PRAMANA
— journal of
physics

Vol. 73, No. 5
November 2009
pp. 913–926

Dust acoustic solitary and shock waves in strongly coupled dusty plasmas with nonthermal ions

HAMID REZA PAKZAD[1,*] and KUROSH JAVIDAN[2]
[1]Department of Physics, Islamic Azad University, Bojnourd Branch, Iran
[2]Department of Physics, Ferdowsi University of Mashhad, Iran and
Center for Theoretical Physics, Ferdowsi University of Mashhad, Iran
*Corresponding author
E-mail: pakzad@bojnourdiau.ac.ir; ttaranomm83@yahoo.com; Javidan@um.ac.ir

MS received 5 January 2009; revised 1 May 2009; accepted 11 June 2009

Abstract. The Korteweg–de Vries–Burgers (KdV–Burgers) equation and modified Korteweg–de Vries–Burgers equation are derived in strongly coupled dusty plasmas containing nonthermal ions and Boltzmann distributed electrons. It is found that solitary waves and shock waves can be produced in this medium. The effects of important parameters such as ion nonthermal parameter, temperature, density and velocity on the properties of shock waves and solitary waves are discussed.

Keywords. Plasma; dust; nonthermal ion; soliton; shock wave.

PACS Nos 52.35.Bj; 52.35.Mw; 52.35.S

1. Introduction

In recent years, the study of nonlinear waves in plasmas has become one of the most important topic in plasma physics [1,2]. Also a number of authors have investigated properties of one-dimensional linear and nonlinear dust acoustic waves (DAW) in coupled unmagnetized dusty plasmas. Rao *et al* theoretically predicted the existence of DAWs in unmagnetized dusty plasmas [3]. Shukla and Mamun have derived Korteweg–de Vries–Burgers (KdV–Burgers) equation by reductive perturbation method and they have studied the properties of the solitons and shock waves for strongly coupled unmagnetized dusty plasmas [4]. Also Mamun *et al* have studied dusty plasma with a Boltzmann electron distribution, a nonisothermal vortex-like ion distribution and strongly correlated grains in a liquid-like state and discussed about the properties of shock wave structures in it [5]. They have derived modified KdV–Burgers equation using a set of generalized hydrodynamic (GH) equations. Ghosh and Gupta have investigated the nonlinear propagation of shock wave in strongly coupled collisional dusty plasma using the GH model incorporating a charging-delay effect [6]. A large-amplitude shock wave and its consequences in the kinetic regime of strongly coupled dusty plasma and the effect of the delayed charging on dust acoustic wave have been investigated in [7]. Strongly coupled plasma is of great interest in science, because of its applications in the interior of heavy planets, plasmas produced by laser compression of matter and nonideal plasmas for industrial applications. Solitons and shock waves would be the fundamental nonlinear coherent structures in dusty plasmas and that has been paid a great deal of interest in recent years [8–10]. In dusty plasmas, if the dissipation is weak at the characteristic dynamical time-scales of the system, then the balance between nonlinear and dispersion effects can result in the formation of symmetrical solitary waves. Also shock waves will be propagated in this system if dissipation effect is strong. Thus solitary and shock waves (oscillatory and monotone types) can be produced in dusty plasmas. Dust acoustic solitary structures in dusty plasma have been investigated widely. For example, Rao *et al* in [11] have investigated shock waves in coupled dusty plasma with Boltzmann distribution of ions. Ghosh *et al* have studied the effect of nonadiabatic dust charge variation on the nonlinear dust acoustic wave with nonisothermal ion [12]. They have shown that the shock strength increases (decreases) with increase in the nonisothermal parameter. The dust acoustic waves in the unmagnetized strongly coupled dusty plasmas with nonthermal distributed ions have been studied in [13]. The effect of nonthermal ions on dust acoustic solitary waves in dusty plasma was investigated in [14]. Mamun *et al* have also studied the multi-dimensional instability of electrostatic solitary structures in magnetized nonthermal dusty plasma [15]. In this paper, the effects of relative density, relative temperature and ion nonthermal distribution on nonlinear dust acoustic waves have been studied by reductive perturbation technique. The GH equations are presented in §2. In §3 we have derived the Korteweg–de Vries–Burgers equation using reductive perturbation method. The solitonic and shock wave solutions for this equation have been studied in §§4 and 5, respectively. In §6 we have derived the modified Korteweg–de Vries–Burgers equation. In this section we have shown that KdV–Burgers solutions cannot be established for critical values of parameters where the nonlinear term is zero. Also new kind of shock wave solutions for the critical values of density has been investigated. Finally the main results of this paper and conclusion are given in §7.

2. Basic equation

We consider an unmagnetized strongly coupled dusty plasma with Boltzmann distributed electrons, nonthermal distributed ions and negatively charged dust grains. We assume that the electrons and ions are weakly coupled compared to the dust grains. The dynamics of the DAW in our coupled dusty plasma are given by GH equations [16–18] as follows:

$$\frac{\partial n_{\mathrm{d}}}{\partial t} + \frac{\partial}{\partial x}(n_{\mathrm{d}} u_{\mathrm{d}}) = 0,$$

$$\left(1 + \tau_{\mathrm{m}} \frac{\partial}{\partial t}\right) \left[n_{\mathrm{d}} \left(\frac{\partial u_{\mathrm{d}}}{\partial t} + u_{\mathrm{d}} \frac{\partial u_{\mathrm{d}}}{\partial x} - \frac{\partial \phi}{\partial x}\right)\right] = \eta_1 \frac{\partial^2 u_{\mathrm{d}}}{\partial x^2},$$

$$\frac{\partial^2\phi}{\partial x^2} = n_{\rm d} + \frac{\mu}{1-\mu}{\rm e}^{\sigma_{\rm i}\phi} - \frac{1}{1-\mu}[1+\beta(\phi+\phi^2)]{\rm e}^{-\phi}, \tag{1}$$

