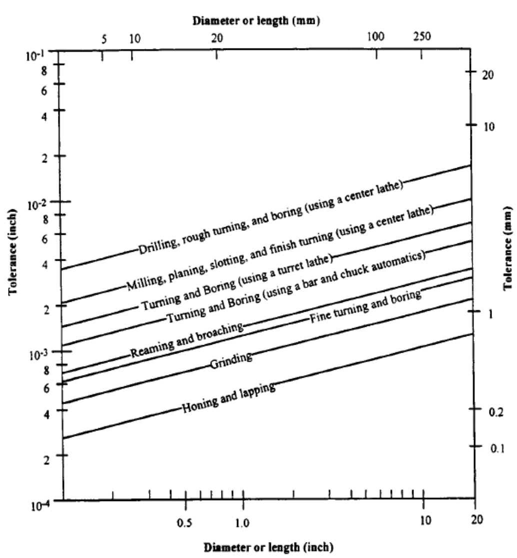

FIGURE 2: Tolerance range of machining processes [6].

TABLE 1. Nominal dimensions and tolerances of input variables (mm).

Manufacturing Variables	r_2	r_3	r_4	r_p	β	θ_1
Nominal Dimensions	250	400	25	104	80°	0°
Tolerances	± 0.3	± 0.2	± 0.02	± 0.15	± 0.5°	± 0.5°

3. DIRECT LINEARIZATION METHOD (DLM)

The Direct Linearization Method (DLM) can be used to determine the position error of a kinematic linkage. In this paper, point C.P. is designed to follow a specific path as the input crank (link 2) is rotated. The nominal position of point C.P. for a given input crank angle, θ_2, is found by solving one closed vector loop equations and one open vector loop equation (see Figure 3).

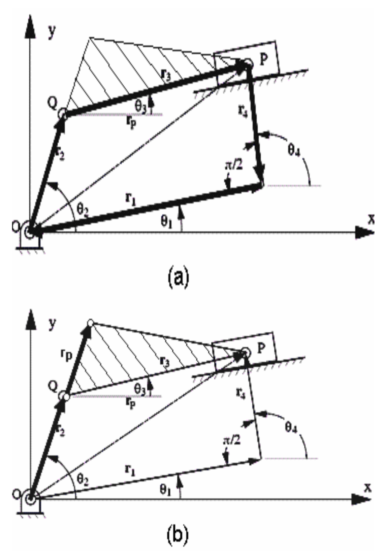

FIGURE 3: (a) Closed vector; (b) loop Open vector loop

Since position of point C.P. is defined by two direction x and y, so each vector loops is separated in to two equation. Closed and open loop equations are shown as follow, respectively:

$$h_x = r_2\cos\theta_2 + r_3\cos\theta_3 - r_4\cos\theta_4 - r_1\cos\theta_1 \quad (1)$$

$$h_y = r_2\sin\theta_2 + r_3\sin\theta_3 - r_4\sin\theta_4 - r_1\sin\theta_1 \quad (2)$$

$$(C.P.)_x = r_2\cos\theta_2 + r_p\cos(\theta_3 + \beta) \quad (3)$$

$$(C.P.)_y = r_2\sin\theta_2 + r_p\sin(\theta_3 + \beta) \quad (4)$$

In this method, the sensitivity matrix is derived using open and closed vector loops. The position error can be predicted by applying statistical approaches. Therefore, it will be assumed that the actual dimensions are normally distributed with a mean equal to the nominal link length with a standard deviation 3σ.
Partial derivatives of equations 1 and 2 with respect to the input variables, give us limit of assembly variables. These equations are then linearized using a first-order Taylor's series expansion [7]. This is written as:

$$[A]\{dX\} + [B]\{dU\} = 0 \tag{5}$$

Where $\{X\}=\{r_2, r_3, r_4, r_p, \beta, \theta_1, \theta_2\}$ is the vector of input variables and $\{U\}=\{\theta_3, r_1\}$ is the vector of assembly variables. $[A]$ and $[B]$ are matrices which represent first-order derivatives of equations (1) and (2) with respect to the manufacturing and assembly variables, respectively, i.e.

$$[A] = \partial h_i \Big/ \partial X_j \tag{6}$$

$$[B] = \partial h_i \Big/ \partial U_j \tag{7}$$

Equation (5) can be rewritten as:

$$\{dU\} = -[B]^{-1}[A]\{dX\} \tag{8}$$

A similar process is applied for open loop equations. Equation (9) expresses the variations of the assembly specification, i.e. C.P., in terms of the manufacturing and assembly variables.

$$\{d(C.P.)\} = [C]\{dX\} + [D]\{dU\} \tag{9}$$

Where [C] and [D] are first-order derivatives of equations (3) and (4) with respect to the manufacturing and assembly variables, respectively, i.e.

$$[C] = \partial(C.P.)_i \Big/ \partial X_j \tag{10}$$

$$[D] = \partial(C.P.)_i \Big/ \partial U_j \tag{11}$$

By substituting equation (8) into (9), the following equation is obtained.

$$\{d(C.P.)\} = ([C] - [D][B]^{-1}[A])\{dX\} = [S_{ij}]\{dX\} \tag{12}$$

where $[S_{ij}]$ is the sensitivity matrix of the assembly variables and can be written as:

$$[S_{ij}] = [C] - [D][B]^{-1}[A] \tag{13}$$

Based on the sensitivity matrix, the influence of each input variable on the assembly specification can be evaluated using Root Sum Square (RSS) statistical approach. The variance of the univariate normal distribution, which expresses the spread of the distribution, is determined using DLM method and computed by equation (14) [8].

$$\sigma_{(C.P.)_i}{}^2 = Var(C.P.)_i = \sum_{j=1}^{n} (S_{ij})^2 \sigma_j{}^2 \tag{14}$$

In the above equation, σ_j^2 is the variance of j-th manufacturing. In the case of multivariate distribution, the variance of input variables is expressed as the variance matrix V, which presents the variance of each input variable along with the correlation between the variables [9]. It is assumed that there is no correlation between the input variables. It is important to note that the matrix of component variances V is diagonal only if the variations of the components are linearly independent. Generally, component variations are assumed to be independent [8]. However, if several component features are produced with a single operation, as with a pattern of holes produced on a gang drill or multiple punch die, the component variations may be correlated. If the component variables are correlated, their covariance terms are placed in the off-diagonal spaces of the V matrix. Therefore, the variance matrix is diagonal and written as follows:

$$[V] = \begin{bmatrix} \sigma_1{}^2 & 0 & \cdots & 0 \\ 0 & \sigma_2{}^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma_n{}^2 \end{bmatrix} \tag{15}$$

4. BIVARIATE NORMAL DISTRIBUTION

Brown estimated the concurrent variation limits of two assembly specifications $d(C.P.)_X$ and $d(C.P.)_Y$ by the following equation [8]:

$$[\Sigma] = [S_{ij}][V][S_{ij}]^T \tag{16}$$

The covariance matrix Σ for bivariate distribution of assembly specification is presented by:

$$[\Sigma] = \begin{bmatrix} V_X & V_{XY} \\ V_{XY} & V_Y \end{bmatrix} \tag{17}$$

The diagonal elements indicate the deviations of each individual variable while the off-diagonal elements describe correlation between variables. The eigenvalues of the covariance matrix indicate the magnitude and direction of greatest variations. These eigenvalues are principle variances that represent the major and minor diameters of the elliptic contour of distribution [9]. The eigenvalues of 2-order variance matrix are determined as follows:

$$V_1 = \frac{V_X + V_Y}{2} + \sqrt{V_{XY}^2 + \left(\frac{V_Y - V_X}{2}\right)^2} \tag{18}$$

$$V_2 = \frac{V_X + V_Y}{2} - \sqrt{V_{XY}^2 + \left(\frac{V_Y - V_X}{2}\right)^2} \qquad (19)$$