where $n_{\rm d}$ is the number density of dust particles normalized by $n_{0{\rm d}}$ and $n_{0{\rm d}}$ is the unperturbed dust particle number density, $u_{\rm d}$ is the velocity of the dust fluid normalized by the dust acoustic speed $C_{\rm d} = \sqrt{Z_{\rm d}T_{\rm i}/m_{\rm d}}$ and $Z_{\rm d}$ is the charged number of dust particles, $T_{\rm i}$ is the temperature of ions and $m_{\rm d}$ is the mass of the dust particles. ϕ is the electrostatic wave potential normalized by $T_{\rm i}/e$ (e is the magnitude of the electron charge). Also $\sigma_{\rm i} = T_{\rm i}/T_{\rm e}$ in which $T_{\rm e}$ is the temperature of the electrons. μ is defined as $\mu = n_{0{\rm e}}/n_{0{\rm i}}$ where $n_{0{\rm e}}$ and $n_{0{\rm i}}$ are the unperturbed number densities of electrons and ions, respectively. $\beta = 4\alpha/(1+3\alpha)$ in which α is a parameter determining the number of fast (nonthermal) ions [15]. If we choose $\alpha = 0$ the medium reduces to the model of Rao *et al* in [11].

$\tau_{\rm m}$ refers to the viscoelastic relaxation time normalized by the dust plasma period $\omega_{\rm pd}^{-1} = \sqrt{m_{\rm d}/(4\pi n_{0{\rm d}}Z_{0{\rm d}}^2e^2)}$ given as

$$\tau_{\rm m} = \eta_1\frac{T_{\rm e}}{T_{\rm d}}\left[1-\mu_{\rm d}+\frac{4}{15}u(\Gamma)\right]^{-1}, \tag{2}$$

where

$$\mu_{\rm d} = 1+\frac{1}{3}u(\Gamma)+\frac{\Gamma}{9}\frac{\partial u(\Gamma)}{\partial\Gamma} \tag{3}$$

is the compressibility [19] in which Γ is the Coulomb coupling parameter [4] and $u(\Gamma)$ is a measure of the excess internal energy of the system. For a weakly coupled plasma with $\Gamma \ll 1$, $u(\Gamma)$ can be written as $u(\Gamma) \cong -(\sqrt{3}/2)\Gamma^{3/2}$ [20]. For $1 < \Gamma < 100$, Slattery *et al* have analytically derived the relation $u(\Gamma) \approx -0.89\Gamma + 0.95\Gamma^4 + 0.19\Gamma^{-1/4} - 0.81$ [21]. η_1 is the normalized viscosity coefficient given as

$$\eta_1 = \frac{1}{m_{\rm d}n_{0{\rm d}}\omega_{\rm pd}\lambda_{D{\rm d}}^2}\left[\eta_{\rm b}+\frac{4}{3}\zeta_{\rm b}\right], \tag{4}$$

where $\eta_{\rm b}$ and $\zeta_{\rm b}$ are transport coefficients of shear and bulk viscosities.

3. KdV–Burgers equation

According to the general method of reductive perturbation theory, we choose the independent variables as

$$\xi = \varepsilon^{1/2}(x-\lambda t), \quad \tau = \varepsilon^{3/4}t, \quad \tau_{\rm m} = \varepsilon^{1/2}\tau_{\rm m0}, \quad \eta_1 = \varepsilon^{1/2}\eta_0, \tag{5}$$

where ε is a small dimensionless expansion parameter which characterizes the strength of nonlinearity in the system and λ is the phase velocity of the wave along the x direction and normalized by dust acoustic velocity. Now we expand dependent variables as follows:

$$n_{\rm d} = 1 + \varepsilon n_{1\rm d} + \varepsilon^2 n_{2\rm d} + \varepsilon^3 n_{3\rm d} + \cdots$$
$$u_{\rm d} = \varepsilon u_{1\rm d} + \varepsilon^2 u_{2\rm d} + \varepsilon^3 u_{3\rm d} + \cdots$$
$$\phi = \varepsilon \phi_1 + \varepsilon^2 \phi_2 + \varepsilon^3 \phi_3 + \cdots . \tag{6}$$

Substituting (6) into (1) and collecting the terms in different powers of ε the following equations can be obtained at the lower order of ε:

$$n_{1\rm d} = -\frac{\phi_1}{\lambda^2}, \quad u_{1\rm d} = -\frac{\phi_1}{\lambda}, \quad \frac{1}{\lambda^2} = \frac{1}{1-\mu}\left(\mu\sigma_{\rm i} + \frac{1-\alpha}{1+3\alpha}\right). \tag{7}$$

At the higher order of ε, we have

$$\frac{\partial n_{1\rm d}}{\partial \tau} - \lambda \frac{\partial n_{2\rm d}}{\partial \xi} + \frac{\partial}{\partial \xi}(u_2 + n_1 u_1) = 0$$
$$\frac{\partial u_{1\rm d}}{\partial \tau} - \lambda \frac{\partial u_2}{\partial \xi} + u_1 \frac{\partial u_1}{\partial \xi} - \frac{\partial \phi_2}{\partial \xi} = \eta_0 \frac{\partial^2 u_1}{\partial \xi^2}$$
$$\frac{\partial^2 \phi_1}{\partial \xi^2} = n_2 + \frac{1}{\lambda^2}\phi_2 + \frac{\mu\sigma_{\rm i}^2 - 1}{2(1-\mu)}\phi_1^2. \tag{8}$$