Also, the rotation angle of principle axes to the *y* axis is given by [10]:

$$\theta_R = \frac{1}{2}\tan^{-1}\left(\frac{2V_{XY}}{V_Y - V_X}\right) \qquad (20)$$

The contour of equal probability can be presented by the following equation in polar coordinates (r,θ) [9].

$$\frac{\cos^2(\theta-\theta_R)}{V_2^2} + \frac{\sin^2(\theta-\theta_R)}{V_1^2} = \frac{n^2}{r^2} \qquad (21)$$

Where n is the sigma-level of the process. The maximum normal-to-path error, which is defined as the maximum perpendicular distance between distribution contour and the nominal C.P. path, is estimated with standard deviation of ±3σ.

5. FEM MODELING OF CRANK SLIDER MECHANISM

In the current work, The C.P position error of a crank slider mechanism with flexible component is analyzed, (see Figure 1). In the case, a vibrations and loading (force and moment), may by changing on tolerance and length of input variables of a assembly, are effected on performance of assembly.
This variation may be by deflection or deformation is created. Hence, affect of material, length and area of section of components, are substantial. In the preceding sections, the normal-to-path error was computed for assemblies with flexible components. In the study, affect of loading and tolerance of component, Simultaneity, on correlation and covariance of C.P. position is determined. The crank slider mechanism under external loading is modeled by the Finite Elements Method (FEM) using CALFEM toolbox. The beam and plate elements are built based on Euler-Bernoulli beam and Constant Strain Triangle theories, respectively. Shape function of all elements is linear we have used two-dimensional elements to approximate the force distribution along rods and plate [11]. Hence, utilizing a linear function for elements, so we use two nodes and three nodes to define an beam element, respectively. Those elements are shown in Figure 4.

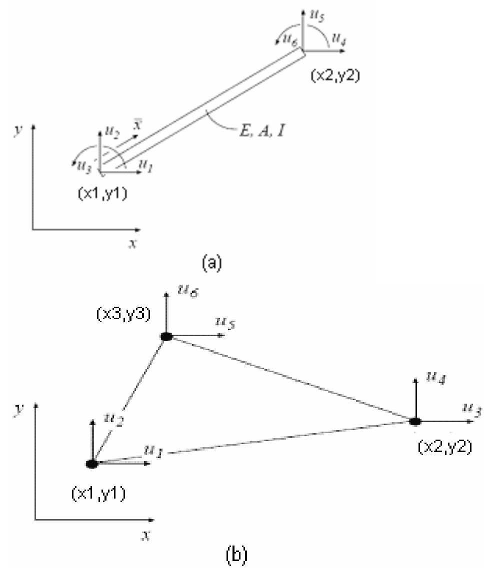

FIGURE 4. (a) Beam element (b) Constant Strain Triangle element [12]

Also contact elements are simulated by spring elements and friction forces in nodes of slider [13]. These elements are given between each nodes of slider and guide surface.(see Figure 5)

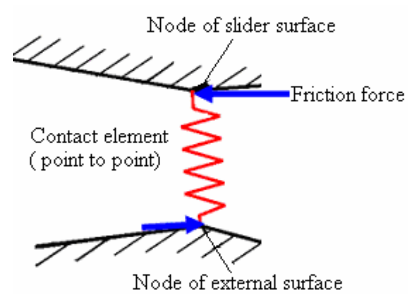

FIGURE 5. One sample of contact element.

Figure 5 demonstrates an FEM model of the crank slider mechanism under loading which is constructed by CALFEM.