Finally substituting (7) in (8) yields KdV–Burgers equation

$$\frac{\partial \phi_1}{\partial \tau} + A\phi_1 \frac{\partial \phi_1}{\partial \xi} + B\frac{\partial^3 \phi_1}{\partial \xi^3} + C\frac{\partial^2 \phi_1}{\partial \xi^2} = 0, \tag{9}$$

where the coefficients are

$$A = -\frac{3}{2}\left[\frac{\mu\sigma_{\rm i} + 3\alpha\mu\sigma_{\rm i} - \alpha + 1}{(1-\mu)(1+3\alpha)}\right]^{+1/2}$$
$$\quad -\frac{1}{2}\left[\frac{\mu\sigma_{\rm i} + 3\alpha\mu\sigma_{\rm i} - \alpha + 1}{(1-\mu)(1+3\alpha)}\right]^{-3/2}\left(\frac{\mu\sigma_{\rm i}^2 - 1}{1-\mu}\right),$$
$$B = \frac{1}{2}\left[\frac{\mu\sigma_{\rm i} + 3\alpha\mu\sigma_{\rm i} - \alpha + 1}{(1-\mu)(1+3\alpha)}\right]^{-3/2}, \quad C = -\frac{\eta_0}{2}. \tag{10}$$

This equation has not known exact solution, but there are some solutions in special cases. If the dissipation term is negligible compared to the nonlinearity and dispersion terms, then solitonic structure will be established by balancing the effects of dispersive and nonlinear terms. On the other hand, if the coupling becomes very strong the shock waves will appear. The dissipative term changes only by changing the parameter η_0. But the nonlinear and dispersive terms are functions of $\mu, \sigma_{\rm i}$ and α. Figure 1 presents A as functions of parameters $\mu, \sigma_{\rm i}$ and α.

4. Solitary waves

We have considered unmagnetized and weakly coupled dusty plasma with nonthermal distributed ions and Boltzmann distributed electrons, and negatively charged

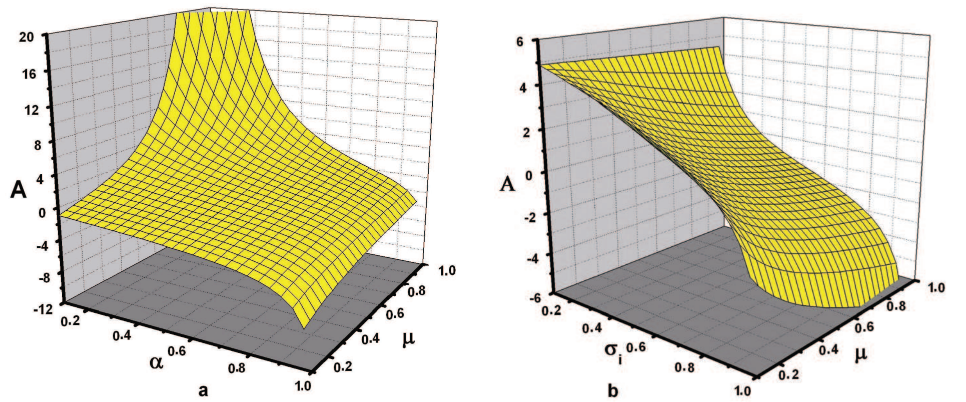

Figure 1. Parameter A as a function of α, μ and $\sigma_{\rm i}$. (**a**) $\sigma_{\rm i} = 0.3$ and (**b**) $\alpha = 0.5$.

dust. If the coupling force is absent, i.e. the dissipation effect is negligible in comparison with that of nonlinearity and dispersion, we will have the KdV equation

$$\frac{\partial \phi_1}{\partial \tau} + A\phi_1 \frac{\partial \phi_1}{\partial \xi} + B\frac{\partial^3 \phi_1}{\partial \xi^3} = 0. \tag{11}$$

The solitary wave solution for this equation is

$$\phi_1 = \phi_0 \,\mathrm{sech}^2\left(\frac{\xi - u_0\tau}{w}\right), \tag{12}$$

where $\phi_0 = 3u_0/A$ is the amplitude of soliton and $w = 2\sqrt{B/u_0}$ is its width. Figure 2 presents the soliton profile for different values for $\mu, \sigma_{\rm i}$ and α with $u_0 = 0.6$. This figure shows that both rarefactive and compressive solitary waves can be produced.

5. Shock waves

Now we discuss the shock wave solutions of (9). Generally shock waves appear when the coefficient C in (9) is not zero. Indeed the coupling force is responsible for the existence of shock wave solutions. By using the co-moving coordinate system that is defined as $\chi = \xi - u_0 t$ in which u_0 is the velocity of the wave, and integrating with respect to the variable χ, we have

$$\frac{\mathrm{d}^2\phi_1}{\mathrm{d}\chi^2} = \frac{u_0}{B}\phi_1 - \frac{A}{2B}\phi_1^2 - \frac{C}{B}\frac{\mathrm{d}\phi_1}{\mathrm{d}\chi}. \tag{13}$$

The constant of integration has been taken equal to zero in (13). Boundary condition is set as

$$\chi \to -\infty:\quad \phi_1 = \frac{\mathrm{d}\phi_1}{\mathrm{d}\chi} = \frac{\mathrm{d}^2\phi_1}{\mathrm{d}\chi^2} = 0$$

$$\chi \to +\infty:\quad \phi_1 = \phi_{\rm c},\quad \frac{\mathrm{d}\phi_1}{\mathrm{d}\chi} = \frac{\mathrm{d}^2\phi_1}{\mathrm{d}\chi^2} = 0. \tag{14}$$

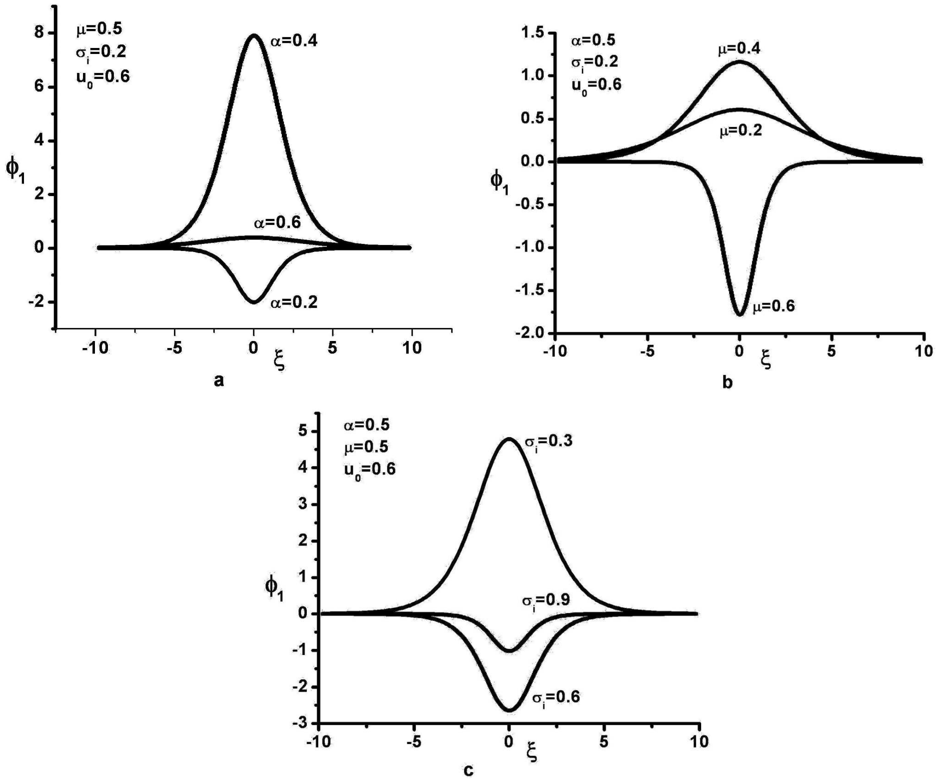

Figure 2. Solitary wave profile with different values for parameters.

Then, we have

$$\phi_c = \frac{2u_0}{A}. \tag{15}$$

By using $\phi_1 = \phi_c + \phi'$, for $|\phi_c| \gg |\phi'|$ (13) can be linearized as [22]

$$\frac{d^2\phi'}{d\chi^2} + \frac{C}{B}\frac{d\phi'}{d\chi} + \frac{u_0}{B}\phi' = 0. \tag{16}$$

It can be seen that the solutions of (16) is proportional to $\exp(H\chi)$ where H is given by

$$H = \frac{C}{B}\left[-1 \pm \sqrt{1 - \frac{4Bu_0}{C^2}}\right]. \tag{17}$$

Equation (17) indicates that for $C^2 < 4Bu_0$ (H is imaginary) the shock wave has an oscillatory profile and for $C^2 > 4Bu_0$ (H is real) we have a monotonic-type shock wave. Thus both types of solutions can be created as follows:

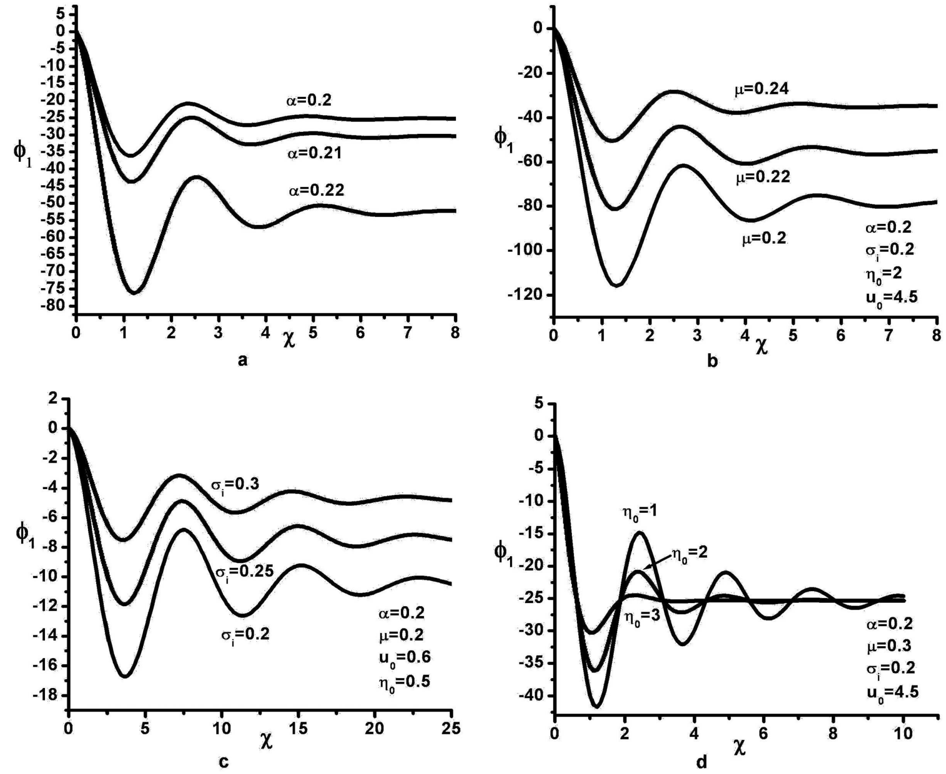

Figure 3. Oscillatory shock wave solutions as functions of medium parameters.

(I) For $C^2 \ll 4Bu_0$ oscillatory shock wave solution is given by [23]

$$\phi_1 = \phi_{\mathrm{c}} + \phi_0 \exp\left(+\frac{C}{2B}\chi\right) \cos\left(\sqrt{\frac{u_0}{B}}\chi\right), \tag{18}$$

where ϕ_0 is a constant. Generally ϕ_0 has been chosen equal to $(-\phi_{\mathrm{c}})$.

(II) For $C^2 > 4Bu_0$ monotonic shock wave solution is given by [24]

$$\phi_1 = \frac{u_0}{A}\left[1 - \tanh\left(\frac{-u_0}{2C}\chi\right)\right] = \frac{u_0}{A}\left[1 - \tanh\left(\frac{u_0}{\eta_0}\chi\right)\right]. \tag{19}$$

Comparing (12) with (18) and (19) one can find that the amplitude of both solutions are the same functions of medium parameters except that the maximum amplitude of monotonic shock wave is 2/3 of the soliton amplitude and oscillatory shock wave has an amplitude exactly 1/3 of the maximum amplitude of soliton solution [25].