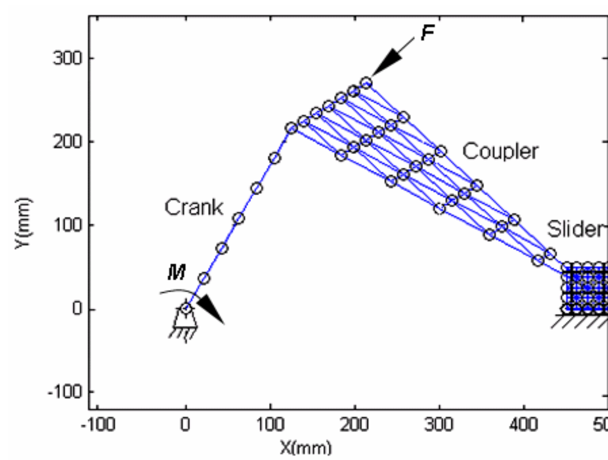

FIGURE 6. FEM model of Crank slider mechanism.

6. DIMENSIONAL VARIATIONS OF INPUT VARIABLES

All parts of an assembly have a nominal dimension with a tolerance limit due to unwanted variations in manufacturing processes. These dimensions and tolerances are varied by stretches, compressions and bendings occurred through part flexibilities. To determine the new dimensions and tolerances, the input variables are considered with their maximum and minimum allowable values, and then the corresponding deflections are computed using FEM. The new tolerance limits are achieved by adding these deflections to the former values. According to Figure 4, the maximum and minimum lengths of each variable are $L+T+\delta_{L+T}$ and $L-T+\delta_{L+T}$, respectively.

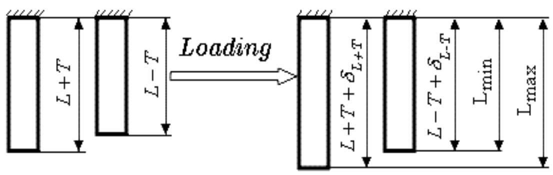

FIGURE 7. The maximum and minimum lengths a variable after loading.

The new mean value and tolerance of each variable are also determined by equations (22) and (23).

$$L = \frac{L_{\max} + L_{\min}}{2} \tag{22}$$

$$T = \frac{L_{\max} - L_{\min}}{2} \tag{23}$$

7. MULTIPLE LINEAR REGRESSION MODEL AND PERCENT CONTRIBUTION

A regression model that contains more than one variable is called a multiple regression model. A multiple linear regression model is defined by the following relationship[14]:

$$y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_K X_K + \in \tag{24}$$

where y represents the maximum normal error, X_i represents the input variables, and ε is a random error term.

The percent contribution chart tells the designer how each dimension contributes to the assembly specification variation. The contribution includes the effect of both the sensitivity and the tolerance. In order to determined the percent contribution of each input variable on the error, β_i are divided by the summation of β_i and the sensitivity of each input variables is determined It is common practice to present the results as a bar chart, sorted according to magnitude [14].

After the percent contribution is defined, the maximum error of mechanism will been decreased by changing tolerance of each variables that have major percent contribution. This variation is effecting on product cost . therefore, to optimization of error and product cost, using optimization algorithm.

8. OPTIMIZATION OF ERROR AND MANUFACTURING COSTS

A promising method of selecting part tolerances is assigning tolerances such that the manufacturing expenses are minimized. This can be accomplished by the cost-tolerance function for each component. Chase et. al. proposed the following general form for this purpose [15]:

$$C = A + B/tol^k \tag{25}$$

Where the constant coefficient A represents fixed costs. It may include setup cost, tooling, material, prior operations, etc. The B term determines the cost of producing a single component dimension to a specified tolerance and includes the machine cost rate. Costs are calculated on a per part basis. In order to reach tighter tolerances, speeds and feeds should be reduced and the number of passes increased, requiring more time and higher costs. The exponent k describes how sensitive the process cost is to changes in tolerance specifications.

Finally, the variation of the maximum error of assembly versus the minimum cost is derived. Based on the computed curve (max error versus min cost), the optimum tolerances for the input variables are determined.

9. CONCLUSION

The first part of the current research deals with background and the theory required for tolerance analysis of flexible mechanism. A flexible crank slider mechanism is chosen in order to introduce the proposed method step-by-step. The last section of this paper states the optimization approach to assign part dimension tolerance with objective of minimization manufacturing costs along with minimization of the maximum error of the assembly specification.

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