Figures 3 and 4 present shock waves in two cases (oscillatory shock wave and monotonic shock wave) for different values of parameters ($\alpha, \sigma_{\mathrm{i}}, \mu$ and η_0). The amplitude of shock waves decreases when the number of fast ions (which is modelled by the parameter α) increases (figure 3a). The amplitude of this type of

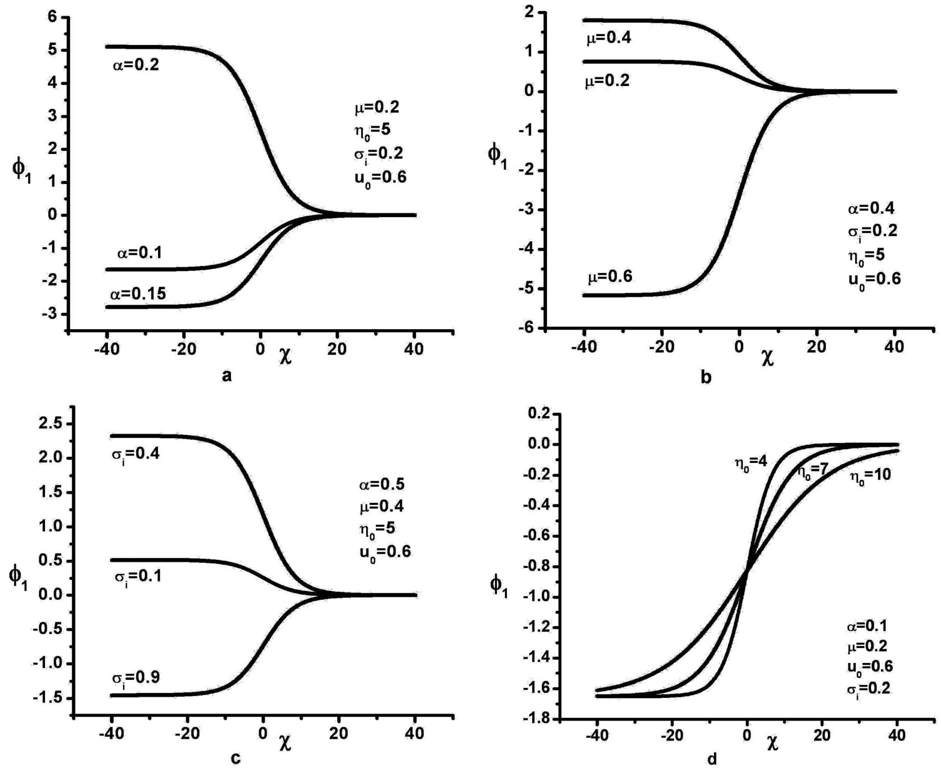

Figure 4. Monotonic shock wave as functions of medium parameters.

solution increases with an increasing μ (figure 3b). Therefore, the amplitude increases when the density of electrons increases or the density of ions decreases. Also the amplitude of oscillatory shock waves increases when $\sigma_{\rm i}$ increases. This means that the amplitude of oscillatory shock waves in the medium increases when the temperature of ions is increased or the temperature of electrons decreases (figure 3c). Figure 3d clearly shows that, the oscillatory type shock wave changes to monotonic type when η_0 increases.

It is obvious from figures 4a–c that the shock wave profile changes from a kink wave structure to an anti-kink type. Therefore, we can conclude that there is a critical value for α (while the other parameters are constant) in which the amplitude of shock wave is zero. According to eq. (19) the structure of monotonic shock wave depends on the sign of the nonlinear term (with a positive u_0). When $A > 0$, a kink wave structure is formed and an anti-kink wave structure is formed when $A < 0$. Therefore, the critical value of α ($\alpha_{\rm c}$) can be determined by $A = 0$. So we have

$$\alpha_{\rm c} = \frac{1 + \mu\sigma_{\rm i} - \sqrt{(1-\mu)(1-\mu\sigma_{\rm i}^2)/3}}{1 - 3\mu\sigma_{\rm i} + 3\sqrt{(1-\mu)(1-\mu\sigma_{\rm i}^2)/3}}.$$

As the dissipation depends mainly on the parameter η_0, we need to study the variation of amplitude with respect to η_0. Comparison of figures 3d and 4d shows

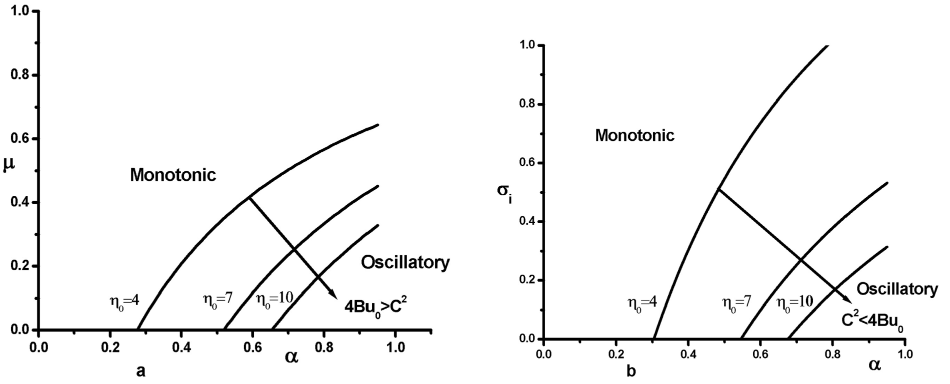

Figure 5. Border between two kinds of shock waves as functions of parameters. (**a**) was plotted with $\sigma_{\rm i} = 0.2$ and (**b**) was plotted with $\mu = 0.3$.

that the amplitude of the shock wave structure becomes steeper when η_0 increases. Figure 5 presents the border between oscillatory- and monotonic-type shock waves as functions of parameters. In the region $C^2 < 4Bu_0$ an oscillatory-type shock wave is formed while in the other region monotonic shock waves are established.

Figure 5a shows the region of oscillatory shock wave and monotonic shock wave for different values of η_0 in μ–α plane with $\sigma_{\rm i} = 0.2$. The region of the oscillatory shock wave decreases when η_0 increases. Also for a fixed η_0, there exists a critical α ($\alpha'_{\rm c}$). When $\alpha < \alpha'_{\rm c}$, the dust acoustic shock waves are monotonic for all values of μ. When $\alpha > \alpha'_{\rm c}$, we can find a critical μ ($\mu'_{\rm c}$) where, in the region of $\mu < \mu'_{\rm c}$, the dust acoustic shock waves are oscillatory. On the contrary, in $\mu > \mu'_{\rm c}$ region the dust acoustic shock waves are monotonic. The value of $\alpha'_{\rm c}$ increases when η_0 increases and the value of $\mu'_{\rm c}$ decreases with increasing η_0. Figure 5b shows two regions of oscillatory shock wave and monotonic shock wave for different values of η_0 in $\sigma_{\rm i}$–α plane for $\mu = 0.3$.

6. Modified Korteweg–de Vries–Burgers equation

Some researchers have studied KP and KdV equations at the critical density and have obtained modified KP and modified KdV equations [14,26,27]. Figure 1 shows that parameter A has both positive and negative values. Therefore, if density reaches a critical value ($\mu_{\rm c}$) the parameter A becomes zero. So the critical density is calculated as

$$\mu_{\rm c} = \frac{6\sigma_{\rm i}(1-\alpha) - (1+3\alpha)(1+\sigma_{\rm i}^2)}{4\sigma_{\rm i}^2(1+3\alpha)} \pm \left[1 + \frac{64\sigma_{\rm i}^2\alpha(1+\alpha)}{[6\sigma_{\rm i}(1-\alpha) - (1+3\alpha)(1+\sigma_{\rm i}^2)]^2}\right]^{1/2}. \tag{20}$$

In this situation the amplitude of dust acoustic solitary wave becomes very large (infinity). Also, monotonic shock waves may be positive or negative in $\chi \to -\infty$.

But figure 4 shows that the amplitude of monotonic shock waves will be damped to zero in $\chi \to +\infty$.

As it was mentioned, $A = 0$ for a critical density $\mu_{\rm c}$. Thus the amplitude of shock wave in (18) and (19) increases to infinity. In this case we use eq. (6) but with a new set of stretching coordinate as follows:

$$\xi = \varepsilon(x - \lambda t), \quad \tau = \varepsilon^{3/2} t, \quad \tau_{\rm m} = \varepsilon \tau_{\rm m0}, \quad \eta_1 = \varepsilon \eta_0. \tag{21}$$

By substituting (21) into (1) and gathering terms with same degrees of ε, at the lowest order of ε we obtain the same results as (7). But for the higher order of ε we have

$$n_{2\rm d} = \frac{3}{2\lambda^4}\phi_1^2 - \frac{1}{\lambda^2}\phi_2, \quad u_{2\rm d} = \frac{1}{2\lambda^3}\phi_1^3 - \frac{1}{\lambda^2}\phi_2. \tag{22}$$

At the next higher order of ε we find

$$\begin{aligned}
&\frac{\partial n_1}{\partial \tau} - \lambda \frac{\partial n_3}{\partial \xi} + \frac{\partial}{\partial \xi}(n_1 u_2 + n_2 u_1 + u_3) = 0,\\
&\frac{\partial u_1}{\partial \tau} - \lambda \frac{\partial u_3}{\partial \xi} + \frac{\partial}{\partial \xi}(u_1 u_2) - \frac{\partial \phi_3}{\partial \xi} = \eta_0 \frac{\partial^2 u_1}{\partial \xi^2},\\
&\frac{\partial^2 \phi_1}{\partial \xi^2} = n_3 + \frac{1}{\lambda^2}\phi_3 + \frac{\mu\sigma_{\rm i}^2 - 1}{2(1-\mu)}\phi_1\phi_2\\
&\qquad + \frac{\mu\sigma_{\rm i}^3 + 3\alpha\mu\sigma_{\rm i}^3 - 15\alpha - 1}{6(1-\mu)(1+3\alpha)}\phi_1^3.
\end{aligned} \tag{23}$$

And finally we derive this equation

$$\frac{\partial \phi_1}{\partial \tau} + D\phi_1^2 \frac{\partial \phi_1}{\partial \xi} + E\frac{\partial^3 \phi_1}{\partial \xi^3} + F\frac{\partial}{\partial \xi}(\phi_{14}\phi_2) + G\frac{\partial^2 \phi_1}{\partial \xi^2} = 0, \tag{24}$$

where

$$\begin{aligned}
D &= \frac{15}{4}\left[\frac{\mu\sigma_{\rm i} + 3\alpha\mu\sigma_{\rm i} - \alpha + 1}{(1-\mu)(1+3\alpha)}\right]^{-3/2} - \frac{1}{2}\left[\frac{\mu\sigma_{\rm i} + 3\alpha\mu\sigma_{\rm i} - \alpha + 1}{(1-\mu)(1+3\alpha)}\right]^{+3/2}\\
&\quad \times \left(\frac{\mu\sigma_{\rm i}^3 + 3\alpha\mu\sigma_{\rm i}^3 - 15\alpha - 1}{2(1-\mu)(1+3\alpha)}\right),\\
E &= \frac{1}{2}\left[\frac{\mu\sigma_{\rm i} + 3\alpha\mu\sigma_{\rm i} - \alpha + 1}{(1-\mu)(1+3\alpha)}\right]^{-3/2},\\
F &= -\frac{3}{2}\left[\frac{\mu\sigma_{\rm i} + 3\alpha\mu\sigma_{\rm i} - \alpha + 1}{(1-\mu)(1+3\alpha)}\right]^{+1/2} - \left[\frac{\mu\sigma_{\rm i} + 3\alpha\mu\sigma_{\rm i} - \alpha + 1}{(1-\mu)(1+3\alpha)}\right]^{-3/2}\\
&\quad \times \left(\frac{\mu\sigma_{\rm i}^2 - 1}{2(1-\mu)}\right),\\
G &= -\frac{\eta_0}{2}.
\end{aligned} \tag{25}$$

For $\mu = \mu_{\rm c}$ we have $A = F = 0$ and (25) reduces to a modified KdV–Burgers equation as follows:

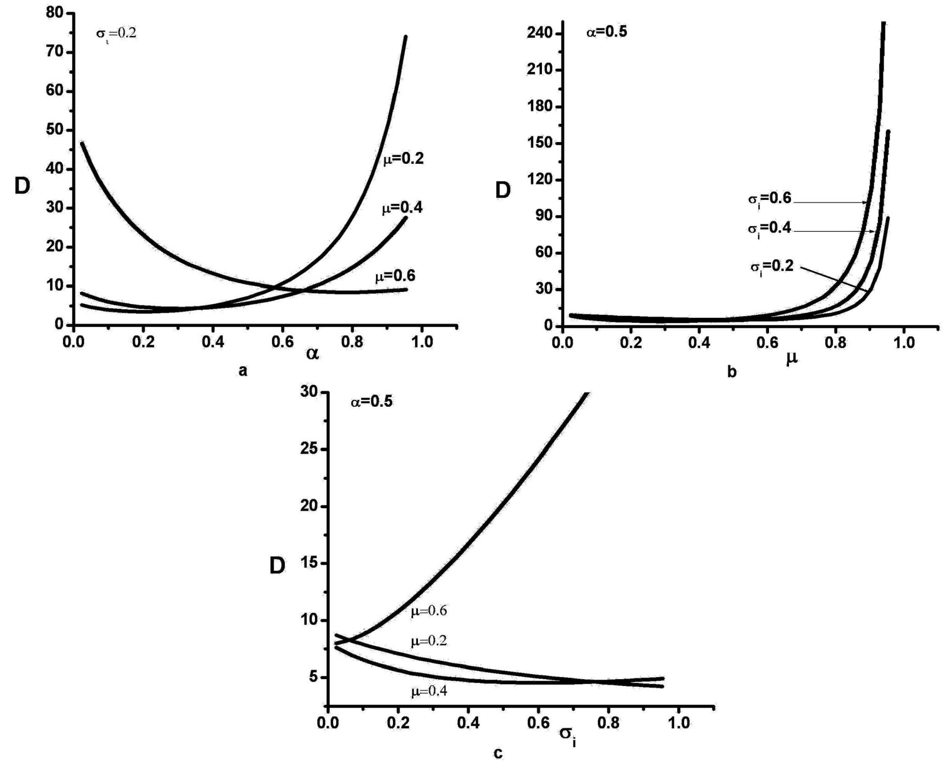

Figure 6. D as functions of plasma parameters.

$$\frac{\partial \phi_1}{\partial \tau} + D\phi_1^2 \frac{\partial \phi_1}{\partial \xi} + E\frac{\partial^3 \phi_1}{\partial \xi^3} + G\frac{\partial^2 \phi_1}{\partial \xi^2} = 0. \tag{26}$$

If the dissipation is neglected, then $G = 0$ and eq. (26) reduces to modified KdV equation as follows:

$$\frac{\partial \phi_1}{\partial \tau} + D\phi_1^2 \frac{\partial \phi_1}{\partial \varsigma} + E\frac{\partial^3 \phi_1}{\partial \xi^3} = 0. \tag{27}$$

The solitary solution for (27) can be written as [28,29]

$$\phi = \pm \phi_0 \,\mathrm{sech}\left(\frac{\xi - u_0 \tau}{w}\right) \tag{28}$$

In which $\phi_0 = \sqrt{6u_0/D}$ and $w = \sqrt{E/u_0}$ are the soliton amplitude and its width, respectively. Note that the solitary wave velocity u_0 is constant. Figure 6 shows parameter D with respect to different values of μ, σ_{i} and α.

We can see that D is positive for all the values of parameters. This means that for $\mu = \mu_{\mathrm{c}}$ only one kind of solitary waves can exist (compressive or rarefactive). Figures 7a–c show the solitary wave profile as functions of μ, σ_{i} and α.

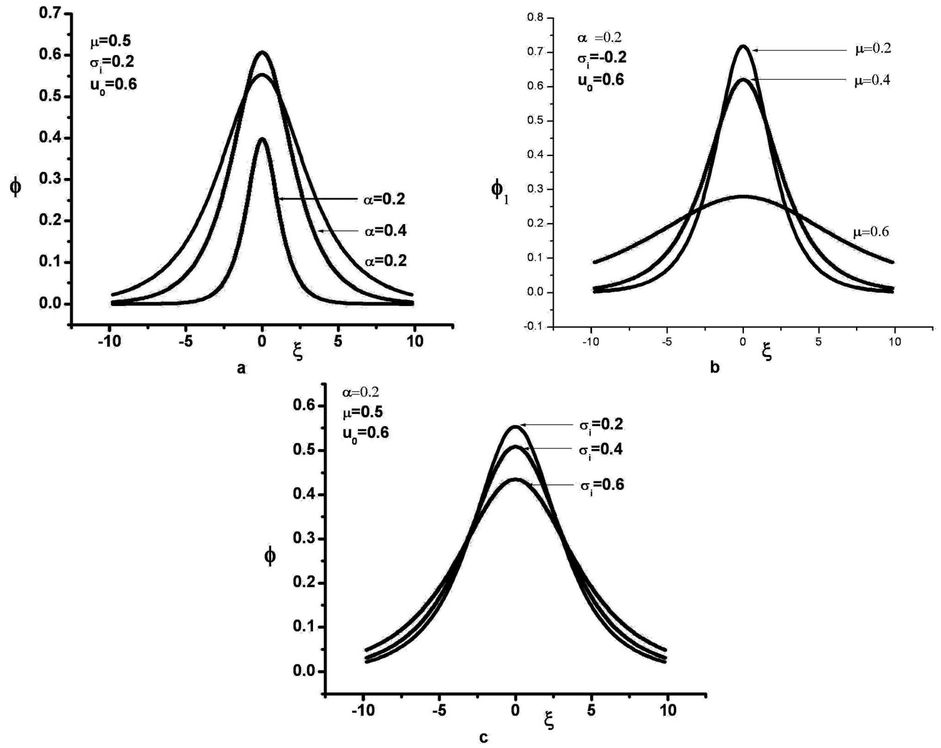

Figure 7. Solitary wave profiles of (28) for different values of parameters.

Equation (26) also has shock wave solutions as [23,24] as follows:
(i) Monotonic shock wave:

$$\phi_1 = \frac{2u}{D}\left[1 - \tanh\left(-\frac{u}{G}\chi\right)\right] \quad \text{for} \quad G^2 > 4Eu. \tag{29}$$

(ii) Oscillatory shock wave:

$$\phi_1 = \sqrt{\frac{3u}{D}} + \phi_0 \exp\left(\frac{G}{2E}\chi\right)\cos\left(\sqrt{\frac{2u}{E}}\chi\right) \quad \text{for} \quad G^2 \ll 4Eu, \tag{30}$$

where ϕ_0 is a constant.

7. Conclusion

In this paper, we studied the propagation of nonlinear waves in unmagnetized and strongly coupled collisionless dusty plasma containing nonthermal ions and Boltzmann distributed electrons. The physical mechanism for the formation of shock structure found in the hydrodynamical case is somewhat similar to that of

[6,30] where shock was collisional in strongly and weakly coupled dusty plasmas. We derived KdV–Burgers and mKdV–Burgers equations and discussed about solitonic solutions and also shock waves. The presence of the Burgers term presents any disturbance from developing into solitons and leads to the formation of a shock wave, whereas the nonthermal ion modifies the nonlinear term. We have shown that if the dissipation is negligible, the solitary waves will appear in the medium if the dispersive and the nonlinear terms are balanced. On the other hand, when dissipative term is noticeable and dissipative, nonlinear and dispersive terms are balanced, we will have shock waves (both monotonic and oscillatory types). The transition from an oscillatory to monotonic wave depends on the magnitude of the dissipation coefficient C [7,12]. With the stronger dissipation, the shock wave structure becomes steeper (monotonic-type) and for weaker dissipation the shock wave has an oscillatory behaviour. Dependency of monotonic and oscillatory shock waves on the medium parameters has been represented by the numerical simulation. Neither solitonic solution nor shock waves can be established when the nonlinearity parameter, A, becomes zero. But in this case we derived mKdV–Burgers equation. The mKdV–Burgers equation has stable solitonic as well as shock wave solutions.

Acknowledgement

The authors would like to thank the referee for constructive suggestions.

References

[1] W S Duan, *Phys. Plasmas* **8**, 3583 (2001)
[2] F Melandso and P K Shukla, *Planet Space Sci.* **43**, 635 (1995)
[3] N N Rao, P K Shukla and M Y Yu, *Planet Space Sci.* **38**, 543 (1990)
[4] P K Shukla and A A Mamun, *IEEE Trans. Plasma Sci.* **29**, 221 (2001)
[5] A A Mamun, B Eliasson and P K Shukla, *Phys. Lett.* **A332**, 412 (2004)
[6] S Ghosh and M R Gupta, *Phys. Plasmas* **12**, 092306 (2005)
[7] S Ghosh, *Phys. Plasma* **13**, 022301 (2006)
[8] Y Nakamura, *Phys. Plasmas* **9**, 440 (2002)
[9] B S Xie *et al*, *Phys. Lett.* **A247**, 403 (1998)
[10] F Verheest, *Space Sci. Rev.* **77**, 267 (1996)
[11] N N Rao, P K Shukla and M Y Yu, *Planet Space Sci.* **32**, 419 (1994)
[12] S Gosh, S Sarkar, M Khan and M R Gupta, *Phys. Plasmas* **9**, 1150 (2002)
[13] R A Carins, A A Mammun, R Bingham, Ro Dendy, R Bostrom and P K Shukla, *Geophys. Res. Lett.* **22**, 2709 (1995)
[14] M M Lin and W S Duan, *Chaos Soliton. Fract.* **33**, 1189 (2007)
[15] A A Mamun *et al*, *Planet Space Sci.* **48**, 163 (2000)
[16] P K Shukla and A A Mamun, *New J. Phys.* **5**, 17 (2003)
[17] P K Kaw and A Sen, *Phys. Plasmas* **5**, 3552 (1998)
[18] P K Kaw, *Phys. Plasmas* **8**, 1878.10 (2001)
[19] S Ichimaru and S Tanaka, *Phys. Rev. Lett.* **56**, 2815 (1986)
[20] S Ichimaru, H Iyetomi and S Tanaka, *Phys. Rep.* **149**, 91 (1987)
[21] W L Slattery, G D Doolen and H E Dewitt, *Phys. Rev.* **A21**, 2087 (1980)
[22] V I Karpman, *Nonlinear waves in dispersive medium* (Pergamon, New York, 1975)

[23] J-K Xue and L-P Zhang, *Chaos, Solitons and Fractals* **32**, 592 (2007)
[24] W M Moslem, *Chaos, Solitons and Fractals* **28**, 994 (2006)
[25] A A Mamun and P K Shukla, *IEEE Trans. Plasma Sci.* **30**, 720 (2002); **33**, 1189 (2002)
[26] M M Lin and W S Duan, *Chaos, Solitons and Fractals* **23**, 929 (2005)
[27] N Yasunori, *Phys. Lett.* **A125**, 320 (1987)
[28] M Wadati, *J. Phys. Soc. Jpn* **38**, 673 (1975)
[29] M Wadati, *J. Phys. Soc. Jpn* **34**, 1289 (1973)
[30] P K Shukla, *Phys. Plasmas* **7**, 1044 (2000)